Lecture 12

Chapter 9

Work and Kinetic Energy

I am sick and tired of your forces!!!

Course website:
http://faculty.uml.edu/Andriy_Danylov/Teaching/PhysicsI
Today we are going to discuss:

**Chapter 9:**

- **Work done by a force:** *Section 9.2; 9.3*
- **Kinetic Energy:** *Section 9.2*
- **Scalar (Dot) product of two vectors:** *Section 9.3*
- **Work-Kinetic Energy theorem:** *Section 9.2*
Since the N. 2nd law is a foundation of our mechanics, we have to “dance” from the N. 2nd law if we want to introduce something new.

So, Let’s get work and energy from N. 2nd law

What is so special about solving problems using energies? Why do we have to go there?

N. 2nd law with forces is great and any problem can be solved. But forces are vectors and it is more difficult to deal with vectors than with scalar quantities. And energy is a scalar quantity.

So, let’s try to make life easier 😊

Read this derivation only if you want
Work-Kinetic Energy Principle

We started from the N. 2nd law

\[ F = ma \]

and got this

\[ \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \int_{x_i}^{x_f} F(x)dx \]

Let’s give “nice names” to new quantities

**Kinetic energy**

\[ K = \frac{1}{2} mv^2 \]

is the energy of motion in a line or trajectory

**Work done by F**

\[ W = \int_{x_i}^{x_f} F(x)dx \]

So, after renaming we have this:

\[ K_f - K_i = \Delta K = W_{net} \]

*The work done by the force is equal to the change in the kinetic energy.*

- Work and energy have units of Nm, or Joules (J), and are **scalars**!!
**Example**

Net Work required to accelerate a 1000kg car from

(a) from 20 m/s to 40 m/s?

\[
W = \Delta K = \frac{m}{2} \left[ v_f^2 - v_i^2 \right] = \frac{1000}{2} \left[ 40^2 - 20^2 \right] = 600,000J = 600kJ
\]

(b) from 40 m/s to zero?

\[
W = \Delta K = \frac{m}{2} \left[ v_f^2 - v_i^2 \right] = \frac{1000}{2} \left[ 0 - 40^2 \right] = -800,000J = -800kJ
\]

➢ If the net work is positive, the kinetic energy increases.
➢ If the net work is negative, the kinetic energy decreases.
Let’s look at work “deeper”

As you noticed, I like to simplify, let’s do it again: assume a **constant force**

\[
W = \int_{x_i}^{x_f} F(x) \, dx = F \int_{x_i}^{x_f} dx = F(x_f - x_i) = Fs
\]

This is true when a force acts along a line of motion, but what if a force makes an angle with a line of motion (more general case)?

**Work done on an object by a force**

\[
W = Fs \cos \theta
\]
Is it possible to do work on an object that remains at rest?

A) yes  
B) no

Work requires that a force acts over a distance. If an object does not move at all, there is no displacement, and therefore no work done (boss’s attitude).

The amount of work you actually do may have little relationship to the amount of effort you apply. For example, if you push on a car stuck in a snow drift, you may exert a lot of force (and effort) but if the car does not budge, you have not done any work! In order for work to be done on an object, the object must move some distance as a result of the force you apply.
A force of 5 N, applied in the direction shown, moves a donkey across a horizontal distance of 10 m. How much work does the force do on the stubborn creature?

By definition

\[ W = F \cdot s \cdot \cos \theta \]

\[ W = 5N \cdot (10m) \cos 60^\circ = 25J \]

Work done by a force can be positive/negative/zero:
Friction acts in the **opposite** direction to the displacement, so the work is **negative**. Or using the definition of work \( W = F \cdot s \cos \theta \), because \( \theta = 180^\circ \), then \( W < 0 \).

**Friction and Work I**

A box is being pulled across a rough floor at a constant speed. What can you say about the work done by friction?

A) friction does no work at all
B) friction does negative work
C) friction does positive work

**ConcepTest**

Friction and Work I
A baggage handler throws a 15 kg suitcase along the floor of an airplane luggage compartment with a speed of 1.2 m/s. The suitcase slides 2.0 m before stopping. Use work and energy to find the suitcase’s coefficient of kinetic friction on the floor.

