Answer all questions, beginning each new question in the space provided. Show all work. Show all formulas used for each problem prior to substitution of numbers. Label diagrams and include appropriate units for your answers. Write your name and section number at the top of each page in the space provided and write the name of your section instructor in the place provided in the cover sheet. You may use an alphanumeric calculator (one which exhibits physical formulas) during the exam as long as you do not program any formulas into memory. By using an alphanumeric calculator you agree to allow us to check its memory during the exam. Simple scientific calculators are always OK!

Score on each problem:

1. (10) ___

2. (20) ___

3. (20) ___

Total Score (out of 50 pts) ___

Total Score (scaled up to 100 pts) ___

Be Prepared to Show your Student ID Card

A Formula Sheet Is Attached To The Back Of This Examination

For your convenience you may carefully remove it from the Exam. Please take it with you at the end of the exam or throw it in a waste basket. Thank you!
Problem 1: (10 points - 2 points each - no partial credit on this problem only – in each question, put a circle around the letter that you think is the best answer.)

1-1. A spring attached to a wall at one end, when stretched by a weight $W$ through a string and pulley, as shown in situation (a), stretches by a distance $x$ from its equilibrium length. How much does the same spring stretch from its equilibrium length in situation (b)?

(a) $x$  
(b) $2x$  
(c) $0$  
(d) $x^2$  
(e) $\sqrt{2}x$

1-2. A block of mass $m$ is pulled along a rough horizontal floor by an applied force $T$ as shown. The vertical component of the force exerted on the block by the floor is:

(a) $mg$  
(b) $mg - T\cos \theta$  
(c) $mg + T\cos \theta$  
(d) $mg - T\sin \theta$  
(e) $mg + T\sin \theta$

1-3. A box sliding on a frictionless flat surface runs into a fixed spring, which compresses a distance $x$ to stop the box. If the initial speed of the box were doubled, how much would the spring compress in this case?

(a) $x$  
(b) $2x$  
(c) $4x$  
(d) $x/2$  
(e) $\sqrt{2}x$

$\text{KE is converted into pot. energy of the spring}$

$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$  

$\frac{1}{2}m(2v)^2 = \frac{1}{2}kx'^2$

$\text{Take a ratio}$

$\frac{(2)}{(1)} \Rightarrow \frac{\frac{1}{2}m(4v^2)}{\frac{1}{2}mv^2} = \frac{1}{2}kx'^2 \Rightarrow \frac{1}{2} \Rightarrow \frac{x'^2}{x^2} \Rightarrow x' = 2x$
1-4. Two boxes of **different masses** start sliding across a rough floor with the **same speed**. Which box will be stopped first by kinetic friction?

(a) the one with the smaller mass  
(b) the one with the larger mass  
**c** both boxes will stop at the same time  
(d) more information is needed to answer this question  
(e) the answer depends on the coefficient of kinetic friction

1-5.  
A planet travels in an elliptical orbit about a star X as shown. The magnitude of the acceleration of the planet is:

(a) greatest at point Q  
(b) greatest at point S  
(c) greatest at point U  
**d** greatest at point W  
(e) the same at all points

\[ \frac{G M}{r^2} \]

\[ \vec{a} = \frac{G M}{r^2} \hat{r} \quad \Rightarrow \quad a = |\vec{a}| = \frac{G M}{r^2} \]

\( a \) is max at the point where \( r \) is min, i.e. at point W.
Problem 2: (20 points) [NO credit for correct answers without appropriate formulae/logic]

A motorcyclist rides with \textit{constant speed} on the inside rim of a vertical circular track of radius \( R = 7.25 \text{ m} \) at the \textit{minimum} speed necessary such that the motorcycle does not lose contact at the top of the loop. The total mass of the motorcycle and rider is 345 kg.

(a) [4 points] Draw separate free-body diagrams at (i) the top, and (ii) the bottom of the circle. [Draw only the \textit{real} forces acting on the free body].

Only \( F_N \) and \( w_p \) are \textit{real} forces. They produce the centripetal force \( \frac{m v^2}{R} = F_N \), which is \textit{not real}.

(b) [4 points] Use Newton’s Laws to set up equations of motion and calculate the \textit{speed} of the motorcyclist from the centripetal acceleration at the \textit{top} of the circle.

\[
N. \text{2nd Law: } m \ddot{a} = \Sigma F
\]

Choose the positive \( y \) direction to be down.

There is only one acceleration in the problem, centripetal, i.e. \( \ddot{a} = \frac{v^2}{R} \)

But \( \ddot{a} \) is from \( N. \text{2nd Law} \)

\[
m \frac{v^2}{R} = \Sigma F (\text{\textit{real}}) = w_p + F_N
\]

at the \textit{min} speed \( \boxed{F_N = 0} \) (the guy is \textit{ready} to start falling down)

so \( m \frac{v^2}{R} = w_p \), so \( v = \sqrt{\frac{g R}{2}} = \sqrt{\frac{9.8 \text{ m/s}^2 \cdot 7.25 \text{ m}}{2}} = 8.4 \text{ m/s} \)

(c) [4 points] Find the contact force from the track on the motorcycle at the \textit{bottom} of the circle (magnitude and direction)

\[
N. \text{2nd Law: } m \ddot{a} = \Sigma F \text{; the positive } y \text{ dir. is upward.}
\]

\[
m \frac{v^2}{R} = F_N - w_p
\]

\[
F_N = m \frac{v^2}{R} + w_p = \sqrt{\frac{w_m}{R}} \text{ we found that } v = \sqrt{\frac{g R}{2}} + w_p = m \left( g + g \right) = 9.8 m
\]

\[
F_N = 2 \cdot 345 \text{ kg} \cdot (9.80 \text{ m/s}) = 6762 \text{ N}
\]
Problem 2: (continued)

If the speed of the motorcycle is doubled, find the magnitude and direction of the new contact forces \( F_N \).

