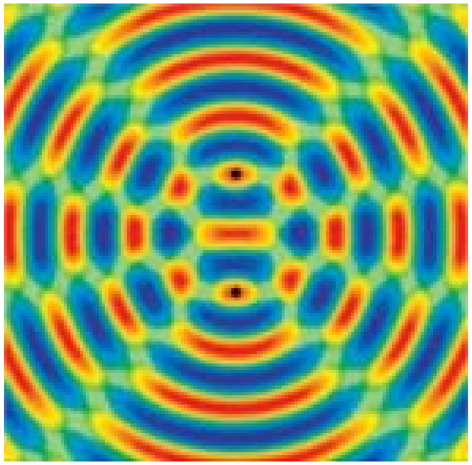
Lecture 22

Physics II

Chapter 21

Interference



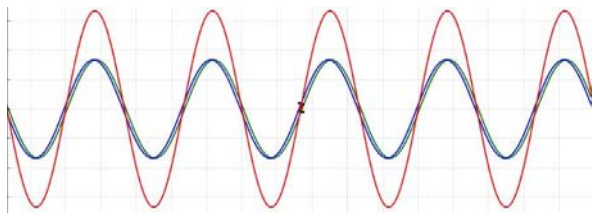
Course website:
http://faculty.uml.edu/Andriy_Danylov/Teaching/PhysicsII

Lecture Capture:
http://echo360.uml.edu/danylov201415/physics2spring.html

Interference

A standing wave is the interference pattern produced when two waves of equal frequency travel in opposite directions.

$y_R = aSin(kx - \omega t)$ $\qquad$ $y_L = aSin(kx + \omega t)$



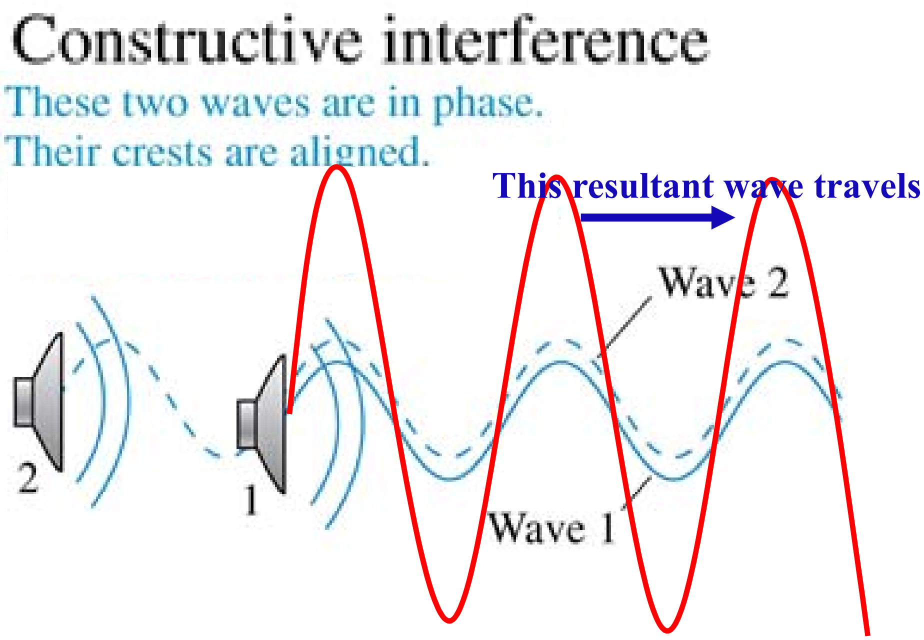
Standing Wave (Demo)

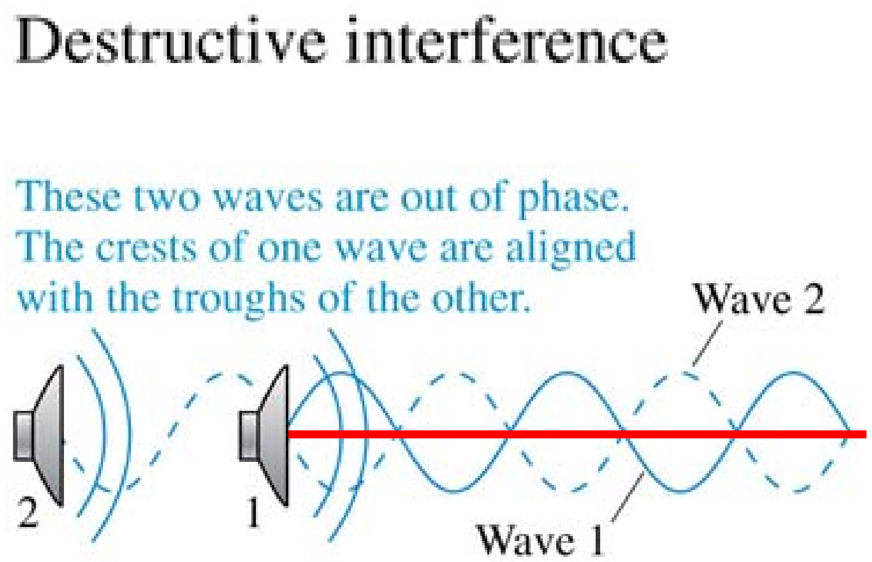
In this section we will look at the interference of two waves traveling in the same direction.

