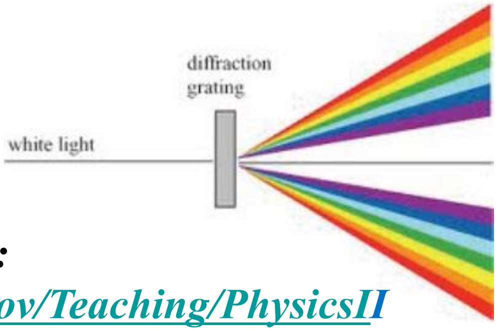
Lecture 22

University of Massachusetts
UMASS Lowell
Physics II

Chapter 22

Wave Optics:

Interference of Light



Course website:
http://faculty.uml.edu/Andriy_Danylov/Teaching/PhysicsII

Wave Motion Interference

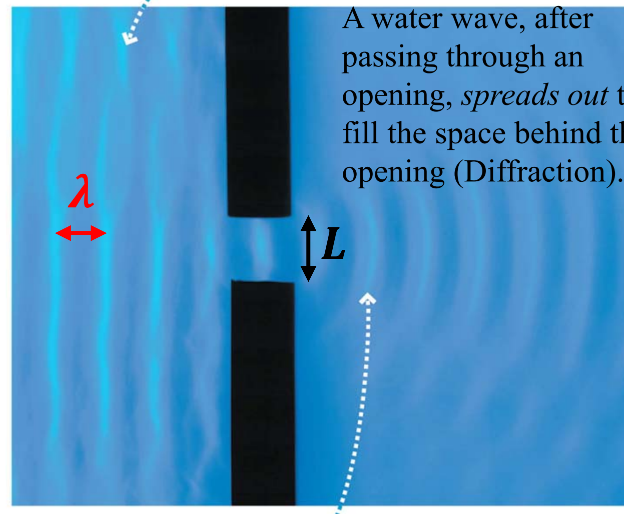
Models of Light

(Water waves are Easy to visualize)

The wave model: Under many circumstances, light exhibits the same behavior as sound or water waves. The study of light as a wave is called *wave optics***.**

$$(\boldsymbol{Wavelength})\ \lambda \sim L\ \ \textit{(object size)}$$

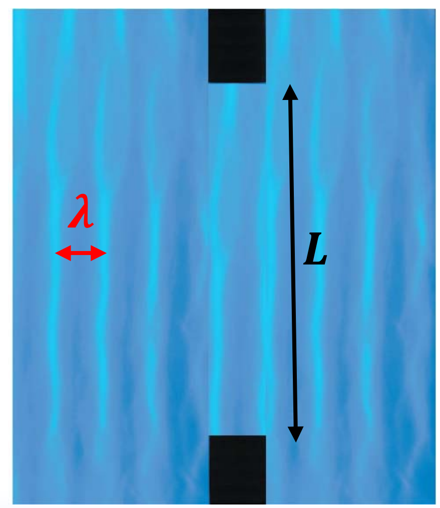
Plane waves approach from the left.



Circular waves spread out on the right.

The ray model: The properties of prisms, mirrors, and lenses are best understood in terms of *light rays*. The ray model is the basis of *ray optics*.

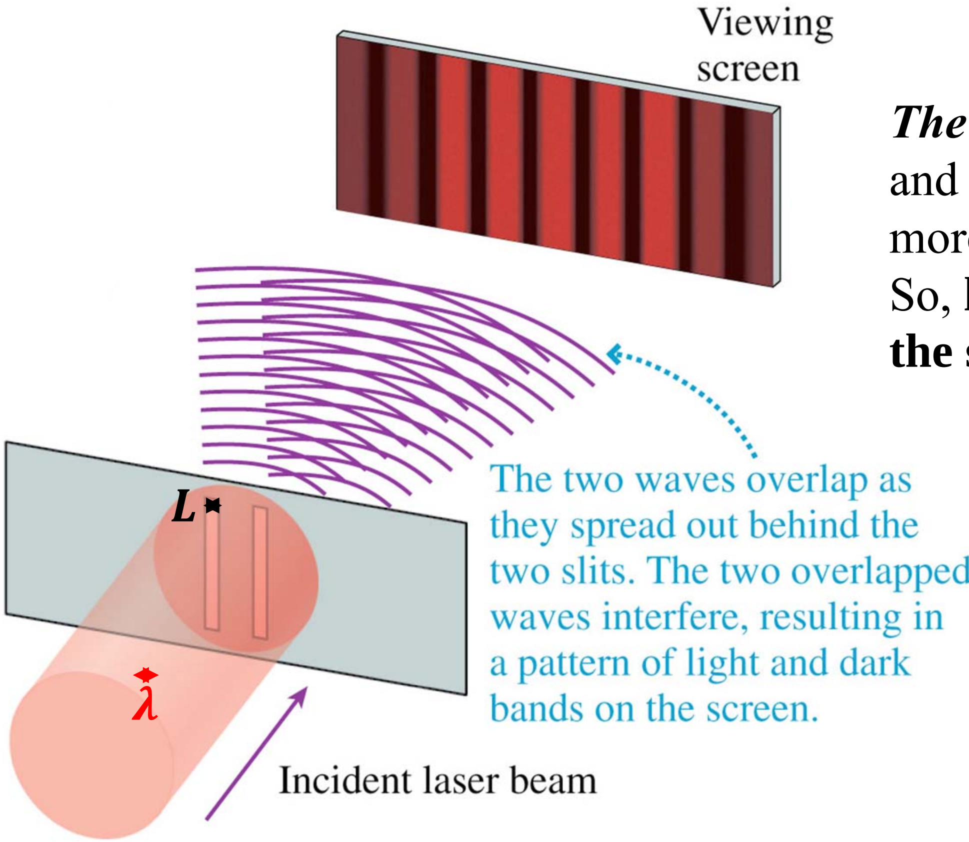
$$(\boldsymbol{Wavelength})\ \lambda \ll L\ \ \textit{(object size)}$$



