Lecture 4

Chapter 24

Conductors in Electrostatic Equilibrium

Course website:
http://faculty.uml.edu/Andriy_Danylov/Teaching/PhysicsII

Department of Physics and Applied Physics
Today we are going to discuss:

Chapter 24:

- Some leftovers from Gauss’s Law
- Section 24.6 Conductors in electrostatic equilibrium
- Skip (Example 24.8)
A total charge $Q$ is spread uniformly throughout a dielectric sphere of radius $R$. What is the electric field outside the sphere ($r>R$)?

Notice a remarkable feature of this result: The field outside the sphere is exactly the same as it would have been if all the charge had been concentrated at the center.

Example 24.3 Uniformly charged dielectric sphere of radius $R$. (Spherical symmetry)

a) Electric field outside the sphere. Apply Gauss's law.

$$
\phi E \cdot dA = \frac{Q_{\text{in}}}{\varepsilon_0}
$$

Gaussian Surface

We have spherical symmetry, so it makes sense to use a sphere $r>R$ as a Gaussian surface. The integration is trivial with this symmetry.

$$
\phi E \cdot dA = \int E \cdot dA = \int E \cdot dA = \frac{Q_{\text{in}}}{\varepsilon_0}
$$

So

$$
E \cdot 4\pi r^2 = \frac{Q_{\text{in}}}{\varepsilon_0}
$$

$Q_{\text{in}}$ - all charges inside the Gaussian surface. The whole charge of sphere is inside. So

$$
Q_{\text{in}} = \frac{Q}{\varepsilon_0} (\text{total charge})
$$

$$
E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} \Rightarrow E(r) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}
$$

It looks like a point charge.
A total charge $Q$ is spread uniformly throughout a dielectric sphere of radius $R$. What is the electric field inside the sphere ($r<R$)?

**Example 24.4** Uniformly charged dielectric sphere of radius $R$. (Spherical symmetry)

a) Electric field inside the sphere.

Apply Gauss's law:

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{in}}}{\varepsilon_0}$$

Gaussian Surface

We have spherical symmetry, so it makes sense to use a sphere $r<R$ as a Gaussian surface. The integration is trivial with this symmetry.

$$\oint \mathbf{E} \cdot d\mathbf{A} = \int \mathbf{E} \cdot d\mathbf{A} = \int_{\text{Gaussian surface}} \mathbf{E} = \int_{\text{Gaussian surface}} E \cdot dA$$

So

$$E \cdot 4\pi r^2 = \frac{Q_{\text{in}}}{\varepsilon_0}$$

$Q_{\text{in}}$ -- all charges inside the Gaussian surface of radius $r<R$.

$$Q_{\text{in}} = \rho \cdot V_{\text{Gauss}} = \rho \cdot \frac{4}{3} \pi r^3 = \left( \frac{Q}{\varepsilon_0} \right) \left( \frac{4}{3} \pi r^3 \right)$$

Volume charge density

$$Q_{\text{in}} = \frac{Q}{\varepsilon_0} \frac{r^3}{R^3}$$

Electric field inside the charged sphere

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} \frac{r^3}{R^3} \Rightarrow E(r) = \frac{Q}{4\pi \varepsilon_0 R^3} r$$

Let's plot it
Find the electric field of an infinitely long line of charge with linear charge density $\lambda$ (C/m).

**Example 24.4** Apply Gauss's Law

This is an example with a cylindrical symmetry where the electric field lines point out from the wire.

Because of the symmetry, we'll use a cylinder of radius $R$ and length $L$ as a Gaussian surface.

\[
\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}
\]

Because \( \mathbf{E} \) is radial, the surface integral is simplified.

\[
\begin{align*}
\oint \mathbf{E} \cdot d\mathbf{A} &= \oint E \, dA \\
S_1 &\quad \text{front surface} \\
S_2 &\quad \text{front surface} \\
S_3 &\quad \text{back surface}
\end{align*}
\]

\[
\begin{align*}
\int E \, dA &= E \int dA \\
S_1 &\quad \text{front surface} \\
S_2 &\quad \text{front surface} \\
S_3 &\quad \text{back surface}
\end{align*}
\]

Now, we need $Q_{\text{enclosed}}$ (total charge enclosed by the Gauss surface).

\[
Q_{\text{enclosed}} = \lambda \cdot L
\]

Finally,

\[
E = \frac{\lambda}{2\pi \varepsilon_0 R}
\]
Example: E field inside of a long, charged wire

Find the electric field of an infinitely long line of charge with volume charge density \( \rho \) (C/m).

Problem 24.50(b)

Let's consider a Gaussian surface (closed) inside the charged cylinder. We'll divide it into three surfaces:
- \( S_1 \): cylinder wall
- \( S_2 \): front face
- \( S_3 \): back face

The electric flux through each surface is:

- For the cylinder wall:
  \[ \mathbf{E} \cdot d\mathbf{A} = \frac{8\pi \rho}{\varepsilon_0} \]

- For the front and back faces:
  \[ \mathbf{E} \cdot d\mathbf{A} = E(2\pi r L) = \frac{8\pi \rho}{\varepsilon_0} \]

Combining these, we get:

\[ E = \frac{\rho}{2\pi \varepsilon_0} \]

Thus, the electric field inside a long, charged wire is:

\[ E(r) = \frac{\rho r}{2\pi \varepsilon_0} \]

This answer matches the boundary condition as it should be:

\[ \text{Bin} = \rho (\frac{r^2 L}{2}) = \frac{1}{4} \]

\[ \rho = \frac{1}{4r^2} \]
Now let’s look at properties of Conductors using Gauss’s law
Electric field inside a conductor

Consider a conductor and apply an external electric field.

