95.141 Exam 1 Spring 2014

Section number ________________

Section instructor ________________

Last/First name ________________

Last 3 Digits of Student ID Number: __ __ __

Answer all questions, beginning each new question in the space provided. Show all work. Show all formulas used for each problem prior to substitution of numbers. Label diagrams and include appropriate units for your answers. Write your name and section number at the top of each page in the space provided and write the name of your section instructor in the place provided in the cover sheet. You may use an alphanumeric calculator (one which exhibits physical formulas) during the exam as long as you do not program any formulas into memory. By using an alphanumeric calculator you agree to allow us to check its memory during the exam. Simple scientific calculators are always OK!

A Formula Sheet Is Attached To The Back Of This Examination

For your convenience you may carefully remove it from the Exam. Please take it with you at the end of the exam or throw it in a waste basket.

Be Prepared to Show your Student ID Card

Score on each problem:

1. (25) __

2. (25) __

3. (25) __

4. (25) __

Total Score (out of 100 pts) __
1. (25 point) Put a circle around the letter that you think is the best answer.

1.1. (5pts) An object is moving with constant velocity in a straight line. Which of the following statements is true?

A) A constant force is being applied in the direction of motion
B) A constant force is being applied in the direction opposite of motion
C) There are no forces acting on the object
D) The net force on the object is zero
E) There is no frictional force acting on the object

1.2. (5pts) What is the sum of $4.3 \cdot 10^3 + 7.1 \cdot 10^4 + 0.005 \cdot 10^6$?

A) $80.3 \cdot 10^3$
B) $8 \cdot 10^4$
C) $8.0 \cdot 10^4$
D) $80.3 \cdot 10^4$
E) None of the above.

1.3. (5pts) Multiply $3.079 \cdot 10^2$ by $0.044 \cdot 10^{-1}$, taking into account significant figures.

A) 1.3.
B) 1.35.
C) 1.4.
D) 1.354.
E) None of the above.
1.4. (5pts) You ride on an elevator that is moving with constant upward acceleration while standing on a bathroom scale. The reading on the scale is

A) equal to your true weight, \( mg \)

\( \boxed{B) \text{ more than your true weight, } mg} \)

C) less than your true weight, \( mg \)

D) could be more or less than your true weight, \( mg \), depending on the magnitude of the acceleration

1.5. (5pts) Refer to the figure below. If you start from the Bakery, travel to the Cafe, and then to the Art Gallery, what is the magnitude of your displacement

![Diagram](image)

A) 6.5 km

\( \boxed{B) \text{ 2.5 km}} \)

C) 10.5 km

D) 9.0 km

E) 1.5 km
2. (25 pts) The position of a particle as a function of time is given by
\[ \vec{r}(t) = \left( -3.0 \, \text{m/s} \right) t \, \hat{i} + (6.0 \, \text{m}) \hat{j} + \left[ 7.0 \, \text{m} - \left( 4.0 \, \text{m/s}^3 \right) t^2 \right] \hat{k} \]
where \( r \) is in meters and \( t \) is in seconds.

a) (5 pts) What is the particle's displacement between \( t_1 = 0 \) and \( t_2 = 2.0 \, \text{s} \)?
b) (5 pts) Determine the particle's instantaneous velocity as a function of time.
c) (5 pts) What is the particle's average velocity between \( t_1 = 0 \, \text{s} \) and \( t_2 = 2.0 \, \text{s} \)?
d) (5 pts) Is there a time when the particle has a velocity of zero?
e) (5 pts) Determine the particle's instantaneous acceleration as a function of time.

\[ \vec{r}(t=2.0 \, \text{s}) = -3.0 \, 2.0 \, \hat{i} + 6.0 \, \hat{j} + \left( 4.0 - 4.8.0 \right) \hat{k} \]
\[ = -6.0 \, \hat{i} + 6.0 \, \hat{j} + \left( -2 \right) \hat{k} \]

\[ \vec{r}(t=0.0 \, \text{s}) = -3.0 \, 0 \, \hat{i} + 6.0 \, \hat{j} + 7.0 \, \hat{k} \]
\[ = 0 \, \hat{i} + 6.0 \, \hat{j} + 7.0 \, \hat{k} \]

\[ \Delta \vec{r} = \vec{r}(2.0 \, \text{s}) - \vec{r}(0 \, \text{s}) = -6.0 \, \hat{i} + \left( 6.0 - 6.0 \right) \, \hat{j} + \left[ -250 - 70 \right] \hat{k} \]
\[ = -6.0 \, \hat{i} + 0 \, \hat{j} - 320 \, \hat{k} \]

b) \( \vec{v}(t) = \frac{d\vec{r}}{dt} = -3.0 \, \hat{i} + 6.0 \, \hat{j} - 4.0 \, 3 \, t^2 \, \hat{k} = -3.0 \, \hat{i} + 6.0 \, \hat{j} - 12.0 \, t^2 \, \hat{k} \)

c) \( \vec{v}(0 \to 2.0 \, \text{s}) = \frac{\vec{r}(2.0 \, \text{s}) - \vec{r}(0 \, \text{s})}{\Delta t} = \frac{\Delta \vec{r}}{\Delta t} = \frac{-6.0 \, \hat{i} + 0 \, \hat{j} - 320 \, \hat{k}}{2.0 - 0} \]
\[ = -3.0 \, \hat{i} + 0 \, \hat{j} - 16.0 \, \hat{k} \]

d) \( |\vec{v}| = \sqrt{(-3.0)^2 + (-12.0 \, t^2)^2} = \sqrt{9 + 144. \, t^4} \]
\[ 9 + 144. \, t^4 = 0 \Rightarrow t^4 = -\frac{9}{144} \]

