Classical Mechanics
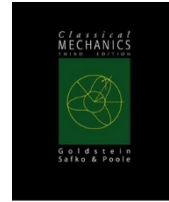**Chapter 9. Canonical Transformations.**
Homework 5
(Due to April 5, 2018).

Problem 5A.   **(10 points)**

Construct from the first principles the Hamiltonian for a 1D harmonic oscillator of mass **m** and spring constant **k**. Determine the value of the constant **C** such that the following equations define a canonical transformation from the old variables (q, p) to the new variables (Q, P):

$$Q = C(p + im\omega q)$$
$$P = C(p - im\omega q)$$

Where $\omega = \sqrt{k/m}$. What is the generating function for this transformation? Find Hamilton's equations of motion for the new variables and integrate them. Hence find the solution to the original problem.

Problem 5B.   **(10 points)**

The Hamiltonian for a particle moving in a vertical uniform gravitational field **g** is

$$H = \frac{p^2}{2m} + mgq$$

where q is the altitude above the ground. We want to find any canonical transformation from old variables *(q,p)* to new variables *(Q,P)* which provides a cyclic coordinate. To do this, define new variables as

$$Q = bp \qquad P = aH$$

*where **a, b** are constants.*
 a) Determine any combination of constants a and b, which provides a canonical transformation.
 b) Find the type 1 generating function, F_1 (q, Q)
 c) Use the relation $F_2(q, P) = F_1 + PQ$ to find the type 2 generating function and check your result by showing that F_2 indeed generates the same transformation
 d) Find the new Hamiltonian K for the new canonical variables Q, P.
 Are there any cyclic variables?
 e) Solve Hamilton equations for the new canonical variables
Find the original variables q,p as a function of time