$y_R = aSin(kx - \omega t)$ $\qquad$ $y_R = aSin(kx - \omega t)$

Interference in One Dimension

The pattern resulting from the superposition of two waves is often called interference. In this section we will look at the interference of two waves traveling in the *same* direction.





The resulting amplitude is $A = 2a$ for *maximum* ***<u>constructive interference</u>***.

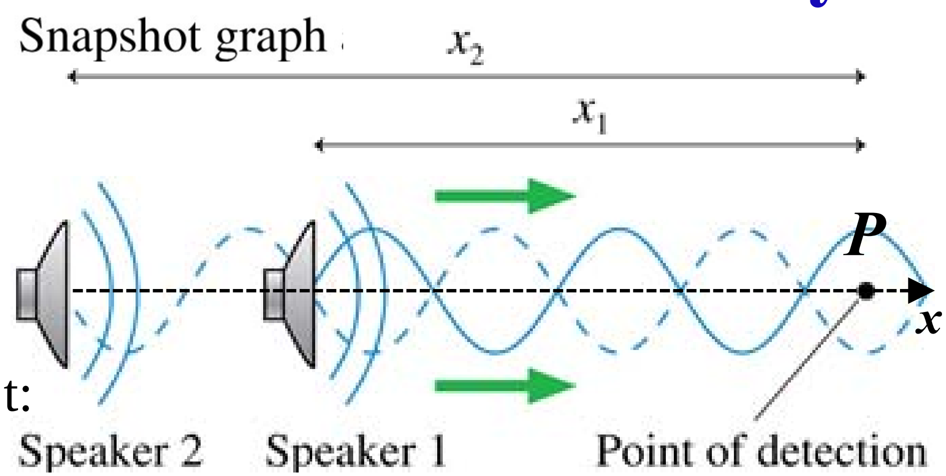
The resulting amplitude is $A = 0$ for *perfect* ***<u>destructive interference</u>***

Let's describe 1D interference mathematically

Consider two traveling waves. They have:

1. The same direction, +x direction
2. The same amplitude, a
3. The same frequency, ω

Let's find a displacement at point P at time t:



$$y(t) = y_1(x_1,t) + y_2(x_2,t) = a\sin(\underbrace{kx_1 - \omega t + \varphi_{10}}_{\varphi_1}) + a\sin(\underbrace{kx_2 - \omega t + \varphi_{20}}_{\varphi_2})$$

The phase of the wave

The phase constant ϕ_0 tells us what the source is doing at $t = 0$.

Using a trig identity: $\sin\varphi_1 + \sin\varphi_2 = 2\cos[\frac{\varphi_1-\varphi_2}{2}]\sin[\frac{\varphi_1+\varphi_2}{2}]$

$$y(t) = 2a\cos[\frac{\Delta\varphi}{2}]\sin[k\left(\frac{x_1+x_2}{2}\right) - \omega t + \left(\frac{\varphi_{10}+\varphi_{20}}{2}\right)]$$

$\Delta\varphi = \varphi_1 - \varphi_2$, $\frac{x_1+x_2}{2} = x_{avg}$, $\frac{\varphi_{10}+\varphi_{20}}{2} = \varphi_{avg}$

$$y(t) = \left(2a\cos\left[\frac{\Delta\varphi}{2}\right]\right)\sin[kx_{avg} - \omega t + \varphi_{avg}{}^{0})$$

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Constructive/destructive interference

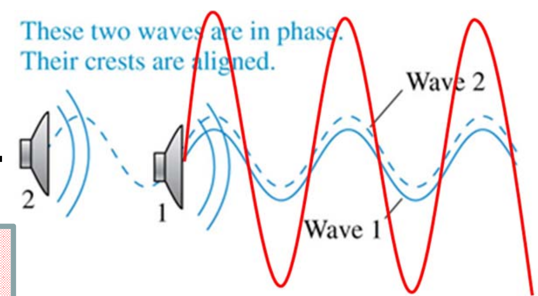
$$y(t) = \left[2a\cos\left(\frac{\Delta\phi}{2}\right)\right]\sin(kx_{avg} - \omega t + (\phi_0)_{avg})$$

It is still a traveling wave

The amplitude: $A = \left|2a\cos\left[\frac{\Delta\varphi}{2}\right]\right|$ where $\Delta\phi = \phi_1 - \phi_2$ is the phase difference between the two waves.

