*Last/First name* A. Danylov

| | |
|---|---|
| 1. (30pt) | ______ |
| 2. (20pt) | ______ |
| 3. (20pt) | ______ |
| 4. (20pt) | ______ |

# *1. Conceptual Questions*

(30 point) Put a circle around the letter that you think is the best answer.

**1.1.** (6pts) An electron is released from rest at x = 2 m in the potential shown. What does the electron do right after being released?

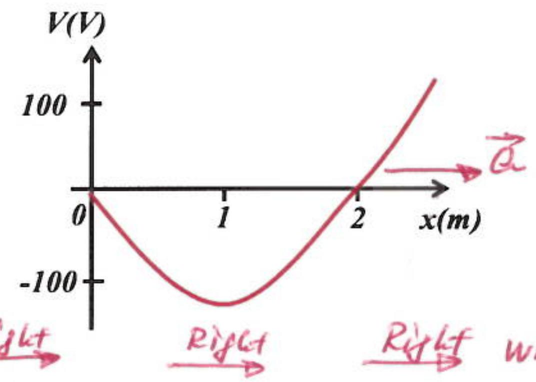
A) Stay at x = 2 m

B) Move to the right (+ x) at steady speed

C) Move to the right with increasing speed

D) Move to the left (– x) at steady speed

E) Move to the left with increasing speed




$V \xrightarrow{E_x = -\frac{dV}{dx}} E \xrightarrow[F_e = qE]{q<0} F_e \xrightarrow{F=ma} a \rightarrow$ direction

**1.2.**(6pts) This is a graph of the x-component of the electric field along the x-axis. The potential is zero at the origin. What is the potential at x=2m?

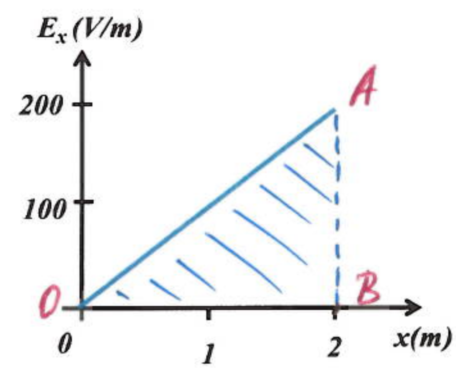
A) 200 V

B) 100 V

C) 0 V

D) -100 V

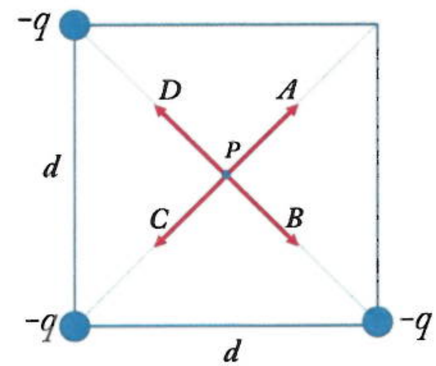
E) -200 V




$\Delta V = V_f - V_i = -\int_i^f E_x dx$ $\quad i \rightarrow x=0$, $f \rightarrow x = 2m$

$V(x=2m) = -Area(OAB) = -\frac{1}{2}\cdot 2m \cdot 200\,V/m = -200\,V$

**1.3.** (6pts) Three equal negative point charges are placed at three of the corners of a square of side d as shown in the figure. Which of the arrows represents the direction of the net electric field at the center of the square?




A) A

B) B

C) C

D) D

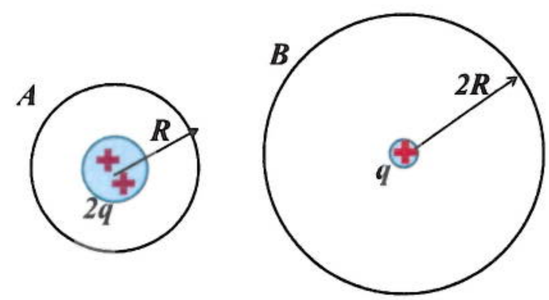
**1.4.** (6pts) Which spherical Gaussian surface has the larger electric flux?

