Lecture 23

Chapter 14

Oscillations
Simple pendulum

Course website:
http://faculty.uml.edu/Andriy_Danylov/Teaching/PhysicsI

Lecture Capture:
http://echo360.uml.edu/danylov2013/physics1spring.html
Exam 3 Results

Average 50.5/100

Total grade distribution

<table>
<thead>
<tr>
<th>Grade</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>89.5-100</td>
<td>7</td>
</tr>
<tr>
<td>84.5-89.5</td>
<td>9</td>
</tr>
<tr>
<td>79.5-84.5</td>
<td>16</td>
</tr>
<tr>
<td>74.5-79.5</td>
<td>14</td>
</tr>
<tr>
<td>69.5-74.5</td>
<td>17</td>
</tr>
<tr>
<td>64.5-69.5</td>
<td>15</td>
</tr>
<tr>
<td>59.5-64.5</td>
<td>18</td>
</tr>
<tr>
<td>54.5-59.5</td>
<td>26</td>
</tr>
<tr>
<td>49.5-54.5</td>
<td>23</td>
</tr>
<tr>
<td>44.5-49.5</td>
<td>26</td>
</tr>
<tr>
<td>39.5-44.5</td>
<td>21</td>
</tr>
<tr>
<td>34.5-39.5</td>
<td>20</td>
</tr>
<tr>
<td>29.5-34.5</td>
<td>18</td>
</tr>
<tr>
<td>24.5-29.5</td>
<td>13</td>
</tr>
<tr>
<td>19.5-24.5</td>
<td>17</td>
</tr>
<tr>
<td>&lt;19.5</td>
<td>40</td>
</tr>
<tr>
<td>absent</td>
<td>40</td>
</tr>
</tbody>
</table>

Section average distribution

<table>
<thead>
<tr>
<th>Section</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>201</td>
<td>65.2381</td>
</tr>
<tr>
<td>202</td>
<td>45.7</td>
</tr>
<tr>
<td>203</td>
<td>57.0435</td>
</tr>
<tr>
<td>204</td>
<td>48.6471</td>
</tr>
<tr>
<td>205</td>
<td>50.2381</td>
</tr>
<tr>
<td>206</td>
<td>53.8947</td>
</tr>
<tr>
<td>207</td>
<td>64.2273</td>
</tr>
<tr>
<td>208</td>
<td>45.4545</td>
</tr>
<tr>
<td>209</td>
<td>50.6842</td>
</tr>
<tr>
<td>210</td>
<td>54.1111</td>
</tr>
<tr>
<td>211</td>
<td>44.8571</td>
</tr>
<tr>
<td>212</td>
<td>42.375</td>
</tr>
<tr>
<td>213</td>
<td>41.6667</td>
</tr>
<tr>
<td>214</td>
<td>41.5556</td>
</tr>
<tr>
<td>215</td>
<td>47</td>
</tr>
</tbody>
</table>
Chapter 14

- Simple Harmonic Motion
- Energy in the Simple Harmonic Oscillator
- The Simple Pendulum
Simple Harmonic Motion

\[ \frac{d^2 x}{dt^2} = -\omega^2 x \]

where \( \omega = \sqrt{\frac{k}{m}} \)

**A Differential Equation**

**displacement** \hspace{1cm} **amplitude** \hspace{1cm} **angular frequency**

\[ x(t) = A \cos(\omega t + \varphi) \]

This function describes a simple harmonic motion

**period of oscillations,** \( T \)

\[ T = \frac{2\pi}{\omega} \quad \rightarrow \quad T = 2\pi \sqrt{\frac{m}{k}} \]

**frequency** \( f = \frac{1}{T} \)
If we know \( x(t) \), we can calculate \( v(t) \) and \( a(t) \)

The velocity, \( v(t) \), and acceleration, \( a(t) \), for simple harmonic motion can be found by differentiating the displacement:

\[
x = A \cos(\omega t + \phi)
\]

\[
v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)
\]

\[
\nu_{\text{max}} = \omega A
\]

\[
a = \frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \phi) = -\omega^2 x
\]

\[
a_{\text{max}} = \omega^2 A
\]
The Phase Angle: the effect of $\varphi$ (I)

Recall calculus.