- The amplitude has a maximum value $A = 2a$ **if**

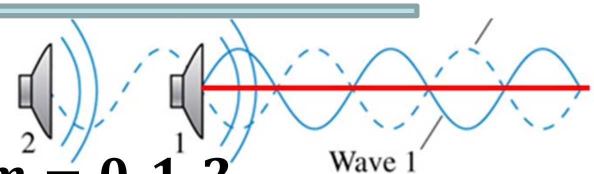
$\cos(\Delta\phi/2) = \pm 1.$ ⇨ $\frac{\Delta\varphi}{2} = m\pi,\quad where\ m = 0, 1, 2, \ldots$



Conditions for **constructive interference:** $\Delta\varphi = 2m\pi$

- Similarly, the amplitude is zero, $A=0$ if

$\cos(\Delta\phi/2) = 0.$ ⇨ $\Delta\varphi/2 = \left(m + \frac{1}{2}\right)\pi,\quad where\ m = 0, 1, 2, \ldots$



Conditions for **destructive interference** $\Delta\varphi = \left(m + \frac{1}{2}\right)2\pi$

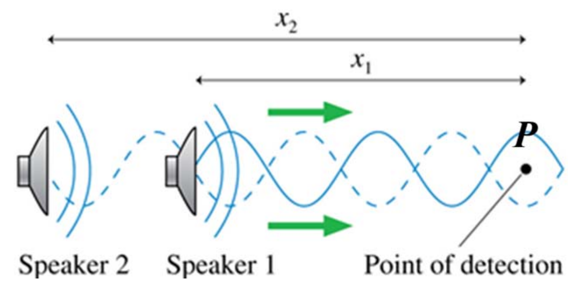
Let's look deeper in Δφ

$\Delta\phi = \phi_2 - \phi_1$ is the phase difference between the two waves.

$$\Delta\varphi = (kx_2 - \cancel{\omega}t + \varphi_{20}) - (kx_1 - \cancel{\omega}t + \varphi_{10}) = k(x_2 - x_1) + (\varphi_{20} - \varphi_{10}) = k\Delta x + \Delta\varphi_0$$

$$k = \frac{2\pi}{\lambda}$$

$$\boxed{\Delta\varphi = \frac{2\pi}{\lambda}\Delta x + \Delta\varphi_0}$$



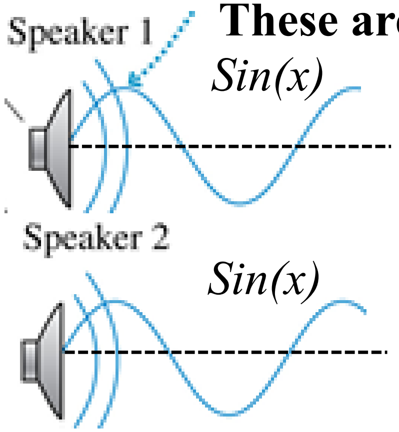
<u>So, there are two contributions to the phase difference:</u>

1. $\Delta x = x_2 - x_1$ – *pathlength difference*
2. $\Delta\varphi_0 = \varphi_{20} - \varphi_{10}$ -- *inherent phase difference*

Inherent phase difference

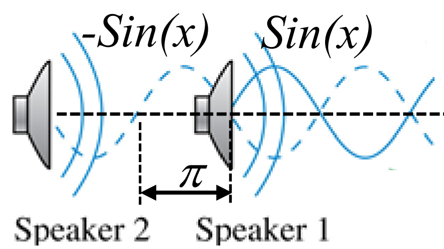
$$\Delta\varphi_0 = \varphi_{20} - \varphi_{10}$$

These are identical sources:



$$\boxed{\Delta\varphi_0 = 0}$$

These are not identical sources: out of phase



We have to shift –Sin by π to get Sin (to overlap them), so

$$\boxed{\Delta\varphi_0 = \pi}$$

Question

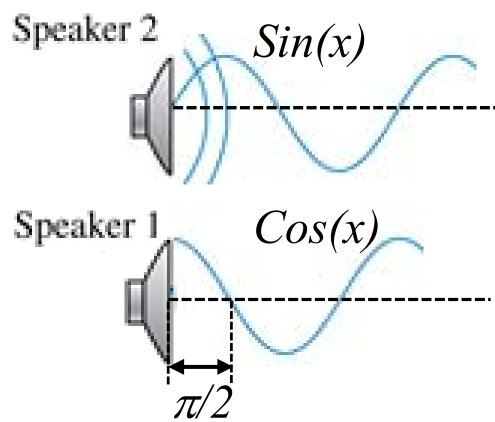
What is the inherent phase difference?

