Lecture 10

Chapter 6

Gravitation and Newton’s Synthesis

Course website:
http://faculty.uml.edu/Andriy_Danylov/Teaching/PhysicsI

Lecture Capture:
http://echo360.uml.edu/danylov2013/physics1spring.html
Exam I.

Wed Feb. 26; 9 - 9.50 am; Olney 150

- Have your student ID
- Remember your recitation section
- Calculators are allowed
- A sheet with formulae will be provided

Students, who need extended time, will be in Olney 136D from 8.30 till …

Review Session Mon 6-8:30 pm, Olney 204
Outline

Chapter 6

- Newton’s Law of Universal Gravitation
- Kepler’s Laws
Newton’s Law of Universal Gravitation

A gravitational force of attraction exists between all objects that have mass.

Gravitational force is central, attractive, proportional to masses, and inversely proportional to the square of the distance.

\[ \vec{F} = -\left( \frac{GmM}{r^2} \right) \hat{r} \]

\[ G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \]

universal gravitational constant
The magnitude of the gravitational constant $G$ can be measured in the laboratory.

100 years later in 1798, Cavendish measured gravitational force between two known masses using a “torsion balance” and calculated $G$. $G$ is the same for all materials, universal

$$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$$
Gravitational attraction to another person

Estimate the attractive Force of Gravity between two average persons 1 m apart.

\[ F = \frac{G m_1 m_2}{R^2} = \frac{6.67 \times 10^{-11} \text{Nm}^2}{\text{kg}^2} \times 70\text{kg} \times 70\text{kg}}{1\text{m} \times 1\text{m}} = 3.3 \times 10^{-7} \text{N} \]

Compare this to the weight (700 N) of each person.

So, Gravitational force between two “small objects” can often be neglected.

Gravitational forces between two objects are only significant when at least one of the objects is very massive.
Superposition of gravitational forces

If there are many particles, the total force is the vector sum of the individual forces:

\[ \vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \cdots + \vec{F}_{1n} = \sum_{i=2}^{n} \vec{F}_{1i}. \]

Example:
Three objects, of equal mass, are at three corners of a square. Draw the gravitational force vectors acting on each block.
ConcepTest 1 Force Vectors

A planet of mass \( m \) is a distance \( d \) from Earth. Another planet of mass \( 2m \) is a distance \( 2d \) from Earth. Which force vector best represents the direction of the total gravitation force on Earth?

The force of gravity on the Earth due to \( m \) is greater than the force due to \( 2m \), which means that the force component pointing down in the figure is greater than the component pointing to the right.

\[
\vec{F} = -\left(\frac{GmM}{r^2}\right)\hat{r}
\]

\[
F_{2m} = \frac{GM_E(2m)}{(2d)^2} = \frac{1}{2} \frac{GMm}{d^2}
\]

\[
F_m = \frac{GM_E m}{d^2} = \frac{GMm}{d^2}
\]
Gravity near the Earth’s Surface

What should we get for the acceleration due to gravity at the Earth’s surface?

Let’s apply Newton’s 2nd law for gravitational force: \( F = \frac{GM_E m}{r^2} \)

\[ F = ma \]

\[ ma = \frac{GM_E m}{r^2} \]

Denote the acceleration due to gravity as:

\[ a = g(r) = \frac{GM_E}{r^2} \]

If \( r = R_E \) (on the surface):

\[ g = \frac{GM_E}{R_E^2} = 9.80 \text{ m/s}^2 \]

\[ M_{\text{EARTH}} = 5.98 \times 10^{24} \text{ kg} \]

\[ R_{\text{EARTH}} = 6.38 \times 10^6 \text{ m} \]

\[ G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \]

\[ F = \frac{GM_E m}{R_E^2} = mg \]
Variations in “g” on the surface

The acceleration due to gravity varies over the Earth’s surface due to altitude, local geology, and the shape of the Earth, which is not quite spherical.

<table>
<thead>
<tr>
<th>Location</th>
<th>Elevation (m)</th>
<th>$g$ (m/s$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>New York</td>
<td>0</td>
<td>9.803</td>
</tr>
<tr>
<td>San Francisco</td>
<td>0</td>
<td>9.800</td>
</tr>
<tr>
<td>Denver</td>
<td>1650</td>
<td>9.796</td>
</tr>
<tr>
<td>Pikes Peak</td>
<td>4300</td>
<td>9.789</td>
</tr>
<tr>
<td>Sydney, Australia</td>
<td>0</td>
<td>9.798</td>
</tr>
<tr>
<td>Equator</td>
<td>0</td>
<td>9.780</td>
</tr>
<tr>
<td>North Pole (calculated)</td>
<td>0</td>
<td>9.832</td>
</tr>
</tbody>
</table>
Two objects are dropped simultaneously from 2m above the ground at the top of Mount Everest and at sea level. Which hits the ground first?