Conductor has tons of free electrons and under the influence of $E_{\text{ext}}$ they will run to the left surface leaving positive charges near the right surface and creating $E_{\text{internal}}$.

How many of them will move?
- The electrons will keep moving until the internal field cancels out the external field inside the conductor.

Thus, the electric field inside a conductor is zero in electrostatic situation.
Charges in a conductor

Consider a positively charged conductor

Where does this excess charge reside in the conductor?

Let’s apply Gauss’s law

Take a Gauss. surface just barely inside the surface of a conductor

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} \]

So \( Q_{in} = 0 \) (inside the Gaussian surface)

Thus, the positive excess charge resides on the external surface of the conductor

In simple words: They just repel each other 😎

The electric field inside the conductor is zero. There’s no net charge inside the conductor. Hence all the excess charge is on the surface.
E (outside) is \perp to the surface of a conductor.

The external electric field right at the surface of a conductor must be perpendicular to that surface.

If it were to have a tangential component, it would exert a force on the surface charges and cause a surface current, and the conductor would not be in electrostatic equilibrium (Proof by contradiction).
E near any conducting surface

Consider a conductor with the surface charge density \( \rho \).

Apply Gauss's law:
\[ \oint E \cdot d\mathbf{A} = \frac{Q_{\text{in}}}{\varepsilon_0} \]

A small cylinder perpendicular to the surface is a Gaussian surface, which consists of three open surfaces, so

\[ \oint E \cdot d\mathbf{A} = \oint E \cdot d\mathbf{A} + \oint E \cdot d\mathbf{A} + \oint E \cdot d\mathbf{A} = E \cdot A = \frac{Q_{\text{in}}}{\varepsilon_0} \]

front facet: \( E \uparrow \uparrow \mathbf{A} \)
back facet: \( E \downarrow \downarrow \mathbf{A} \)
side: \( \mathbf{E} \perp d\mathbf{A} \)

\( Q_{\text{in}} = \rho \cdot A \), so
\[ E \cdot A = \frac{\rho \cdot A}{\varepsilon_0} \Rightarrow E = \frac{\rho}{\varepsilon_0} \]

and, as we know, it's \( \perp \) to the surface.
A point charge $q$ is located distance $r$ from the center of a neutral metal sphere. The electric field at the center of the sphere is:

- A) $\frac{q}{4\pi \varepsilon_0 r^2}$
- B) $\frac{q}{4\pi \varepsilon_0 R^2}$
- C) $\frac{q}{4\pi \varepsilon_0 (R-r)^2}$
- D) 0
- E) It depends on what the metal is.
Charged conductor with a hole inside

Are there any charges on an interior surface?

Let’s apply Gauss’s law

Place a Gaussian surface around the hole.

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} = 0 \]

The electric flux is zero through the Gaussian surface, since \( E = 0 \) inside the conductor.

So \( Q_{in} = 0 \) (inside the Gaussian surface), i.e. there is no charge on the surface of the hole.

Any excess charge resides on the exterior surface of a conductor, not on any interior surfaces.
The use of a conducting box, or Faraday cage, to exclude electric fields from a region of space is called **screening**.

Sensitive instruments are often enclosed in a "Faraday cage" to shield them from unwanted radio frequencies (electromagnetic waves). The field pushes electrons toward the left, leaving a net negative charge on the left side and a net positive charge on the right side. The result is that the net electric field inside the box is zero.
Screening/Faraday cage II

Shielding essentially prevents electromagnetic pulses from interfering with the proper functioning of electronics.

- conductive fabrics
- metallic inner shields
- Vacuum metallization
- Conductive paints

A Faraday cage protects its contents by preventing electromagnetic energy from getting inside.
Let’s put a charge inside the hole

Are there any charges on an interior surface?

Charged (+Q) conductor

Let’s apply Gauss’s law

Place a Gaussian surface around the hole.

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} = 0 \]

The electric flux is zero through the Gaussian surface, since \( E=0 \) inside the conductor.

So \( Q_{in} = 0 \) (the net charge inside the Gaussian surface)

But, we know that there is \(+q\) inside, it means that there must be \(-q\) on the interior surface

(+q charge induced –q on the surface)

Let’s count all charges inside the conductor to find the amount of charge on the exterior surface

\[ -q + Q_{outer\ surface} = +Q \]

\[ Q_{outer\ surface} = +Q + q \]

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Charge $+3 \text{ nC}$ is in a hollow cavity inside a large chunk of metal that is electrically neutral. The total charge on the exterior surface of the metal is

A) $0 \text{ nC}$.
B) $+3 \text{ nC}$.
C) $-3 \text{ nC}$.
D) Can’t say without knowing the shape and location of the hollow cavity.
An uncharged conducting sphere of radius $R$ contains two spherical cavities. Point charge $Q_1$ is placed within the first cavity (not necessarily at the center) and $Q_2$ is placed within the second one. Find the charge on the outer surface.
Thank you
See you next time
Find the electric field of an infinite nonconducting plane of charge with surface charge density \( \eta \) (C/m\(^2\)).