A) 0

B) $\pi/2$

C) π

D) 2π



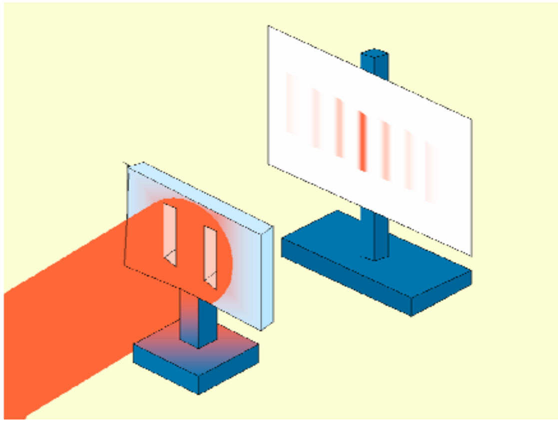
We have to shift Cos by $\pi/2$ to get Sin (to overlap them), so

$$\boxed{\Delta\varphi_0 = \pi/2}$$

Sources are very often identical

($\Delta\varphi_0=0$)

(like the double slit experiment in Optics)



So, let's prepare expressions for these cases:

Pathlength difference for constructive interference

Assume that the sources are identical $\Delta\varphi_0 = 0$. Let's separate the sources with a pathlength Δx

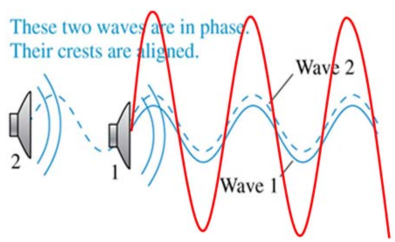
$$\Delta\varphi = \frac{2\pi}{\lambda}\Delta x + \cancelto{0}{\Delta\varphi_0}$$

Conditions for
constructive interference:

$$\Delta\varphi = 2m\pi$$

$$\frac{2\pi}{\lambda}\Delta x = 2m\pi$$

$$\boxed{\Delta x = m\lambda}$$



Question

Are the sources identical?

A) yes

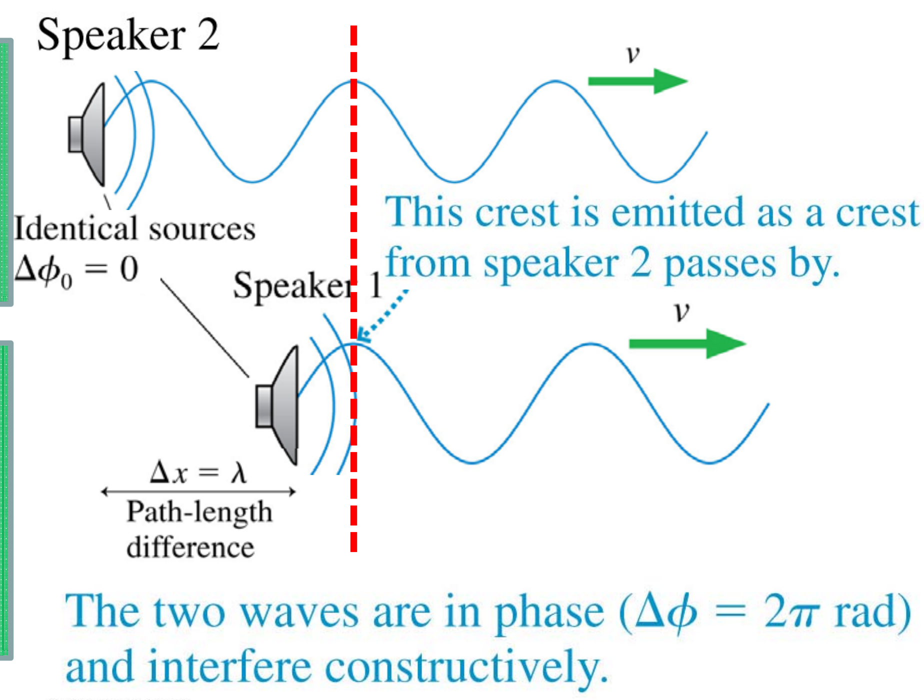
B) no

Question

What is the pathlength difference?

