95.141 Exam 3 Spring 2014

Section number

Section instructor

Last/First name

Last 3 Digits of Student ID Number: ___ ___ ___

Answer all questions, beginning each new question in the space provided. Show all work. Show all formulas used for each problem prior to substitution of numbers. Label diagrams and include appropriate units for your answers. Write your name and section number at the top of each page in the space provided and write the name of your section instructor in the place provided in the cover sheet. You may use an alphanumeric calculator (one which exhibits physical formulas) during the exam as long as you do not program any formulas into memory. By using an alphanumeric calculator you agree to allow us to check its memory during the exam. Simple scientific calculators are always OK!

A Formula Sheet Is Attached To The Back Of This Examination
For your convenience you may carefully remove it from the Exam. Please take it with you at the end of the exam or throw it in a waste basket.

Be Prepared to Show your Student ID Card

Score on each problem:

1. (25) ___

2. (25) ___

3. (25) ___

4. (25) ___

Total Score (out of 100 pts) ___
1. (25 point) Put a circle around the letter that you think is the best answer.

1.1. (5 pts) Two masses (1 kg and 2 kg) are mounted on a light rod, which mass can be ignored, as shown in the figure. Calculate the moment of inertia of the system when rotated about an axis shown in the figure.

![Diagram of a rod with two masses](image)

A) 7 kgm²  
B) -1 kgm²  
C) 11 kgm²  
D) 18 kgm²  
E) None of the above

1.2. (5 pts) An inelastic collision of two objects is characterized by the following.

A) Total linear momentum of the system is conserved.  
B) Total energy of the system remains constant.  
C) Total kinetic energy of the system remains constant.  
D) Only A and B are true.  
E) A, B, and C are all true.

1.3. (5 pts) If a constant net torque is applied to an object, that object will

A) rotate with constant linear velocity.  
B) rotate with constant angular velocity.  
C) rotate with constant angular acceleration.  
D) having an increasing moment of inertia.  
E) having an decreasing moment of inertia.
1.4. (5 pts) The axle of a horizontal wheel is mounted on supports that rest on a rotating vertical disc as shown in the figure. The wheel has angular velocity \( \omega_1 = 4 \, \text{rad/s} \) about its axle, and the disc has angular velocity \( \omega_2 = 3 \, \text{rad/s} \) about a horizontal axis. (Note arrows showing these motions in the figure) What is the magnitude of the wheel's resultant angular velocity, as seen by an outside observer, at the instant shown?

\[
\omega_0 = \sqrt{\omega_1^2 + \omega_2^2} = \sqrt{16 + 9} = 5 \, \text{rad/s}
\]

A) 3 rad/s  
B) 4 rad/s  
C) 5 rad/s  
D) 7 rad/s  
E) None of the above

1.5. (5 pts) The total angular momentum of the system is a conserved quantity if

A) no net external force acts on a system  
B) no net external torque acts on a system  
C) no net internal force acts on a system  
D) no net internal torque acts on a system
2. (25 pts) A ball of mass \( m \) slides with velocity \( v \) on a frictionless horizontal surface and hits a large wooden block of mass \( M \) which lies on the same frictionless surface. The block \( M \) is suspended like a pendulum and initially at rest. The ball has become embedded in the block after the collision.

a) (3pts) What is conserved during the collision?

Linear momentum is conserved.

Why is that conserved? (Provide a short explanation)

\[ \Sigma F_{ext} = 0 \]

b) (10pts) What is the speed of the block just after the ball has become embedded in terms of \( m, M, \) and \( v \)?

\[ p_i = p_f \]

\[ m v_i = (m+M) v_f \]

\[ v_f = \left( \frac{m}{m+M} \right) v \]

As a result of the collision, the block and the ball swing up to a maximum height \( h \).

c) (2pts) What is conserved during the swing? Mechanical energy.

