95.612. Classical Mechanics.

Final Exam. May 3, 2013

Open book exam.

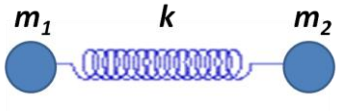
1. **(10 points)** Consider a particle m moving under an influence of the inverse-square law which is described by the Hamiltonian

$$H = \frac{1}{2m}(p_1^2 + p_2^2/q_1^2) - \frac{\lambda m}{q_1}$$

Where q_1 and q_2 are the generalized coordinates.

a) *What is the Hamilton-Jacobi equation for this system?*
b) *Solve this equation to find Hamilton's principal function, S.*
c) *Deduce the dynamical motion of the system (you need not evaluate any integrals).*

2. **(10 points)** Two masses m_1 and m_2 are connected by a massless spring of force constant k and unstretched length a. Oscillations are small.



a) *Write the Lagrangian of the system*
b) *Find the normal mode frequencies*
c) *Find the normal mode eigenvectors and the general solution*
d) *Construct the modal matrix* ***A***
e) *Find the normal coordinates.*

3. **(10 points)** The Hamiltonian for a particle moving in a vertical uniform gravitational field ***g*** is

$$H = \frac{p^2}{2m} + mgq$$

where q is the altitude above the ground. We want to find any canonical transformation from old variables *(q,p)* to new variables *(Q,P)* which provides a cyclic coordinate. To do this, define new variables as

$$Q = bp \qquad P = aH$$

*where **a, b** are constants.*

a) Determine any combination of constants a and b, which provides a canonical transformation.

b) Find the type 1 generating function, F_1 (q, Q)

c) Use the relation $F_2(q, P) = F_1 + PQ$ to find the type 2 generating function and check your result by showing that F_2 indeed generates the same transformation

d) Find the new Hamiltonian K for the new canonical variables Q, P. Are there any cyclic variables?

e) *Solve Hamilton equations for the new canonical variables*

f) *Find the original variables q,p as a function of time*

4. **(10 points)** An inextensible massless string of length $\boldsymbol{l}$ passes through a hole in a frictionless table. A point mass $\boldsymbol{m_1}$ at one end moves on the frictionless table and a point mass $\boldsymbol{m_2}$ hangs from the other end. Assume that $\boldsymbol{m_1}$ experiences only a radial motion and $\boldsymbol{m_2}$ moves only in a vertical line.

*a) Write down the Lagrangian of the system **Using the method of undetermined multipliers***

b) Write down the modified Lagrange equations and solve them for $\ddot{q}_i$ and the Lagrange multiplier. Write magnitude of the tension of the string.