A) λ/2

B) λ



Thus, for a constructive interference of two identical sources with A = 2a, we need to separate them by an integer number of wavelength

Pathlength difference for destructive interference

Assume that the sources are identical $\Delta\varphi_0 = 0$. Let's separate the sources with a pathlength Δx

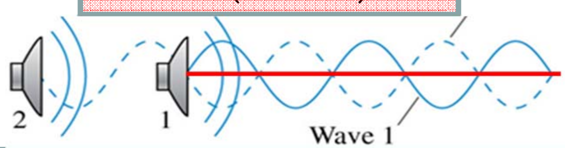
$$\boxed{\Delta\varphi = \frac{2\pi}{\lambda}\Delta x + \cancel{\Delta\varphi_0}^{\,0}}$$

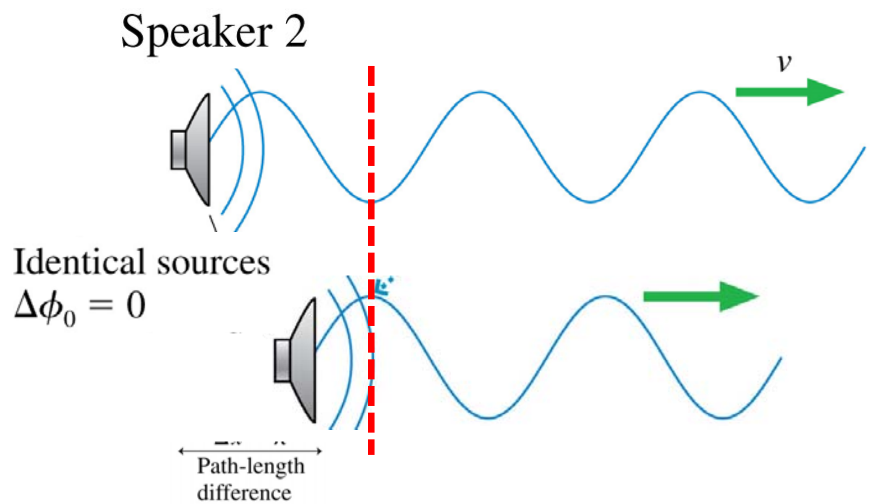
Conditions for
destructive interference:

$$\Delta\varphi = \left(m + \frac{1}{2}\right)2\pi$$

$$\frac{2\pi}{\lambda}\Delta x = \left(m + \frac{1}{2}\right)2\pi$$

$$\boxed{\Delta x = \left(m + \frac{1}{2}\right)\lambda}$$





$\Delta x = \lambda/2$

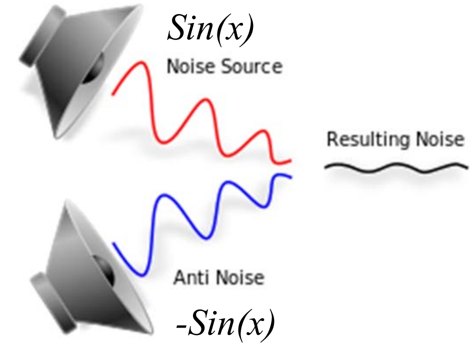
Thus, for a constructive interference of two identical sources with A =0, we need to separate them by an integer number of wavelength

$$\Delta\varphi = \frac{2\pi}{\lambda}\Delta x + \Delta\varphi_0$$

Applications

Noise-cancelling headphones

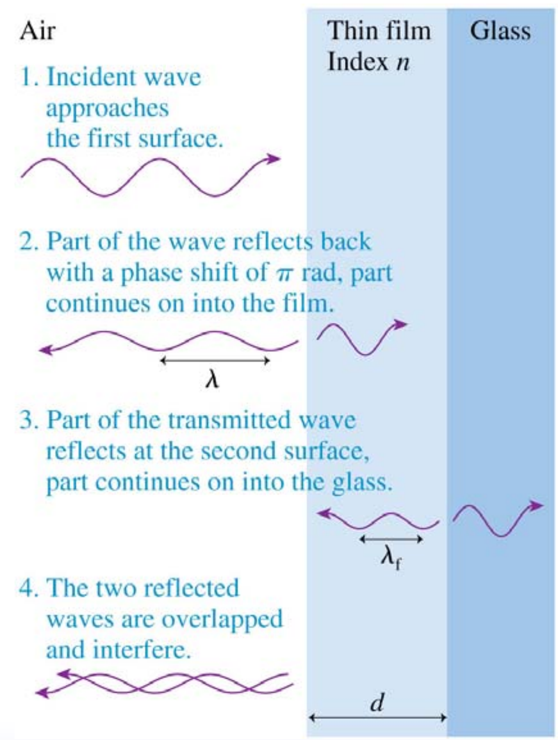
It allows reducing unwanted sound by the addition of a second sound specifically designed to cancel the first (destructive interference).