A) Surface A

B) Surface B

C) They have the same flux

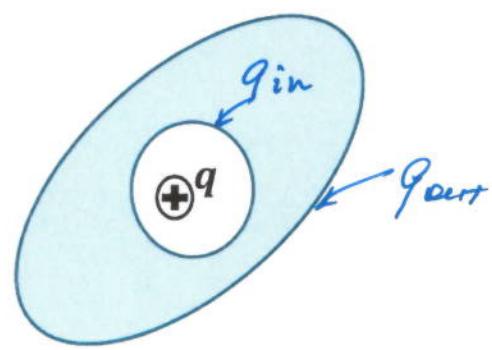
D) Not enough information to tell




Flux $\Phi_E = \oint \vec{E} \cdot d\vec{A} = Q_{in}/\varepsilon_0$

$Q_{in}^{(A)} = 2q$ ; $Q_{in}^{(B)} = q \Rightarrow \Phi_A > \Phi_B$

**1.5.** (6pts) A +20 nC point charge is inside a hole in a conductor.
The conductor has a net charge of +50 nC.




a) What is the total charge on the inside surface of the conductor?

Answer: $q_{in} = -20\,nC$

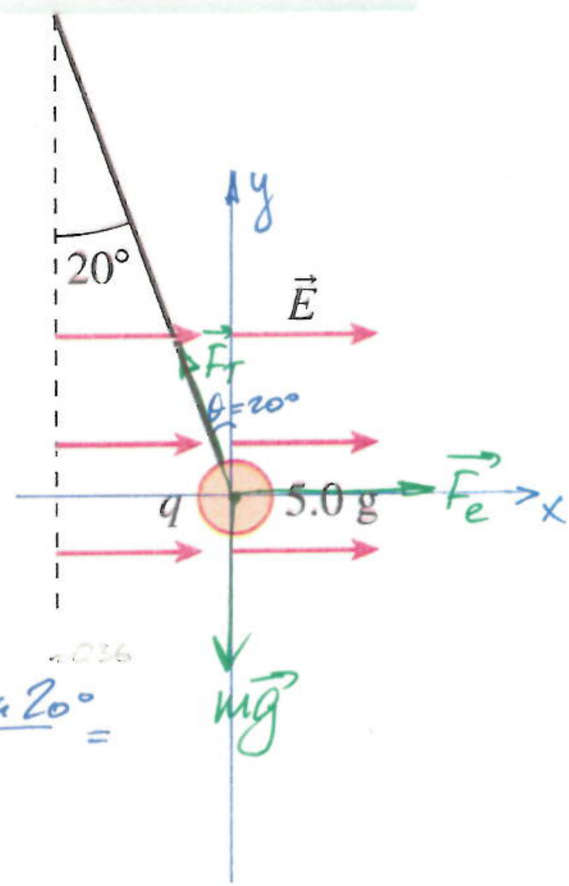
b) What is the total charge on the outside surface of the conductor?
Answer:

$Q_{total} = q_{in} + q_{out}$ ; $Q_{tot} = +50\,nC$

$q_{out} = Q_{total} - q_{in} = +50\,nC - (-20\,nC) = 70\,nC$

***Problem 2. (20 pts)***

***An electric field $\vec{E} = 100,000\ \hat{\imath}\ N/C$ causes the 5.0 g point charge to hang at a 20° angle. What is the charge on the ball?***




Static equilibrium $\vec{a} = 0$

N. 2nd law: $\sum_i \vec{F} = m\vec{a}$, so

$$\sum_i \vec{F} = 0 \Rightarrow \begin{cases} \sum_i F_y = 0 \\ \sum_i F_x = 0 \end{cases}$$

$$\begin{cases} F_T \cdot \cos\theta - mg = 0 \rightarrow F_T = mg/\cos\theta \\ F_e - F_T \cdot \sin\theta = 0 \leftarrow F_e = qE \end{cases}$$

$$qE = \left(\frac{mg}{\cos\theta}\right)\sin\theta$$

$$q = \frac{mg}{E} \cdot \tan\theta = \frac{(5 \cdot 10^{-3} kg) \cdot (9.8\ m/s^2) \cdot \tan 20°}{100,000\ N/C} =$$

$$q = 1.783 \cdot 10^{-7} C$$

***Problem 3. (20 pts)***

***A very long solid nonconducting cylinder of radius $R_0$ and length L ($R_0$<<L) possesses a uniform volume charge density ρ (C/m³). Determine the electric field as a function of r for***