Why is that conserved? (Provide a short explanation)

Gravitational force is conservative.
There is no friction.

d) (10pts) Determine the maximum height \( h \) in terms of \( m, M, \) and \( v \).

\[ E_i = E_f \]

\[ K_i + U_i = K_f + U_f \]

\[ \frac{1}{2} (m+M) v_i^2 = (m+M) g h \]

\[ h = \frac{v_i^2}{2g} = \left( \frac{m}{m+M} \right)^2 \frac{v^2}{2g} = h \]
3. (25 pts) A cat of mass $m$ is dropped from an edge of a roof. The edge of the roof is $l$ meters away from the wall of the building. The origin of the coordinate system is placed at point $O$ as shown in the figure. Ignore air resistance

a) (10pts) Find the angular momentum of the cat in terms of unit vectors $(\hat{i}, \hat{j}, \hat{k})$ relative to point $O$ at the moment shown in the figure.

\[ \vec{\mathbf{L}} = \vec{r} \times \vec{p} \; ; \; \vec{p} = m\vec{v} \]

\[ \vec{\mathbf{L}} = (mvl) \hat{k} \quad \text{(apply right-hand rule)} \]

\[ \vec{\mathbf{L}} = 0\hat{i} + 0\hat{j} + (mvl)\hat{k} \]

b) (2pts) Write rotational Newton second law in terms of angular momentum and name each of the physical quantities involved in the expression.

\[ \Sigma \vec{\mathbf{F}} = \frac{d\vec{\mathbf{L}}}{dt} \]

$\vec{\mathbf{L}}$ - torque; $\vec{\mathbf{L}}$ - angular momentum.

c) (10pts) Find the torque on the cat in terms of unit vectors $(\hat{i}, \hat{j}, \hat{k})$.

\[ \vec{\mathbf{T}} = \frac{d\vec{\mathbf{L}}}{dt} = \frac{d}{dt} (mvl\hat{k}) = ml \left( \frac{dv}{dt} \right) \hat{k} = (mgl)\hat{k} \]

\[ \vec{\mathbf{T}} = 0\hat{i} + 0\hat{j} + (mgl)\hat{k} \]

Torque is produced by $\vec{\mathbf{F}}$.

d) (3pts) Is the angular momentum of the cat conserved? Justify your answer?

The angular momentum is not conserved because there is an external torque produced by $\vec{\mathbf{F}}$. 

4. A disc whose moment of inertia $I = 1 \text{ kg} \cdot \text{m}^2$ is rotating at angular velocity $\omega = 100 \text{ rad/s}$. This disc is pressed against a similar disc that is able to rotate freely but is initially at rest. The two discs stick together and rotate as a unit.

a) Find the final angular velocity of the combination.

b) Find the kinetic energies before and after "collision".

c) How much energy was lost to friction when the discs were brought together?

\[ \text{a) Angular momentum is conserved.} \quad \sum I_i = \sum I_f. \]

\[ \omega \]

\[ I \]

\[ \omega = 0 \quad \text{at rest} \]

\[ I \]

\[ 2I \]

\[ \omega_3 - ? \]

\[ I \cdot \omega = 2I \cdot \omega_3 \]

\[ \omega_3 = \frac{\omega}{2} = \frac{100 \text{ rad/s}}{2} = 50 \text{ rad/s} \]

\[ \text{b) } K_i = \frac{1}{2} I \cdot \omega^2 + \frac{1}{2} I \cdot \omega_1^2 = \frac{1}{2} \cdot (1 \text{ kg} \cdot \text{m}^2) \cdot (100 \text{ rad/s})^2 = 5000 \text{ J} \]

\[ K_f = \frac{1}{2} (2I) \omega_3^2 = \frac{1}{2} \cdot 2 \cdot (1 \text{ kg} \cdot \text{m}^2) \cdot (50 \text{ rad/s})^2 = 2500 \text{ J} \]

\[ \text{Energy lost } = \Delta K = K_i - K_f = 5000 \text{ J} - 2500 \text{ J} = 2500 \text{ J} \]

\[ \text{is lost.} \]