1. **(10 points)** Consider the motion of a particle P of mass m moving in the plane under the influence of a force of magnitude $\alpha m/r^2$ directed towards a fixed point O, where r is the distance from O to P. Where α *is* a constant. Assume that the potential energy is zero as $r \to \infty$.

    a) Find a Lagrangian.

    b) Find a Hamiltonian corresponding to this Lagrangian.

    c) What quantities are conserved?

    d) Find the equations of motion in the Hamiltonian formulation.

    e) Write down the equation for r

2. **(15 points)** Derive the equations of motion governing the free vibrations of the system shown in the figure. Assume the springs and the rigid bar to be massless. Consider only small oscillations.
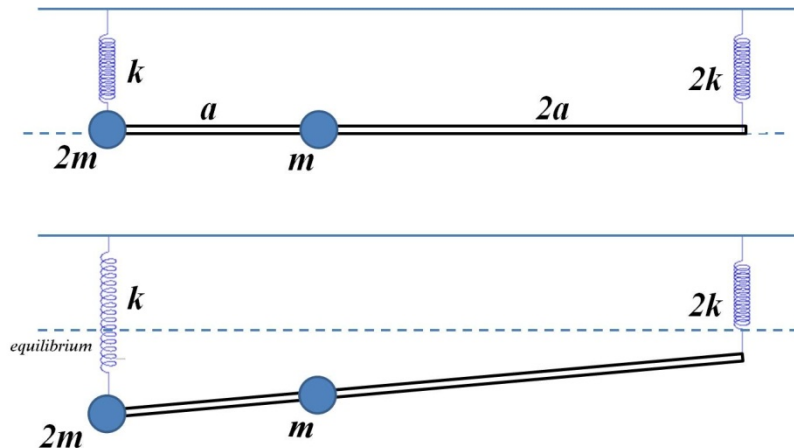
    a) *Write the Lagrangian of the system*

    b) *Find the normal mode frequencies*

        $\omega_1 = \sqrt{k/2m}$; $\omega_2 = \sqrt{22k/m}$; *(assume that $\sqrt{1881} \approx 43$)*

    c) *Find the normal mode eigenvectors and the general solution*

    d) *Construct the modal matrix **A***