Problem 1: (10 points - 2 points each - no partial credit on this problem only – in each question, put a circle around the letter that you think is the best answer.)

1-1. A 1.0-kg ball moving at 2.0 m/s perpendicular to a wall rebounds from the wall at 1.5 m/s. The change in the momentum of the ball is:

(a) zero
(b) 0.5 N.s away from wall
(c) 0.5 N.s toward wall
(d) 3.5 N.s away from wall
(e) 3.5 N.s toward wall

\[ \Delta \vec{p} = \vec{p}_f - (\vec{p}_i) = p_{fx} + p_{iy} = (p_{fx} - p_{ix}) \hat{x} \]

\[ = (1.5 \cdot 1.5 \cdot \hat{x}) + (2.0 \cdot 1.0 \cdot \hat{y}) = (3.5 \text{N.s}) \hat{x} \]

1-2. Cart A, with a mass of 0.2 kg, travels on a horizontal air-track at 3 m/s and hits cart B, which has a mass of 0.4 kg and is initially at rest. After the collision, the center of mass of the two-cart system has a speed, in m/s, of:

(a) 0
(b) 1
(c) 2
(d) 3
(e) need to know if collision elastic or inelastic

\[ \vec{v}_{cm} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2} \]

\[ = \frac{0.2 \cdot 3 \cdot \hat{x} + 0.4 \cdot 0 \cdot \hat{x}}{0.2 + 0.4} = \frac{1}{2} \hat{x} \]

1-3. A rod rests on frictionless ice (viewed from above in the diagram). Horizontal forces, equal in magnitude and opposite in direction are simultaneously applied to its ends as shown. The quantity that vanishes is its

(a) net torque about its center of mass
(b) angular acceleration
(c) total linear momentum
(d) total kinetic energy
(e) rotational inertia about its center of mass

1-4. Three identical objects of mass \( M \) are fastened to a massless rod of length \( L \) as shown. The moment of inertia of this array about one end of the rod is:

(a) \( \frac{ML^2}{2} \)
(b) \( ML^2 \)
(c) \( 3ML^2 / 2 \)
(d) \( 5ML^2 / 4 \)
(e) \( 3ML^2 \)

1-5. A wheel initially has an angular velocity of 18 rad/s but it is slowing at a rate of 2 rad/s². By the time it stops, it will have turned through about:

(a) 13 revolutions
(b) 26 revolutions
(c) 39 revolutions
(d) 52 revolutions
(e) 65 revolutions

\[ \omega^2 = \omega_0^2 + 2 \cdot \theta \cdot \theta \]

\[ \theta = \frac{\omega^2 - \omega_0^2}{2 \cdot \theta} = \frac{- \omega_0^2}{2 \cdot \theta} = \frac{(18 \text{ rad/s})^2}{2 \cdot 2 \cdot 18 \text{ rad/s}^2} = 81 \text{ rad} \]

\[ \# \text{ revol.} = \frac{81 \text{ rad}}{2\pi \text{ rad/rev}} = 12.89 \text{ rev} \]
Problem 2: (20 points) [NO credit for correct answers without appropriate formulae/logic]
A wheel of radius 0.15 m has a massless string wrapped around it, one end of which is attached to the wheel, the other end to a block of mass 3.0 kg. At t = 0 s the block is released from rest, and is observed to accelerate at 0.55 m/s².

\[ m = 3.0 \text{ kg}; \ R = 0.15 \text{ m}; \ V_0 = 0; \ a = 0.55 \text{ m/s}^2. \]

(a) [4 points] What is the tension in the string?

Apply Newton's 2nd law for \( m \):
\[ ma = \Sigma F \]
\[ ma = mg - T = T = m(g - a) \]
\[ T = (3.0 \text{ kg})(9.80 \text{ m/s}^2 - 0.55 \text{ m/s}^2) = 24.75 \text{ N} = 28 \text{ N} \]

(b) [4 points] What is the magnitude of the torque on the wheel?

Assume clockwise rotation is positive. The torque is created by \( T \).
\[ \tau = R_+F = \frac{F \times T}{R_+} = RT = R \cdot m(g - a) = (0.15 \text{ m})(87.75 \text{ N}) = 4.16 \text{ mN} \]
\( R_+ \) - distance between a line of force (\( T \)) and the axis of rotation.