- Thin transparent films, placed on glass surfaces, such as lenses, can control reflections from the glass.
- Antireflection coatings on the lenses in cameras, microscopes, and other optical equipment are examples of thin-film coatings.

$$\Delta x \neq 0$$

$$\Delta\varphi_0 = 0$$



ConcepTest 1D interference

- Two loudspeakers emit waves with $\lambda = 2\ m$.
- What, if anything, can be done to cause constructive interference between the two waves?

A) Move speaker 1 forward by 0.5 m

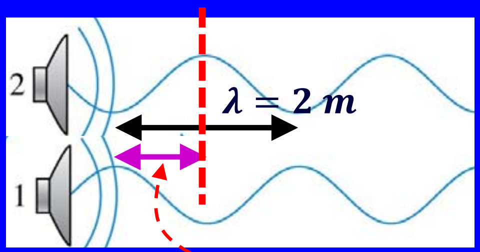
B) Move speaker 1 forward by 1.0 m

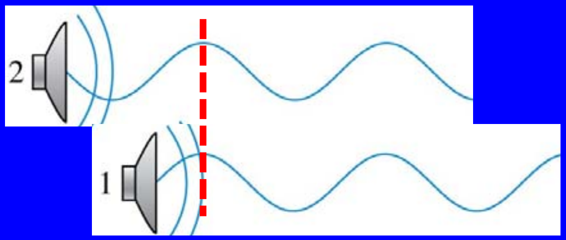
C) Move speaker 1 forward by 2.0 m

D) Do nothing

The sources are out of phase, $\Delta\phi_0 = \pi$ rad.

We have to compensate the inherent phase difference with a pathlength difference





It has to be moved by $\Delta x = \lambda/2$ to align crests

Interference in two and three dimensions

A Circular or Spherical Wave

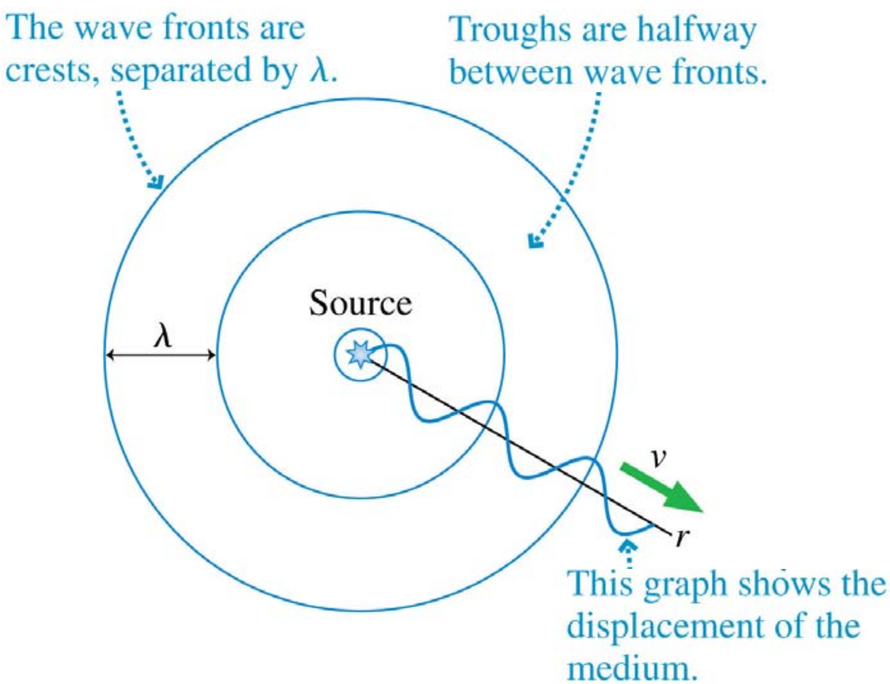
A linear (1D) wave can be written

$$y(x,t) = a\sin(kx - \omega t + \varphi_0)$$

A circular (2D) or spherical (3D) wave can be written

$$D(r,t) = a\sin(kr - \omega t + \varphi_0)$$

where r is the distance measured outward from the source.