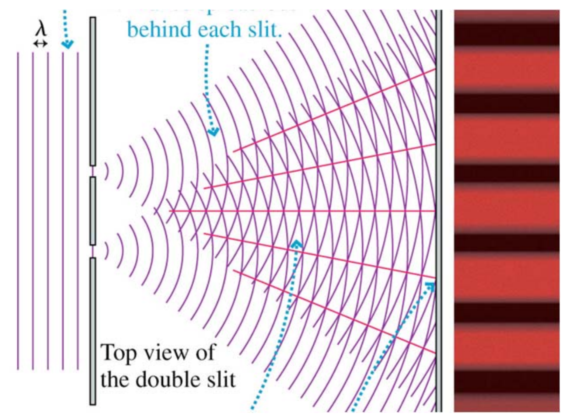
But today we will only focus on the wave optics.

Young's Double-Slit Experiment (*wave optics)*

In 1801, Thomas Young demonstrated that the wave theory of light was correct

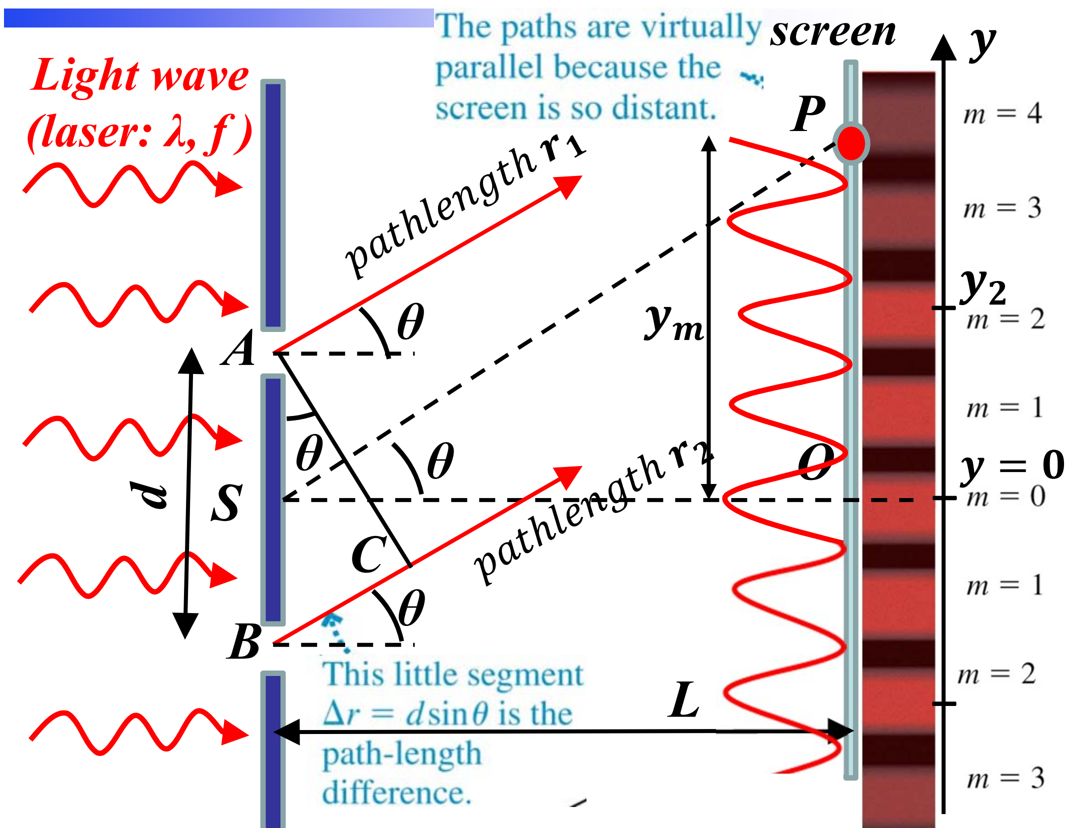


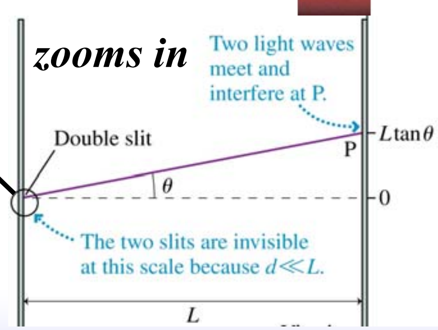
The IDEA: The whole space is filled with bright and dark spots (C/D interference). But it is much more convenient to see these spots on a screen. So, **let's rewrite our interference equations for the screen.**



(***Wavelength***) $\lambda \sim L$ ***(object size)*** ⇨ *wave optics*

Analyzing Double-Slit Interference





Since L>>d, the paths from the two slits at angle θ are <u>ALMOST parallel</u> $(r_1 \uparrow\uparrow r_2)$

$$\boxed{\Delta\varphi = \frac{2\pi}{\lambda}\Delta r + \cancel{\Delta\varphi_0}^{\,0}} = m2\pi$$

Inherent phase difference: $\Delta\varphi_0 = 0$

Constructive if $\Delta r = m\lambda$

The wave from the lower slit travels an extra distance: $\Delta\, ABC$

$$\Delta r = d\sin\theta_m = m\lambda$$

(angles of bright fringes)

It's easier to measure distances than angles: replace θ → y

$$\Delta\, SPO: \quad \boxed{\begin{aligned} \tan\theta_m &= \frac{y_m}{L} \\ d\sin\theta_m &= m\lambda \end{aligned}}$$

Since angles are small: $\sin\theta_m \approx \theta_m$, $\tan\theta_m \approx \theta_m$

$$\frac{y_m}{L} = \frac{m\lambda}{d} \Rightarrow \boxed{y_m = m\frac{\lambda L}{d}}, \quad m = 0, 1, 2,$$

Conditions for constructive interference

(positions of bright fringes)

Analyzing Double-Slit Interference (cont.)