***a)*** outside the cylinder (***$r > R_0$***)

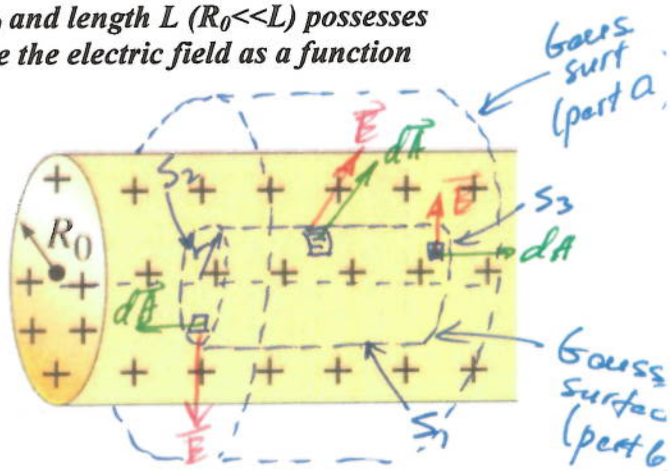
*Draw a Gaussian surface in the figure;*

*Draw electric field and area vectors;*

***b)*** inside the cylinder (***$r < R_0$***)

*Draw a Gaussian surface in the figure;*

(in one of the cases show how you handle a linear integral)




b) Gauss's law $\oint \vec E \cdot d\vec A = \frac{Q_{in}}{\varepsilon_0}$

$$\oint \vec E \cdot d\vec A = \int_{S_1} \vec E \cdot d\vec A + \int_{S_2} \vec E \cdot d\vec A^{\,0} + \int_{S_3} \vec E \cdot d\vec A^{\,0} = \int_{S_1} E\,dA = \left\| \begin{matrix} E = \text{const} \\ \text{on the Gauss surface} \end{matrix} \right\| = E \int_{S_1} dA =$$

$\vec E \uparrow\uparrow d\vec A$ $\quad \vec E \perp d\vec A$ $\quad \vec E \perp d\vec A$

$$= E \cdot 2\pi r \cdot L = Q_{in}/\varepsilon_0 \; ; \quad Q_{in} = \rho \cdot V_r = \rho \cdot \pi r^2 \cdot L$$

$$E \cdot 2\pi r \cdot L = \frac{\rho \cdot \pi r^2 \cdot L}{\varepsilon_0} \Rightarrow \left\| E(r) = \frac{\rho \cdot r}{2 \cdot \varepsilon_0}, \; r \le R_0 \right\|$$

a) similar, the el. flux is

$$E \cdot 2\pi r \cdot L = \frac{Q_{in}}{\varepsilon_0} \; ; \quad Q_{in} = \rho \cdot V_R = \rho \cdot \pi R^2 \cdot L$$

$$E \, 2\pi r \cdot L = \rho \frac{\cdot \pi R^2 \cdot L}{\varepsilon_0}$$

$$\left\| E(r) = \frac{\rho \cdot R^2}{2 \cdot \varepsilon_0 \cdot r}, \; r \ge R_0 \right\|$$

***Problem 4. (20 pts)***
***A proton's speed as it passes point A is 200,000 m/s. It follows the trajectory shown in the figure with a solid line. The dashed lines in the figure are equipotential lines. What is the proton's speed at point B?***

Conserv. of energy:

$$E_a = E_b$$

$$K_a + U_a = K_b + U_b$$

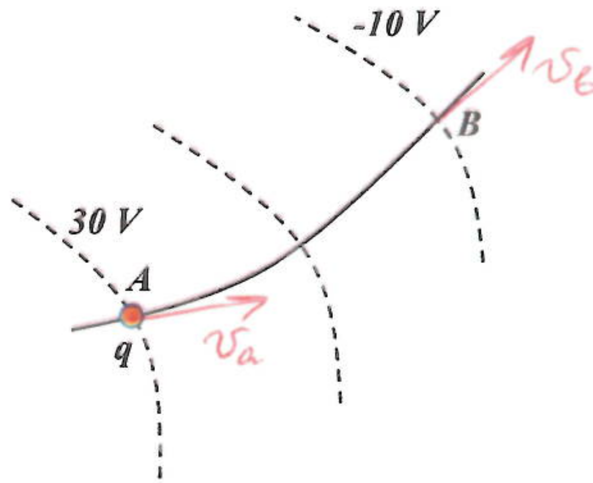
$$U = q \cdot V$$

$$K_a + qV_a = K_b + qV_b$$

$$K_b = K_a + q(V_a - V_b)$$

$$\frac{1}{2} m v_b^2 = \frac{1}{2} m v_a^2 + q(V_a - V_b)$$

$$v_b = \sqrt{v_a^2 + \frac{2q}{m}(V_a - V_b)} = \sqrt{(200{,}000\ m/s)^2 + \frac{2(1.6 \cdot 10^{-19}\,C)}{1.67 \cdot 10^{-27}\,kg} \cdot (30V - (-10V))}$$




$$v_b = 218{,}300\ m/s$$