Transition from 1D to 2D/3D interference

- The mathematical description of interference in two or three dimensions is very similar to that of one-dimensional interference.
- The conditions for constructive and destructive interference are:

one-dimensional

Constructive:

$$\boxed{\Delta\varphi = \frac{2\pi}{\lambda}\Delta x + \Delta\varphi_0} = 2m\pi$$

Destructive:

$$\Delta\varphi = \frac{2\pi}{\lambda}\Delta x + \Delta\varphi_0 = \left(m + \frac{1}{2}\right)2\pi$$

two or three dimensions

Maximum constructive interference:

$$\Delta\phi = 2\pi\frac{\Delta r}{\lambda} + \cancelto{0}{\Delta\phi_0} = m \cdot 2\pi$$

Perfect destructive interference:

$$\Delta\phi = 2\pi\frac{\Delta r}{\lambda} + \cancelto{0}{\Delta\phi_0} = \left(m + \frac{1}{2}\right) \cdot 2\pi$$

where Δr is the *path-length difference*. $m = 0, 1, 2, \ldots$

If the sources are identical ($\Delta\varphi_0 = 0$), the interference is

Constructive if $\Delta r = m\lambda$

Destructive if $\Delta r = (m + \frac{1}{2})\lambda$

Example of 2D interference

Constructive if $\Delta r = m\lambda$

Destructive if $\Delta r = (m + \frac{1}{2})\lambda$

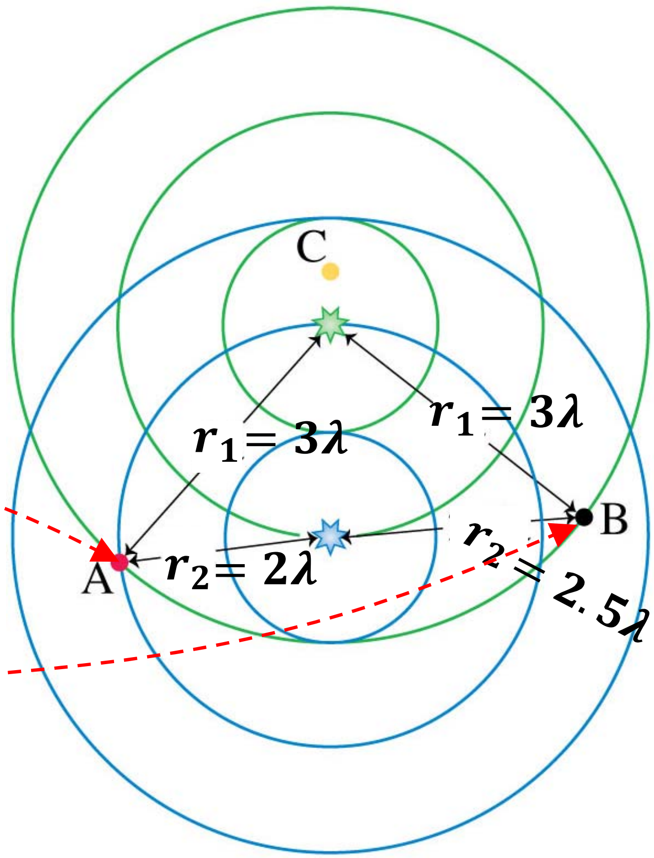
- The figure shows two identical sources that are in phase.
- The path-length difference Δr determines whether the interference at a particular point is constructive or destructive.

$$\Delta r_A = r_1 - r_2 = \lambda$$

- At A, $\Delta r_A = \lambda$, so this is a point of constructive interference.

$$\Delta r_B = r_1 - r_2 = \frac{1}{2}\lambda$$

- At B, $\Delta r_B = \frac{1}{2}\lambda$, so this is a point of destructive interference.



ConcepTest *2D* Interference

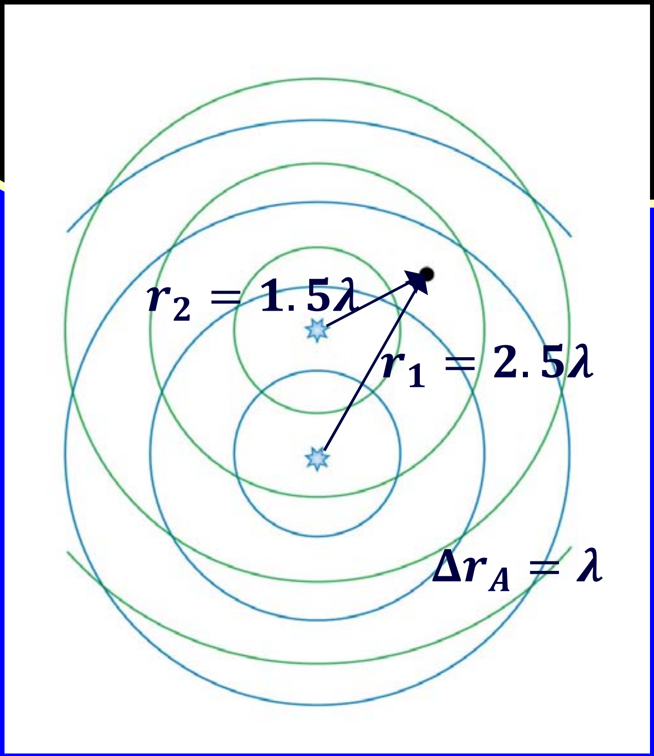
- Two in-phase sources emit sound waves of equal wavelength and intensity. At the position of the dot,