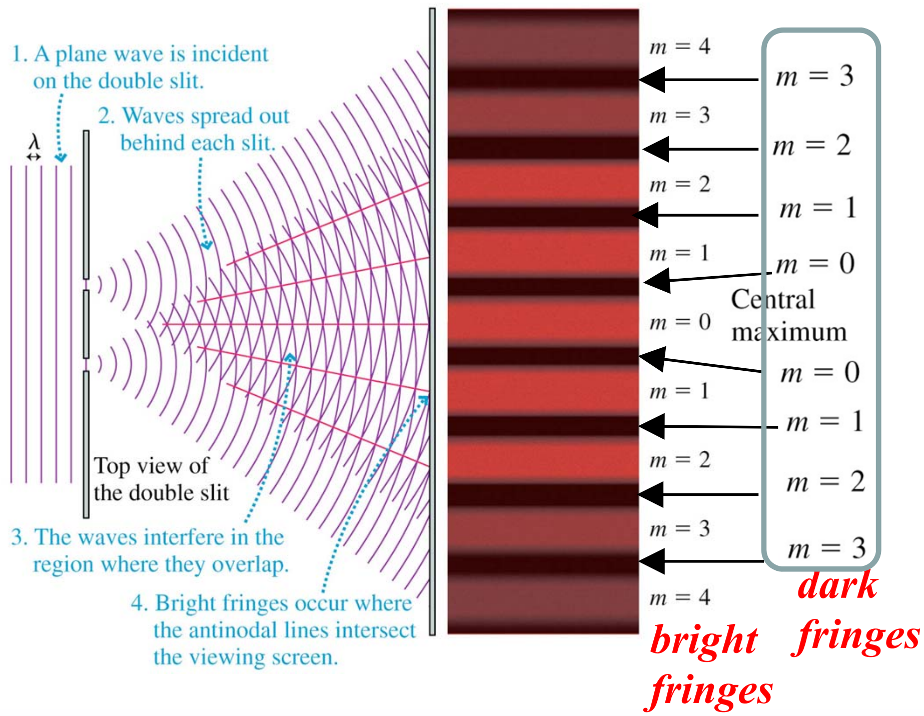
Similar for dark fringes $\text{Destructive if} \quad \Delta r = (m + \frac{1}{2})\lambda$

$$y_m = (m + \frac{1}{2})\frac{\lambda L}{d}, \quad m = 0,1,2,\ldots$$

Conditions for destructive interference (positions of dark fringes)

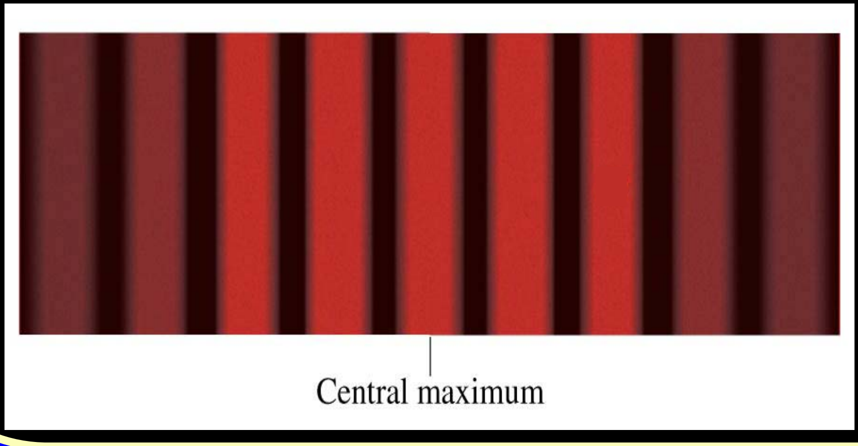
Let's find distance between fringes: $\Delta y_m = y_{m+1} - y_m = (m+1)\frac{\lambda L}{d} - m\frac{\lambda L}{d} = \frac{\lambda L}{d}$

There is no dependence on m, so they are equally spaced



ConcepTest Double-Slit Interference I

A laboratory experiment produces a double-slit interference pattern on a screen. If the slits are moved closer together, the bright fringes will be



A) Closer together

B) In the same positions

C) Farther apart

D) There will be no fringes because the conditions for interference won't be satisfied.

distance between fringes

$$\Delta y = \frac{\lambda L}{d} \text{ and } d \text{ is smaller, so } \Delta y \text{ Is larger}$$

Intensity of the Double-Slit Interference

Remember (Ch.34) that the intensity of a wave is proportional to square of a wave amplitude