**Example Problem 9.33**

Using the Work-Kinetic Energy Principle:

\[ W_{\text{net}} = \Delta K \]

There are three forces \((N, F_x, F_k)\) which do work.

\[ F_x = mgs; \quad N = mg; \quad F_k = \mu N = \mu mgs \]

\[ W_x = F_x \cdot \Delta x = mg \cdot s \cdot \cos 90^\circ = 0 \]

\[ W_N = N \cdot \Delta x = N \cdot s \cdot \cos 90^\circ = 0 \]

\[ W_{friction} = F_k \cdot \Delta x = \mu mgs \cdot 90^\circ = -\mu mgs \]

\[ W_{\text{net}} = \Delta K \Rightarrow -\mu mgs = k_f - k_i = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = -\frac{1}{2} m v_i^2 \]

\[ \mu = \frac{v_i^2}{2gs} = \frac{(1.2 m/s)^2}{2 \cdot 0.98 m/s^2 \cdot 2 \cdot 0.4 m} = 0.037 \]
ConcepTest

A box is being pulled up a rough incline by a rope connected to a pulley. How many forces are doing work on the box?

A) one force
B) two forces
C) three forces
D) four forces
E) no forces are doing work

Any force not perpendicular to the motion will do work:

- $N$ does *no work*
- $T$ does *positive work*
- $f_{fr}$ does *negative work*
- $mg$ does *negative work*
In a baseball game, the catcher stops a 90-mph pitch. What can you say about the work done by the catcher on the ball?

A) catcher has done positive work
B) catcher has done negative work
C) catcher has done zero work

The force exerted by the catcher is opposite in direction to the displacement of the ball, so the work is negative. Or using the definition of work ($W = F \cdot s \cos \theta$), because $\theta = 180^\circ$, then $W < 0$. Note that because the work done on the ball is negative, its speed decreases.
ConcepTest Tension and Work

A ball tied to a string is being whirled around in a circle. What can you say about the work done by tension?

A) tension does no work at all
B) tension does negative work
C) tension does positive work

No work is done because the force acts in a perpendicular direction to the displacement. Or using the definition of work ($W = F \cdot s \cos \theta$), because $\theta = 90^\circ$, then $W = 0$.

Follow-up: Does the Earth do work on the Moon?
Let’s digress from work for a few slides and then, we will get “back to work.”

In order to simplify the work expression, let’s introduce very useful

**Dot Product of Vectors**

**or**

**Scalar product**
Scalar(Dot) Product of Two Vectors

The scalar product of two vectors is:

In a magnitude/angle form:

\[ \mathbf{A} \cdot \mathbf{B} = AB \cos \theta \]

In a component form:

\[ \mathbf{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \]
\[ \mathbf{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k} \]
\[ \mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z \]

Therefore, we can say that

*work is a scalar product of force and displacement*
A constant force $\mathbf{F}$ acts on an rabbit as it moves from position $\mathbf{r}_1$ to $\mathbf{r}_2$. What is the work done by this Force?

\[
\mathbf{W} = \mathbf{F} \cdot \mathbf{s}
\]

\[
\mathbf{F} = -4\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}
\]

\[
\mathbf{r}_1 = 2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k} \quad \rightarrow \quad \mathbf{r}_2 = 5\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}
\]

\[
\mathbf{s} = \Delta \mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1 = 3\mathbf{i} - 7\mathbf{j} + 9\mathbf{k}
\]

\[
W = \mathbf{F} \cdot \mathbf{s} = (-4)(3) + (5)(-7) + (2)(9) = -29 \text{ J}
\]
(a) Determine the work a hiker must do on a 15.0-kg backpack to carry it up a hill of height $h = 10.0$ m, as shown. Determine also

(b) the work done by gravity on the backpack, and

(c) the net work done on the backpack. 

*For simplicity, assume the motion is smooth and at constant velocity (i.e., acceleration is zero).*
**Example**

Work on a backpack

Wherever, the net force acting on an object is zero, the net work will be zero as well.

- Net force acting on the backpack: $F_H - mg$
- Net work done by $F_H$: $W_H = F_H \cdot d \cdot \cos \theta = F_H \cdot h$
- Net work done by $mg$: $W_g = mg \cdot d \cdot \cos (180^\circ - \theta) = -mgd \cdot \cos \theta$
- Total work: $W_t = W_H + W_g = mgh - mgd = 0$

It is not an accident that the net work is zero. This is a fundamental principle in physics.

**End of class**
ConcepTest  Work and KE

A child on a skateboard is moving at a speed of 2 m/s. After a force acts on the child, her speed is 3 m/s. What can you say about the work done by the external force on the child?

1) positive work was done
2) negative work was done
3) zero work was done

The kinetic energy of the child increased because her speed increased. This increase in KE was the result of positive work being done.

Follow-up: What does it mean for negative work to be done on the child?