(c) [4 points] What is the moment of inertia of the wheel?

According to Fig. 10-20(c)
\[ I = \frac{1}{2} MR^2, \] but we don't know \( M \), so we need to find \( I \) from equs of motion. Let's apply rotary N. 2nd law
\[ \tau = I \cdot \alpha \] (\( \tau \) - we found in (6))
\[ \alpha = \frac{a}{R} \Rightarrow \tau = I \cdot \frac{a}{R} \Rightarrow I = \tau \cdot \frac{R}{a} = \left[ R \cdot m(g - a) \right] \cdot \frac{R}{a} = mR^2 \left[ \frac{g}{a} - 1 \right] \]
\[ = (4.16 \text{ mN}) \left( \frac{0.15 \text{ m}}{0.55 \text{ m/s}^2} \right) = 1.13 \text{ kg.m}^2 \]
(d) [4 points] How much work does gravity perform on the system when the block drops 0.5 m?

\[ W = \int F \, dl = \int F_{\text{mg}} \, dl = F \cdot \cos \theta = 0.0 \rightarrow F \cdot l \]

\[ W = u \cdot g \cdot l = (3.0 \, \text{kg})(9.80 \, \text{m/s}^2)(0.5 \, \text{m}) = 14.7 \, \text{J} \]

(e) [4 points] What percentage of this work done by gravity appears as rotational kinetic energy of the wheel?

\[ \Delta K_{\text{rot}} = K_{\text{f}} - K_{\text{i}} = K_{\text{rot}} \]

work-energy theorem

\[ W = \Delta K = \Delta K_{\text{rot}} + \Delta K_{\text{trans.}} = \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2 = \| V = \omega R \| = \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2 = \frac{1}{2} (I + mR^2) \cdot \omega^2 \]

\[ \omega^2 = \frac{2W}{I + mR^2} \]

now, the percentage

\[ \frac{\Delta K_{\text{rot}}}{W} = \frac{1}{2} I \omega^2 = \frac{I \cdot \left(\frac{2W}{I + mR^2}\right)}{W} = \frac{I}{I + mR^2} = \frac{1}{1 + \frac{mR^2}{I}} \]

\[ = \frac{1}{1 + \frac{3 \, \text{kg} \cdot (0.15 \, \text{m})^2}{1.13 \, \text{kg} \cdot \text{m}^2}} = 0.194, \, \text{so} \quad 19.4\% \]
Problem 3: (20 points) [NO credit for correct answers without appropriate formulae/logic]

Two small spheres of putty, A and B, of mass $M_1 = 0.25 \text{ kg}$ and $M_2 = 0.30 \text{ kg}$, respectively, hang from the ceiling on strings of equal length. Sphere A is drawn aside and raised to a height $h_1 = 0.15 \text{ m}$ as shown and then released. Sphere A collides with sphere B. They stick together and swing to a maximum height $h_f$.

(a) [4 points] If the zero reference of the gravitational potential energy is chosen to be at the bottom of the swing where B is located before collision, what is the gravitational potential energy of A at the time of release?

$$U_{iA} = m_1 g \cdot h_1 = (0.25 \text{ kg}) \cdot (0.15 \text{ m}) = 0.375 \text{ J} \approx 0.37 \text{ J}$$

(b) [4 points] What is the velocity of A at the instant it collides with B?

Conservation of mechanical energy: $E_i = E_f \Rightarrow U_i + K_i = U_f + K_f$

$$m_1 g h_1 = \frac{1}{2} m_1 V_{iA}^2$$

$$V_{iA} = \sqrt{2 g h_1} = \sqrt{2 \cdot (0.25 \text{ m/s}^2) \cdot (0.15 \text{ m})} = 1.415 \text{ m/s} \approx 1.4 \text{ m/s}$$

(c) [4 points] What is the common velocity of A and B immediately after collision.