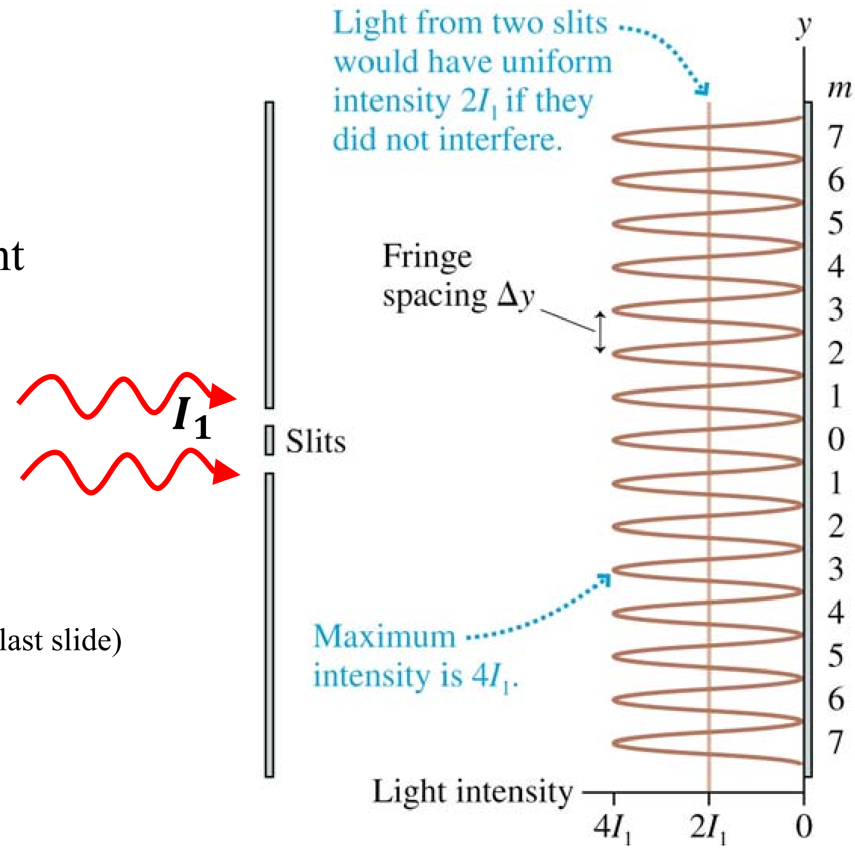
$$\boxed{I = \frac{E_0^{\,2}}{2c\mu_0}} \Rightarrow I \sim A^2$$

We have two waves in the double-slit experiment and an expression for an amplitude

$$A = \left|2a\cos\left[\frac{\Delta\varphi}{2}\right]\right|$$

$$I_{\text{double}} = 4I_1 \cos^2\left(\frac{\pi d}{\lambda L}y\right)$$

(For a derivation see the last slide)



$$I_{max} = 4I_1$$

So, the intensity is *quadrupled*.

(At the expense of what did we quadrupole the intensity?)

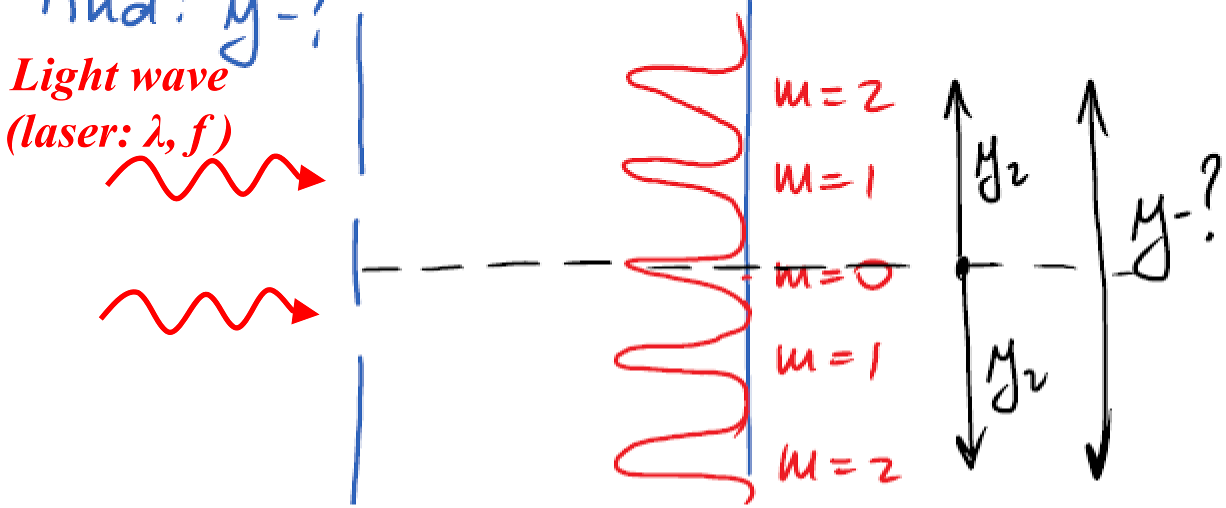
Example with a demonstration

EXAMPLE 22.1 Double-slit interference of a laser beam

Light from a helium-neon laser ($\lambda = 633$ nm) illuminates two slits spaced 0.5 mm apart. A viewing screen is 2.5m behind the slits. What are the distances between the two $m = 2$ bright fringes ?

Given: $\lambda = 633\ nm$; $d = 0.50 \cdot 10^{-3}\ m$; $L = 2.5\ m$; $m = 2$

Find: y - ?



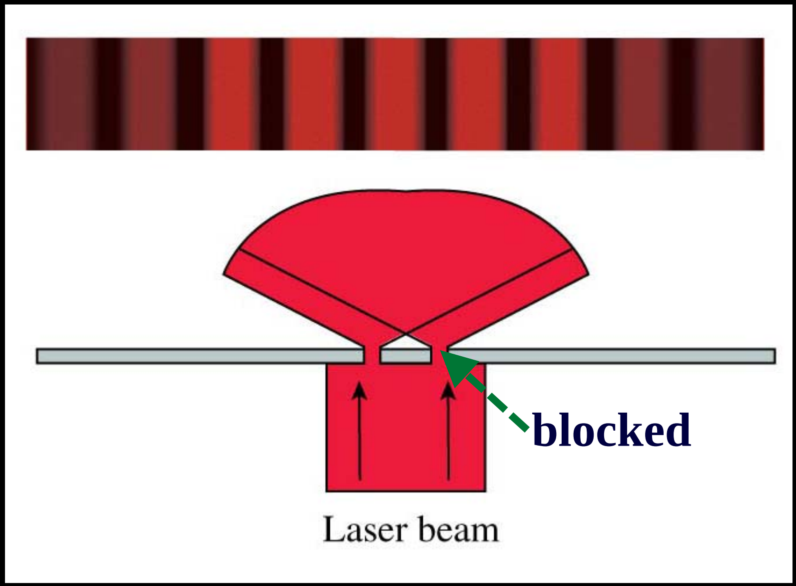
Eq. 22-6: $y_m = \dfrac{m \lambda L}{d}$