Let us start with a function,

$y = x^2$

We can move it up by adding a constant to the $y$-value:

$y = x^2 + c$

You can move it left or right by adding a constant to the $x$-value

$y = (x + c)^2$
What does the Phase Angle do? The effect of $\phi$

- **$\phi = 0$**
  \[ x = A \cos(\omega t + \phi) \]

- **$\phi > 0$**
  \[ x = A \cos(\omega t + \phi) = A \cos\left[ \omega(t + \frac{\phi}{\omega}) \right] \]
  \[ \Delta t = \frac{\phi}{\omega} \]  
  Positive phase moves the curve left by $\phi/\omega$

- **$\phi < 0$**
  \[ x = A \cos(\omega t - \phi) = A \cos\left[ \omega(t - \frac{\phi}{\omega}) \right] \]
  \[ \Delta t = \frac{\phi}{\omega} \]  
  Negative phase moves the curve right by $\phi/\omega$

Value of phase, $\phi$, doesn’t affect the shape of the curve $x(t)$
Example: SHM

\[ x(t) = A \cos(\omega t + \phi) \]

\[ A = 10 \text{ m} \]
\[ T = 2 \text{ s} \Rightarrow T = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \text{ rad/s} \]

How we used to find \( \phi \) (phase):

Apply the initial conditions:

at \( t = 0 \) \( x = -5 \text{ m} \) (see figure), so

\[ x(t=0) = -5 = A \cos(\omega t + \phi) = 10 \cos(\phi) \]

\[ -5 = 10 \cos \phi \]

\[ \cos \phi = -\frac{1}{2} \Rightarrow \phi = \frac{2\pi}{3} \]

\[ x(t) = (10 \text{ m})\cos(3.14t + 2\pi/3) \]

- Let's find max velocity, \( v_{\text{max}} \):

\[ v_{\text{max}} = 4\omega = 10 \cdot 3.14 \cdot \frac{21.4}{15} \text{ m/s} \]

- If \( \omega = 10 \text{ rad/s} \), let's find \( k \):

\[ \omega^2 = \frac{k}{m} \Rightarrow k = m \omega^2 = 10 \cdot (3.14 \cdot \frac{21.4}{15})^2 \text{ m} \]
ConcepTest 1  Spring constant

Two identical blocks oscillate on different horizontal springs. Which spring has the larger spring constant?

A) The red spring.
B) The blue spring.
C) There’s not enough information to tell.

\[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \]

\[ f \propto \sqrt{k} \]

since \( f_{\text{red}} > f_{\text{blue}} \),
then \( k_{\text{red}} > k_{\text{blue}} \)
Demonstrations (mass/spring system).

http://stopwatch.onlinedclock.net/

$$T = 2\pi \sqrt{\frac{m}{k}}$$

The period is independent of Amplitude!

A spring/mass system can be used in space to measure mass of an object
Energy in Simple Harmonic Motion

**Potential energy stored in spring**

\[ U = \frac{1}{2} kx^2 \]

**Kinetic energy of mass**

\[ K = \frac{1}{2} mv^2 \]

\[ x = A \cos(\omega t + \varphi) \]

\[ U = \frac{1}{2} kA^2 \cos^2(\omega t + \varphi) \]

\[ v = \frac{dx}{dt} = -\omega A \sin(\omega t + \varphi) \]

\[ K = \frac{1}{2} kA^2 \sin^2(\omega t + \varphi) \]

\[ E = K + U = \frac{1}{2} kA^2 \]

It is a constant!

Total mechanical energy is conserved
Oscillations. Summary

\[ E = U = \frac{1}{2} kA^2 \]

\[ E = K = \frac{1}{2} m v_{\text{max}}^2 \]

\[ E = U = \frac{1}{2} kA^2 \]

\[ F = F_{\text{max}} = ma \]

\[ x = -A \quad \nu = 0 \quad a_{\text{max}} = \omega^2 A \]

\[ x = 0 \quad \nu_{\text{max}} = \omega A \quad a = 0 \]

\[ x = A \quad \nu = 0 \quad a_{\text{max}} = \omega^2 A \]

\[ x = A \cos(\omega t + \phi) \]

\[ \nu = -\omega A \sin(\omega t + \phi) \]

\[ a = -\omega^2 A \cos(\omega t + \phi) \]

\[ T = \frac{2\pi}{\omega} \]

\[ \omega = \sqrt{\frac{k}{m}} \]
Thank you

See you on Wednesday