A) The interference is constructive.

B) The interference is destructive

C) The interference is somewhere between constructive and destructive

D) There's not enough information to tell about the interference.



Constructive if $\Delta r = m\lambda$

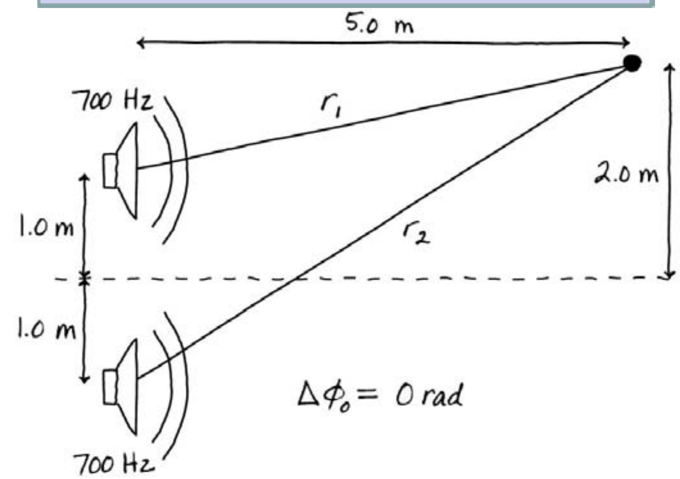
Destructive if $\Delta r = (m + \frac{1}{2})\lambda$

EXAMPLE 21.10 Two-dimensional interference between two loudspeakers

Two loudspeakers in a plane are 2.0 m apart and in phase with each other. Both emit 700 Hz sound waves into a room where the speed of sound is 341 m/s. A listener stands 5.0 m in front of the loudspeakers and 2.0 m to one side of the center. Is the interference at this point maximum constructive, perfect destructive, or in between? How will the situation differ if the loudspeakers are out of phase?

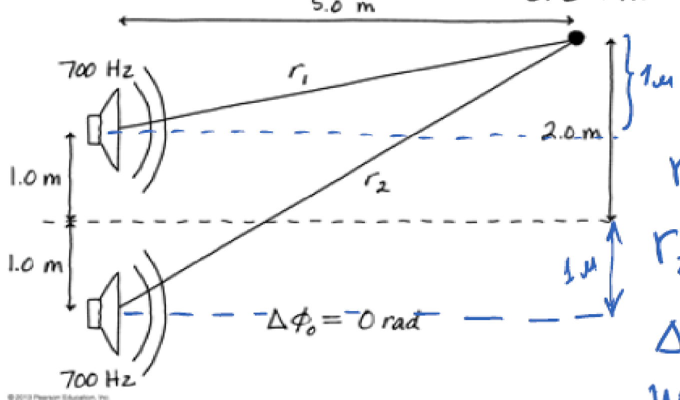
Constructive if $\Delta r = m\lambda$

Destructive if $\Delta r = (m+\frac{1}{2})\lambda$



Two speakers are in phase ($\varphi_{01}=\varphi_{02}, \Delta\varphi_0=0$)

listener



We need to find Δr and compare with λ.

$$r_1 = \sqrt{(5.0m)^2+(1.0m)^2} = 5.1m$$

$$r_2 = \sqrt{(5.0m)^2+(3.0m)^2} = 5.83m$$

$$\Delta r = r_2 - r_1 = 5.83m - 5.1m = 0.73m$$

We know that $f_1 = f_2 = 700Hz$ (sources are identical)

$$\lambda = \frac{v}{f} = \frac{340\ m/s}{700\ Hz} = 0.486\ m$$

$$\frac{r}{\lambda} = \frac{0.73m}{0.486m} = 1.5 = \frac{3}{2} = (\overset{=1}{m}+\frac{1}{2})$$

So, the conditions are for destructive interference.

What you should read

Chapter 21 (Knight)

Sections

- ***21.5***
- ***21.6***
- ***21.7***

Thank you

See you on Friday