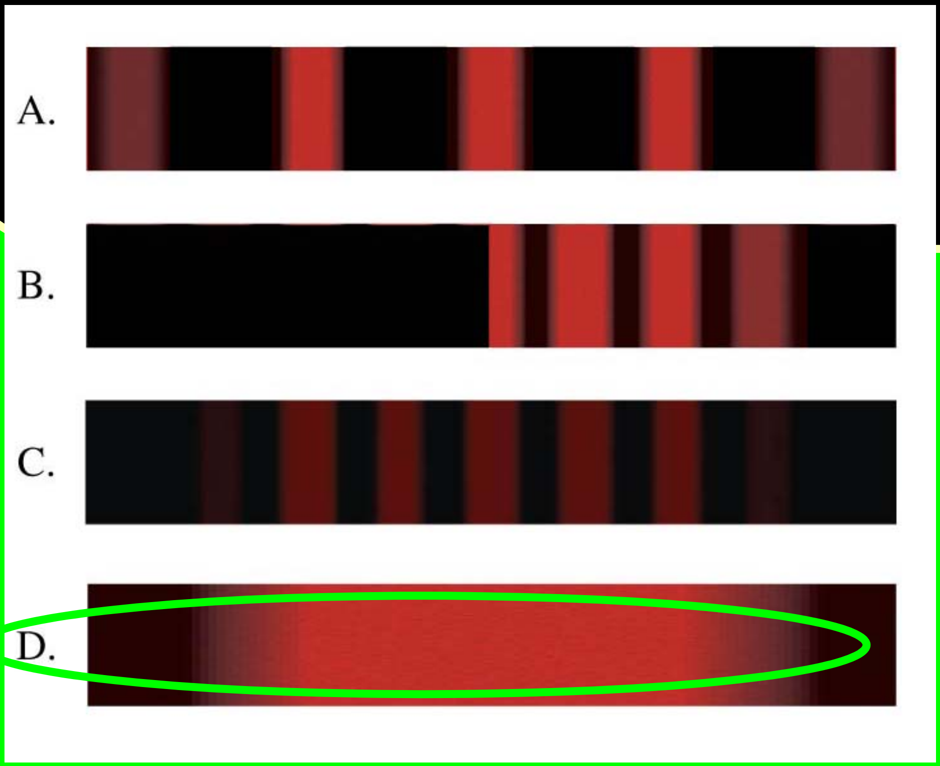
$$y_2 = \frac{2 \lambda \cdot L}{d} = \frac{2 \cdot 633 \cdot 10^{-9}\ m \cdot 2.5\ m}{0.50 \cdot 10^{-3}\ m} = 6.3 \cdot 10^{-3}\ m = 6.3\ mm$$

$$y = 2 \cdot y_2 = 12.6\ mm$$

ConcepTest Double-slit interference III

A laboratory experiment produces a double-slit interference pattern on a screen. If the left slit is blocked, the screen will look like





Diffraction Grating

Let's improve (more convenient to use) results of a double-slit system. How?

Spacing between bright spots: $\Delta y_m = \frac{\lambda L}{d}$

We saw in the demo that the spacing between bright spots is inconveniently small (~ mm), but we can ***increase the spacing by reducing d***

Intensity of bright spots:

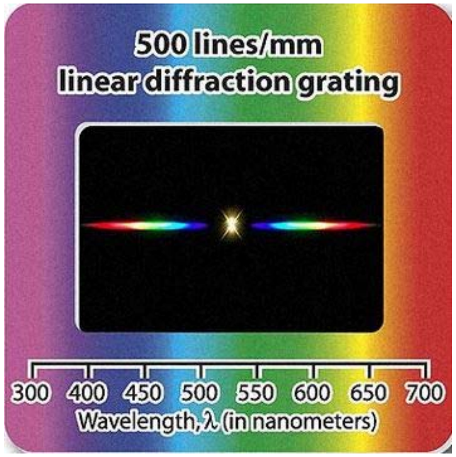
$$I_{double\ slit} = 2^2 I_1 \cos^2[\frac{\pi d}{\lambda L} y]$$

Number of slits

We saw in the demo that the intensity of the bright spots is not bright enough, but we can ***increase brightness by increasing number of slits (N)***

Thus, we can replace the double slit with an opaque screen that has N closely spaced slits.

A large number of equally spaced parallel slits is called a diffraction grating.



The Diffraction Grating

- The figure shows a diffraction grating in which *N* slits are equally spaced a distance *d* apart.

