95.612. Classical Mechanics.

Midterm Exam 1. March 22, 2013

Open book exam.

1. (a) Determine the trajectory $r(\theta)$ for a particle of mass ***m*** moving in an oscillator potential $V(r) = kr^2/2$, where ***k*** is a constant.

(b) Demonstrate by converting to Cartesian coordinates that the trajectory is an ellipse with semi-major axis $a = r_0/\sqrt{1-\varepsilon}$, where

$$\varepsilon = \sqrt{1 - (kl^2/mE^2)}; \quad r_0 = l/\sqrt{mE}$$

(make a convenient choice of a constant of integration $\theta_0 = \pi/4$)

(c) Sketch the trajectory. Show parameters (semi-major/semi-minor axis, the force center, and r_0) **(10 points)**

$$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b}\arcsin\left(\frac{bx}{a}\right)$$

2. A particle of mass m is constrained to move without friction on the inside surface of a vertical paraboloid of revolution $\rho^2 = kz$ (polar coordinates ρ, φ, z are used), where k is a constant. The particle is subject to a gravitational force. Using Lagrange's method of undetermined multipliers, find the equations of motion of the particle and the Lagrange multiplier λ.

(you may leave the answers in integral forms as a function of $\rho(t)$. **(10 points)**