During collision energy is not conserved (totally inelastic collision), but momentum is conserved, since the net external force equals zero ($T_i$, cancels $m_1 g$; $T_f$ cancels $m_2 g$). So,

$$m_1 V_{iA} = (m_1 + m_2) V_3 \Rightarrow \text{ now both masses move with same velocity } V_3$$

$$V_3 = \frac{m_1}{m_1 + m_2} \cdot V_{iA} = \frac{0.25 \text{ kg}}{0.25 \text{ kg} + 0.30 \text{ kg}} \cdot 1.415 \text{ m/s} = 0.479 \text{ m/s} = 0.78 \%$$
(d) [4 points] What fraction (or percentage) of the initial mechanical energy remains in the two-mass system after the collision?

\[
\text{Initial amount of energy } \ U_{ini} = m_1g\cdot h_1 \quad \text{(potential)}
\]
\[
\text{Final amount of energy (after collision) } \ U_f = \frac{1}{2} (m_1+m_2) v_3^2
\]

Lost energy is \( U_{ini} - U_f \), but we need a different thing.

\[
\frac{U_f}{U_{ini}} = \frac{\frac{1}{2} (m_1+m_2) v_3^2}{m_1g\cdot h_1} = \frac{\frac{1}{2} \cdot (0.25kg + 0.30kg) \cdot (0.779m/s)^2}{0.3675J} = 0.454
\]

So, only \( \approx 45\% \) will stay in a form of mechanical energy.
The rest, \( 55\% \), will be lost due to a totally inelastic collision.

(e) [4 points] What is the maximum height \( h_f \), reached by A+B after collision?

\[
\text{After the collision, mechanical energy will be conserved (two bodies move together now)}
\]
\[
E_{ini} = E_f
\]
\[
\frac{1}{2} (m_1+m_2) v_3^2 = (m_1+m_2) g h_f
\]
\[
h_f = \frac{v_3^2}{2g} = \frac{(0.779 m/s)^2}{2 \cdot (9.80 m/s^2)} = 0.03096 m \approx 0.031 m
\]
Trig:
\[
\begin{align*}
\sin \theta &= a/c \\
\cos \theta &= b/c \\
\tan \theta &= a/b \\
c^2 &= a^2 + b^2
\end{align*}
\]

Quadratic Formula:
A\(x^2 + Bx + C = 0\) has solutions:
\[
x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}
\]

Misc Formulas:
Circumference of a circle = \(2\pi r\)
Area of a circle = \(\pi r^2\)
Surface Area of a Sphere = \(4\pi r^2\)
Volume of sphere = \((4/3)\pi r^3\)
Volume of cylinder = \(\pi r^2 h\)

Differentiation:
\[
dx^n/dx = nx^{n-1} \quad (n \neq 0)
\]
dcos(x)/dx = -sin(x) \quad (x \text{ in radians)}

dsin(x)/dx = cos(x) \quad (x \text{ in radians)}

d(f(x) + g(x))/dx = df(x)/dx + dg(x)/dx

Integration:
\[
\int x^n dx = \frac{x^{n+1}}{n+1} + C
\]

Units:
Newton: \(N \text{ (kgm/s}^2\)
Velocity: \(dx(t)/dt\) or \(v\) (m/s)
Acceleration: \(d^2x(t)/dt^2\) or \(a\) (m/s\(^2\))

1-D Motion:
displacement = \(\Delta x\)
\[
\begin{align*}
\text{velocity average: } \Delta x/\Delta t &= (x_2 - x_1)/(t_2 - t_1) \\
\text{acceleration average: } \Delta v/\Delta t &= (v_2 - v_1)/(t_2 - t_1)
\end{align*}
\]

Given \(x(t)\)
\[
\begin{align*}
v(t) &= dx(t)/dt \text{ (instantaneous)} \\
a(t) &= dv(t)/dt = d^2x(t)/dt^2 \text{ (instantaneous)}
\end{align*}
\]

1-D Motion with Const. Acc.:
\[
\begin{align*}
x(t) &= \left[1/2\right] a t^2 + v_0 t + x_0 \\
v(t) &= a t + v_0 \\
v^2 &= v_0^2 + 2a(x - x_0)
\end{align*}
\]