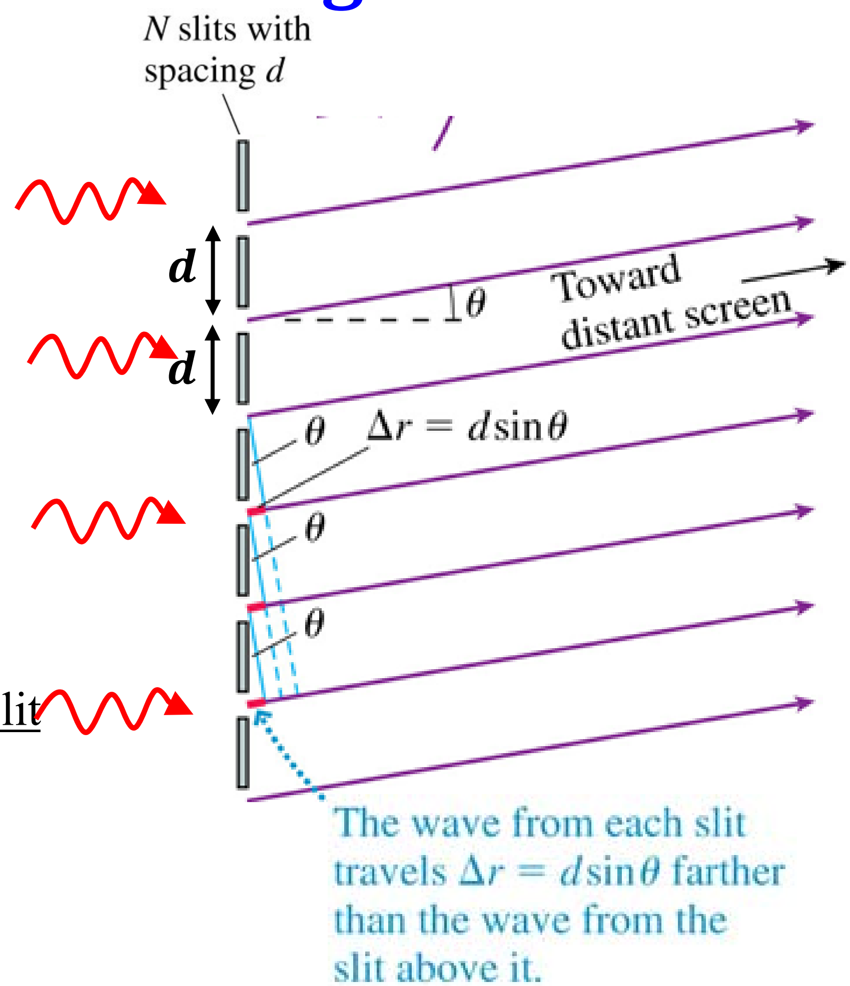
- When illuminated from one side, each of these slits becomes the source of a light wave that diffracts, or spreads out, behind the slit. A practical grating will have hundreds or even thousands of slits.

- <u>Physics and math are the same as for a double-slit experiment</u>

$$\Delta\varphi = \frac{2\pi}{\lambda}\Delta r + \cancel{\Delta\varphi_0}\ 0$$

- Bright fringes will occur at angles θ_m, such that:

$$\Delta r = d\sin\theta_m = m\lambda \qquad m = 0, 1, 2, 3, \ldots$$

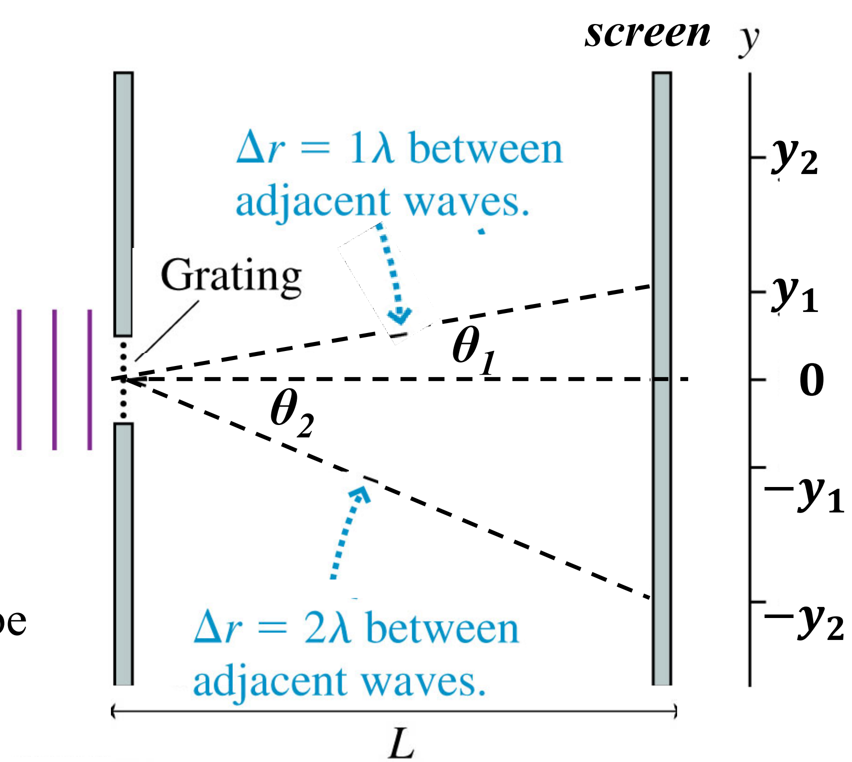


The Diffraction Grating

It's easier to measure distances on the screen than angles: $\boldsymbol{\theta \rightarrow y}$

The y-positions of these fringes will occur at:

$$y_m = L \tan\theta_m$$ (positions of bright fringes)



$$\boxed{\begin{aligned} d\sin\theta_m &= m\lambda \\ y_m &= L\tan\theta_m \end{aligned}}$$

$m = 0, 1, 2, 3, \ldots$

The integer m is called the **order** of the diffraction.

However, these angles are NOT small and cannot be simplified like we did for 2slit experiment.

Thus, using these two equations we can find positions of the bright spots.