It's impossible. So there's no time where \( |\vec{v}| = 0 \).
e) \( \vec{a}(t) = \frac{d\vec{v}}{dt} = 0 \, \hat{i} + 0 \, \hat{j} - 24.0 \, \hat{k} \)

It's not constant.
3. (25 pts) A child throws a ball with an initial speed of 8.00 m/s at an angle of 40.0° above the horizontal. The ball leaves her hand 1.00 m above the ground.

A) (8 pts) How long is the ball in flight before it hits the ground?
B) (6 pts) How far from where the child is standing does the ball hit the ground?
C) (7 pts) Find the x and y components of velocity just before the ball strikes the ground.
   What is the magnitude of the ball's velocity just before it hits the ground?
D) (5 pts) At what angle below the horizontal does the ball approach the ground?

\[ Y = Y_0 + V_{0y}t - \frac{1}{2}gt^2 \]

\[ V_{0y} = V_0 \sin \theta = 8.00 \text{ m/s} \cdot \sin 40° = 5.142 \text{ m/s} \]

\[ Y = -h \quad Y_0 = 0; \]

\[ -h = V_{0y}t - \frac{1}{2}gt^2 \]

\[ 4.9t^2 - 5.142t + 1.00 = 0 \quad \Rightarrow \quad t_1 = -0.1676 \text{ s} \approx -0.17 \text{ s} \]

\[ t_2 = 1.217 \text{ s} \approx 1.22 \text{ s} \]

\[ x = V_{0x} \cdot t_2 = 6.1283 \text{ m/s} \cdot 1.217 \text{ s} = 7.458 \text{ m} \approx 7.46 \text{ m} \]

\[ V_{0x} = V_0 \cdot \cos \theta = 8.00 \text{ m/s} \cdot \cos 40° = 6.1283 \text{ m/s} \approx 6.13 \text{ m/s} \]

\[ V_x = V_{0x} = 6.13 \text{ m/s} \]

\[ V_y = V_{0y} - gt_2 = 5.142 \text{ m/s} - 9.8 \text{ m/s}^2 \cdot 1.217 \text{ s} = -6.7846 \text{ m/s} \approx -6.8 \text{ m/s} \]

\[ V = \sqrt{V_x^2 + V_y^2} = \sqrt{(6.1283 \text{ m/s})^2 + (-6.7846 \text{ m/s})^2} = 9.14 \text{ m/s} \]

\[ \theta_f = \arctan \frac{V_y}{V_x} = \frac{-6.7846}{6.128} \quad \Rightarrow \quad \theta_f = 47.9° \approx 48° \]
4. (25 pts) The Atwood's machine with masses $m_1=0.6\text{ kg}$ and $m_2=0.4\text{ kg}$ is shown in the drawing. Ignore friction and the mass of the pulley. The system is released from rest with mass $m_1$ 1 m above the floor.

A) (5 pts) Draw free-body diagrams for mass $m_1$ and mass $m_2$.

B) (8 pts) Calculate the magnitude of the acceleration of the system

C) (7 pts) Calculate the tension in the string

D) (5 pts) Find the time required for mass $m_1$ to reach the floor

\[ (1) \quad m_1g - F_T = m_1a \]

\[ (2) \quad F_T - m_2g = m_2a \]

\[(1) + (2) \Rightarrow m_1g - m_2g = (m_1 + m_2)a \]

\[ a = \frac{m_1 - m_2}{m_1 + m_2} g = \frac{0.6\text{ kg} - 0.4\text{ kg}}{0.6\text{ kg} + 0.4\text{ kg}} \cdot 9.8\text{ m/s}^2 = 1.96\text{ m/s}^2 \approx 2.0\text{ m/s}^2 \]

\[ (2) \Rightarrow F_T = m_2a + m_2g = (m_2 + m_2)g = (1.96\text{ m/s}^2 + 0.8\text{ m/s}^2) \cdot 0.4\text{ m/s}^2 = 4.7\text{ N} \]

\[ \text{d') } y = y_0 + v_0t + \frac{a}{2}t^2 \text{ (for } m_1) \]

\[ y = h; \ y_0 = 0; \ v_0 = 0 \]

\[ h = \frac{a}{2}t^2 \Rightarrow t = \sqrt{\frac{2h}{a}} = \sqrt{\frac{2 \cdot 1.0\text{ m}}{1.96\text{ m/s}^2}} = 1.01\text{ s} \approx 1.0\text{ s} \]