Projectile Motion:
\[
\begin{align*}
x(t) &= v_{0x} t + x_0 \\
v_x(t) &= v_{0x} \\
a_x(t) &= 0 \\
y(t) &= \left[1/2\right] a y t^2 + v_{0y} t + y_0 \\
v_y(t) &= a_y t + v_{0y} \\
a_y(t) &= a_y
\end{align*}
\]

For motion over level ground
Range = \([v_0 \cos(\theta)]/g\)

Acceleration due to gravity:
g = 9.8 \text{ m/s}^2 \text{ downward}

Newton's Second Law:
\[
\sum \vec{F}_{net} = \sum \vec{F}_{ext} = m\vec{a}
\]

Circular Motion:
\[
T = 1/f \quad v = 2\pi R/T \\
a_{centripetal} = v^2/R
\]

Frictional Forces:
\[
f_s \leq \mu_s F_N \\
f_k = \mu_k F_N \\
\mu_s > \mu_k
\]

Work and Kinetic Energy
\[
W = F\Delta x \\
W = \int_0^r \vec{F} \cdot d\vec{r} \\
K = (1/2)mv^2 \\
W_{net} = \Delta K \\
\Delta K = K_f - K_i
\]

Potential Energy
\[
\Delta U = U(x) - U(x_0) = -\int_{x_0}^x Fdx
\]
\[ F(x) = -dU(x)/dx \]

For gravity on earth's surface:
\[ F_g = mg \]
\[ U(y) = U_0 + mgy \]

For gravity in general:
\[ F_y = -\frac{GmM_y}{R^2} \]
\[ U(r) = -\frac{GmM_y}{r} \]

For springs:
\[ F = -kx \]
\[ U(x) = (1/2)kx^2 \]

With conservative forces only:
\[ E_{tot} = K + U \quad \text{(a constant)} \]
\[ \Delta E_{tot} = \Delta K + \Delta U = 0 \]

With non-conservative forces:
\[ \Delta E_{tot} = \Delta K + \Delta U = W_{NC} \]

Gravity
Kepler's third law:
\[ T^2/R^3 = 4\pi^2/GM_{\text{cm}} \]
\[ G = 6.67 \times 10^{-11} \text{N.m}^2/\text{kg}^2 \]

Power
\[ P = dW/dt \]
\[ P = \vec{F} \cdot \vec{v} \]

Momentum and Impulse
\[ \vec{p} = mv \]
\[ \dot{\vec{p}} = dp/dt \]
\[ \vec{J} = \int \vec{F} \, dt = \vec{F}_{\text{tot}} \Delta t \]
\[ J = \Delta p \]

For 1-D Collisions:
\[ \vec{p}_A + \vec{p}_B = \vec{p}'_A + \vec{p}'_B \]

For 1-D Elastic Collisions:
\[ \frac{1}{2} m_A v^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_B v_B'^2 \]

Center of Mass
\[ \vec{r}_{\text{cm}} = \Sigma m_i \vec{r}_i / M \]
\[ \Sigma \vec{F}_{\text{ext}} = M \vec{a}_{\text{cm}} \]

Rotation
\[ \omega = d\theta/dt \]
\[ \alpha = d\omega/dt \]
\[ 1/T = 1/f \]
\[ \omega = 2\pi f \]

For constant angular acceleration
\[ \theta(t) = \left(1/2\right)\alpha t^2 + \omega_0 t + \theta_0 \]
\[ \omega(t) = \alpha t + \omega_0 \]
\[ \omega^2 = \omega_0^2 + 2\alpha (\theta - \theta_0) \]

\[ K_{\text{rot}} = (1/2)I_1\omega^2 \]
\[ K_{\text{rot}} = (1/2)I_{CM}\omega^2 + (1/2)Mv_{CM}^2 \]
\[ I = \Sigma m_i R_i^2 \]

\[ \Sigma \tau = I\alpha \]
\[ \vec{L} = \vec{r} \times \vec{p} \]
\[ \vec{\tau} = \vec{r} \times \vec{F} \]
\[ \tau = r \cdot F \cdot \sin\theta \]

\[ v_{\text{tangential}} = R\omega \]
\[ a_{\text{tangential}} = R\alpha \]
\[ a_r = \omega^2 R \]