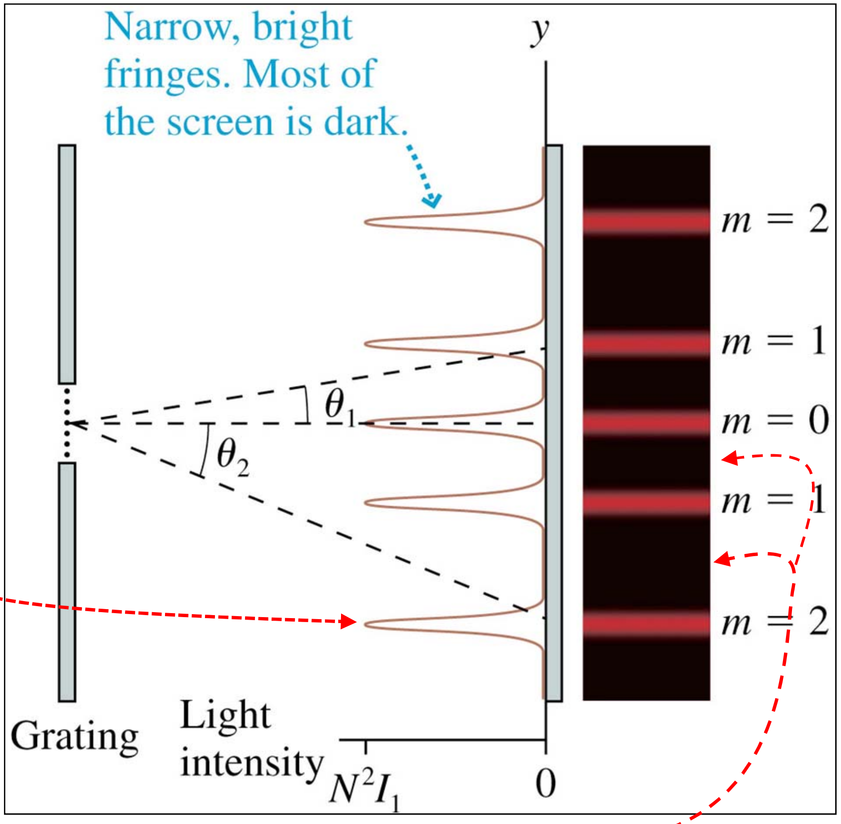
Bright spot intensity

Now with N slits, the wave amplitude at the points of constructive interference is ***Na***.

Because intensity depends on the square of the amplitude, the intensities of the bright fringes are:

$$\boldsymbol{I_{max} = N^2 I_1}$$

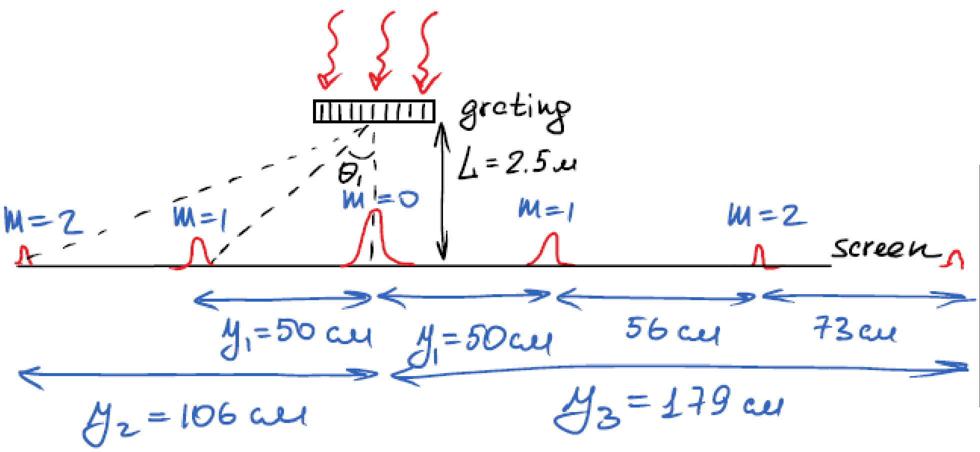
Not only do the fringes get brighter as N increases, they also get narrower.



The bright spots are no longer equally spaced.

Measuring wavelength emitted by a diode laser *(with a demonstration)*

Light from a diode laser passes through a diffraction grating having 300 slits per millimeter. The interference pattern is viewed on a wall 2.5 m behind the grating. Calculate the wavelength of the laser.



$$\begin{cases} d\cdot\sin\theta_m = m\lambda \\ y_m = L\cdot\tan\theta_m \end{cases} \qquad m = 1;\ y_1 = 0.5\,m$$

$$\tan\theta_1 = \frac{y_1}{L} = \frac{0.5\,m}{2.5\,m} \Rightarrow \theta_1 = 0.197\ \text{rad}$$

What is d? If a 1 mm length of the grating has 300 slits, then the spacing between slits