A) Object at sea level
B) Object on Mt. Everest
C) They hit simultaneously

The acceleration due to gravity varies over the Earth’s surface due to altitude and it is smaller on Mt. Everest.
Newton’s Cannon on a Mountain

http://waowen.screaming.net/revision/force&motion/ncananim.htm
Geosynchronous Orbit

- A satellite in a *geosynchronous* orbit stays at the same position with respect to Earth as it orbits.
- Such satellites are used for TV and radio transmission, for weather forecasting, and as communication relays.

http://sphere.ssec.wisc.edu/images/geo_orbit.animated.gif
Geosynchronous Orbit

To calculate the height of a satellite of mass $m$ in geosynchronous orbit:

Gravitational force provides centripetal acceleration

$$ F = \frac{GM_E m}{R^2} $$

Satellite’s velocity

$$ v = \frac{2\pi R}{T} $$

$$ F = \frac{mv^2}{R} = m\left(\frac{2\pi R}{T}\right)^2 $$

$$ F = \frac{4\pi^2 R m}{T^2} = \frac{GM_E m}{R^2} $$

Distance from the Earth’s center

$$ T = 1 \text{ day} = 24 \text{ hrs} = 86,400 \text{s} $$

$$ R = 3\sqrt{\frac{GM_E T^2}{4\pi^2}} $$

Final caution: Height is often given by

$$ h = R - R_E $$

$R = 35,800 \text{ km}$
Satellite in an orbit just above ground level

If a cannonball is launched horizontally just above ground level, what speed would make it travel in a circular orbit at the surface of the Earth?

(Radius of earth: 6400 km, acceleration due to gravity at earth’s surface: 10 m/s²)

Gravitational force provides centripetal acceleration

\[ F_G = mg = m \frac{v^2}{R_E} \]

\[ g = \frac{v^2}{R_E} \quad \Rightarrow \quad v = \sqrt{g R_E} \quad \Rightarrow \quad v = \sqrt{10 \times 6400 \times 10^3} = 8000 \text{ m/s} \]
“Weightlessness”

Weight is defined as the magnitude of the force of gravity on an object. At the surface of Earth this is \( mg \).

But we measure weight, by the force a mass exerts on, say, a spring scale.

The “weightlessness” in a freely falling elevator is because everything is accelerating equally independent of mass. So, there is no normal force.

She does have a gravitational force acting on her, though!
“Weightlessness”

- The “weightlessness” in orbit is exactly the same, everything is accelerating equally independent of mass.

- The satellite and all its contents are in free fall, except with a huge tangential velocity. So, there is no normal force. This is what leads to the experience of weightlessness.

They do have a gravitational force acting on them, though!
“Weightlessness”

More properly, this effect is called \textit{apparent} weightlessness, because the gravitational force still exists. It can be experienced on Earth as well, but only briefly:
Kepler’s Laws

A classic example of scientific advancement

Observations → pattern → model

Tycho Brahe 1546-1601

Johannes Kepler 1571-1630

Isaac Newton 1642-1727
Kepler’s Laws: 1

Planets move in planar elliptical paths with the Sun at one focus of the ellipse.


s → semi-major axis
b → semi-minor axis
Kepler’s Laws: 2

During equal time intervals the radius vector from the Sun to a planet sweeps out equal areas.

http://phys23p.sl.psu.edu/phys_anim/astro/kepler_2.avi
Kepler’s Laws: 3

If a planet makes one revolution around the Sun in time $T$, and if $R$ is the semi-major axis of the ellipse (R is radius if orbit circular):

$$T^2 \propto R^3$$

or

$$\frac{T^2}{R^3} = C$$

where the constant $C$ is the same for all planets.
Kepler’s Third Law

Look at circular orbit for simplicity

Gravitational force provides centripetal acceleration

\[ \frac{GMm}{r^2} = \frac{mv^2}{r} \]

Period of rotation

\[ T = \frac{2\pi r}{v} \]

\[ \frac{T^2}{r^3} = \frac{4\pi^2}{v^2} \]

Constant for all planets

\[ \frac{T^2}{r^3} = \frac{4\pi^2}{GM} \]
Thank you

See you on Wednesday
Astronauts in the space shuttle float because they are in “free fall” around Earth, just like a satellite or the Moon. Again, it is gravity that provides the centripetal force that keeps them in circular motion.
The Moon does not crash into Earth because of its high speed. If it stopped moving, it would, of course, fall directly into Earth. With its high speed, the Moon would fly off into space if it weren’t for gravity providing the centripetal force.

ConcepTest 3

Averting Disaster

The Moon does not crash into Earth because:

A) it’s in Earth’s gravitational field
B) the net force on it is zero
C) it is beyond the main pull of Earth’s gravity
D) it’s being pulled by the Sun as well as by Earth
E) none of the above