(d) [4 points] at the top of the circle.
\[
\frac{m v^2}{R} = m g + F_N \quad \Rightarrow \quad F_N = m \left( \frac{v^2}{R} - g \right) = m \left( \frac{4gR}{R} - g \right) = 3mg
\]

(e) [4 points] at the bottom of the circle.
\[
\frac{m v^2}{R} = F_N - m g \quad \Rightarrow \quad F_N = m \left( \frac{v^2}{R} + g \right) = m \left( \frac{4gR}{R} + g \right) = 5mg
\]
Problem 3: (20 points) [NO credit for correct answers without appropriate formulae/logic]

A block of mass \( M = 2.0 \text{ kg} \) is released from rest at the top of a ramp of height \( h = 3.0 \text{ m} \). The top half of the ramp is frictionless. The bottom half has a coefficient of kinetic friction \( \mu_k = 0.4 \) between block and ramp. For the bottom half of the ramp, the block moves at constant velocity.

(a) [4 points] Draw a free-body diagram of all the forces on the block as it slides down the bottom half of the ramp from B to C.

(b) [4 points] Use the information about the motion in the bottom half of the ramp (and Newton's Laws) to calculate the slope angle \( \theta \) of the ramp.

\[
\begin{align*}
\sum F &= \Sigma F^v \\
F_N - \mu_k N \cos \theta &= 0 \\
F - \mu_k N \sin \theta &= 0
\end{align*}
\]

\[
\begin{align*}
F &= \mu_k N \\
N &= \frac{mg}{\cos \theta}
\end{align*}
\]

\[
\theta = \tan^{-1}(\mu_k) \approx 21.8^\circ
\]

(c) [4 points] Use conservation of energy to find the speed of the block at the middle of the ramp (point B)?

\[
E_1 = E_2 \Rightarrow m_1 + k_1 = m_2 + k_2
\]

\[
\begin{align*}
\frac{1}{2}m_1 v_1^2 + 0 &= \frac{1}{2}m_2 v_2^2 + mgh \\
\frac{1}{2}m_2 v_2^2 &= \frac{1}{2}m_1 v_1^2 + mgh \\
\frac{1}{2}m_2 v_2^2 &= \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_1 v_1^2 + mgh \\
\frac{1}{2}m_2 v_2^2 &= \frac{1}{2}m_1 v_1^2 + mgh \\
\frac{1}{2}m_2 v_2^2 &= \frac{1}{2}m_1 v_1^2 + mgh \\
V_2 &= \sqrt{2gh - gh} = \sqrt{gh}
\end{align*}
\]
Problem 3: (continued)

(d) [4 points] What is the net work done by friction on the block as it slides from the point B to point C.

\[ W = \int \vec{F}_{fr} \cdot d\vec{s} = \int \left[ \frac{F_{fr}}{\cos \theta} \right] d\ell \]

\[ \text{where } \theta = \text{angle between } \vec{F}_{fr} \text{ and } \vec{d} \text{ such that } \sin \theta = 0.9777 \]

\[ l = \text{length of the ramp (total)} \]

\[ l = \frac{h}{\sin \theta} \]

\[ W = -mg \cdot \cos \theta \cdot \left( \frac{h}{2 \cdot \sin \theta} \right) = -\frac{mg}{2 \cdot \tan \theta} = -\frac{mg}{2} \]

**It does not depend on \( \mu_c \)!!**

\[ W = 29.4 \text{ J} \]

(e) [4 points] What is the power delivered by the force of gravity as the block slides from point B to point C?

\[ P = \frac{W}{t} \]

\[ W \text{- work done by the gravitational force can be found in two different ways:} \]

1. Since acceleration = 0 on the rough surface, it means that \( W_{net} = 0 \) (see my notes, problem 11, at the end).

\[ W_{net} = 0 = W_{fr} + W_{gr} = -\frac{\mu_k m g}{2} + \frac{m g h}{2} \]

2. \( W_{gr} = \vec{F}_{gr} \cdot \vec{d} = m g \cdot \frac{1}{2} \cdot \cos \left( \frac{\pi}{2} - \theta \right) = m g \cdot \frac{1}{2} \cdot \sin \theta = \frac{h}{2 \cdot \sin \theta} = m g \cdot \sin \theta \cdot \left( \frac{h}{2 \cdot \sin \theta} \right) \]

\[ \text{displacement} \]

\[ = \frac{m g h}{2} = W_{fr} \text{ same.} \]

Now we need \( t \)? Easy, since \( v_2 = \text{constant} = \sqrt{\frac{k}{h}} \) - found in part (c) \( t = \frac{h}{v_2} = \frac{h}{\sqrt{\frac{k}{h}}} = \frac{h}{\sqrt{\frac{k}{2 \cdot Vgh}}} \]

\[ t = \frac{1}{2} \cdot \sqrt{\frac{h}{Vgh}} \]

**Finally:** \( P = \frac{W_{fr}}{t} = \left( \frac{m g h}{2} \right) \cdot \frac{1}{\sqrt{\frac{h}{2 \cdot Vgh}}} = m g \cdot Vgh^3 \cdot \sin \theta = m g \cdot \frac{h}{2} \cdot \frac{1}{2 \cdot \sin \theta} \] [Watts]