$$d = \left(\frac{1}{300}\right) mm = 0.33\cdot 10^{-5}\,m$$

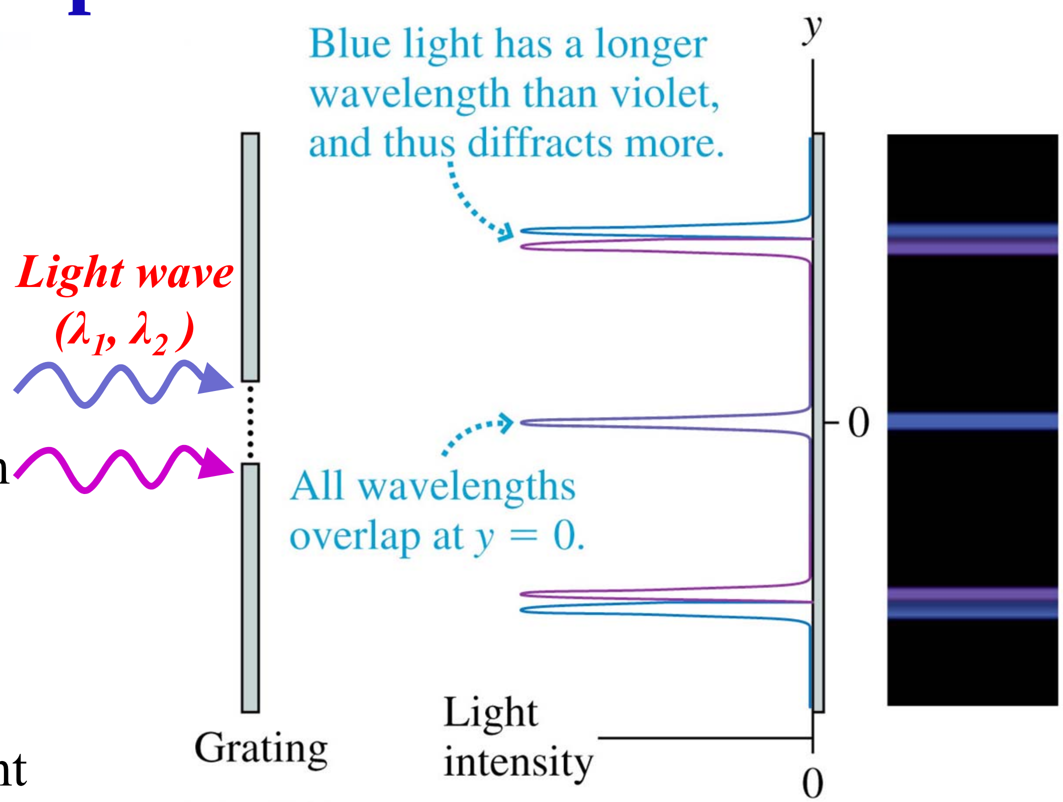
$$\lambda = d\cdot\sin\theta_1 = 0.33\cdot 10^{-5}\,m \cdot \sin(0.197) =$$

$$\approx 0.0653\cdot 10^{-5}\,m = 653\ nm$$

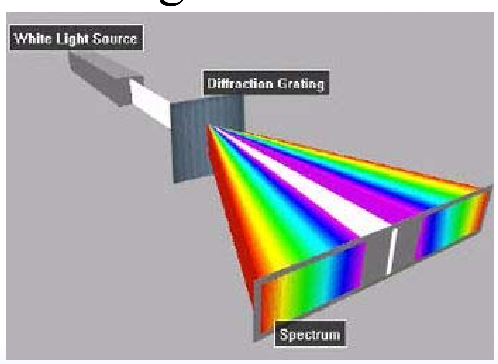
Grating Spectrometer

$$\boxed{\begin{aligned} d\sin\theta_m &= m\lambda \\ y_m &= L\tan\theta_m \end{aligned}} \qquad m = 0, 1, 2, 3, \ldots$$

- If the incident light consists of two slightly different wavelengths, each wavelength will be diffracted at a slightly different angle.

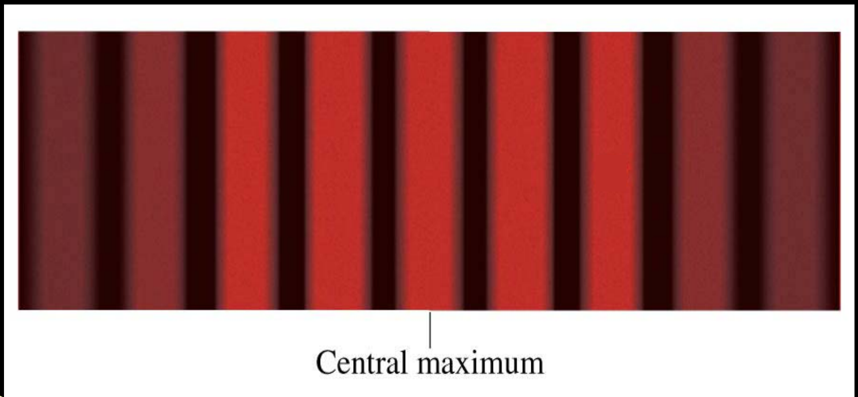


- Diffraction gratings are used for measuring the wavelengths of light (Grating Spectrometer)



ConcepTest Double-Slit Interference II

A laboratory experiment produces a double-slit interference pattern on a screen. If the screen is moved farther away from the slits, the fringes will be



A) Closer together

B) In the same positions

C) Farther apart

D) Fuzzy and out of focus.

distance between fringes

$\Delta y = \frac{\lambda L}{d}$ and d is smaller, so Δy Is larger

What you should read

Chapter 22 (Knight)

Sections

- ***22.1***
- ***22.2***
- ***22.3***

Thank you

See you on Tuesday

Intensity of the Double-Slit Interference

Remember (Ch.34) that the intensity of a wave is proportional to square of a wave amplitude

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} \Rightarrow S_{aver} = \boxed{I = \frac{E_0{}^2}{2c\mu_0}} \Rightarrow I = bA^2$$

We got in the previous class $A = \left|2a\cos\left[\frac{\Delta\varphi}{2}\right]\right|$

$$\boxed{\Delta\varphi = \frac{2\pi}{\lambda}\Delta r + \cancelto{0}{\Delta\varphi_0}} = \frac{2\pi}{\lambda}\frac{dy}{L}$$

the previous slide: $\Delta r = d\sin\theta_m \approx d\theta_m \approx d\tan\theta_m = \frac{d\,y}{L}$

$$A = \left|2a\cos\left[\frac{\pi d}{\lambda L}y\right]\right|$$

$$I = bA^2 = 4\overbrace{ba^2}^{=I_1}\cos^2[\frac{\pi d}{\lambda L}y] = 4I_1\cos^2[\frac{\pi d}{\lambda L}y]$$

The intensity of the double-slit interference pattern at position y is: $\boxed{I_{\text{double}} = 4I_1\cos^2\left(\frac{\pi d}{\lambda L}y\right)}$