Chapter 24 Lecture
IN THIS CHAPTER, you will learn about and apply Gauss’s law.
What is Gauss’s law?

Gauss’s law is a general statement about the nature of electric fields. It is more fundamental than Coulomb’s law and is the first of what we will later call Maxwell’s equations, the governing equations of electricity and magnetism.

Gauss’s law says that the electric flux through a closed surface is proportional to the amount of charge $Q_{\text{in}}$ enclosed within the surface. This seemingly abstract statement will be the basis of a powerful strategy for finding the electric fields of charge distributions that have a high degree of symmetry.

**LOOKING BACK** Section 22.5 The electric field of a point charge Section 23.2 Electric field lines
What good is symmetry?

For charge distributions with a high degree of symmetry, the symmetry of the electric field must match the symmetry of the charge distribution. Important symmetries are planar symmetry, cylindrical symmetry, and spherical symmetry. The concept of symmetry plays an important role in math and science.
What is electric flux?
The amount of electric field passing through a surface is called the **electric flux**. Electric flux is analogous to the amount of air or water flowing through a loop. You will learn to calculate the flux through open and closed surfaces.

« LOOKING BACK  Section 9.3  Vector dot products
How is Gauss’s law used?

Gauss’s law is **easier to use** than superposition for finding the electric field both inside and outside of charged spheres, cylinders, and planes. To use Gauss’s law, you calculate the electric flux through a **Gaussian surface** surrounding the charge. This will turn out to be much easier than it sounds!
What can we learn about conductors?

Gauss’s law can be used to establish several properties of conductors in electrostatic equilibrium. In particular:

- Any excess charge is all on the surface.
- The interior electric field is zero.
- The external field is perpendicular to the surface.
Chapter 24 Content, Examples, and QuickCheck Questions
Electric Field of a Charged Cylinder

- Suppose we knew only two things about electric fields:
  1. The field points away from positive charges, toward negative charges.
  2. An electric field exerts a force on a charged particle.
- From this information alone, what can we deduce about the electric field of an infinitely long charged cylinder?
- All we know is that this charge is positive, and that it has cylindrical symmetry.
Cylindrical Symmetry

- An infinitely long charged cylinder is symmetric with respect to:
  - Translation parallel to the cylinder axis.
  - Rotation by an angle about the cylinder axis.
  - Reflections in any plane containing or perpendicular to the cylinder axis.

- The symmetry of the electric field must match the symmetry of the charge distribution.
Could the field look like the figure below? (Imagine this picture rotated about the axis.)

The next slide shows what the field would look like reflected in a plane perpendicular to the axis (left to right).

Is this a possible electric field of an infinitely long charged cylinder? Suppose the charge and the field are reflected in a plane perpendicular to the axis.
Electric Field of a Charged Cylinder

- This reflection, which does not make any change in the charge distribution itself, does change the electric field.
- Therefore, the electric field of a cylindrically symmetric charge distribution cannot have a component parallel to the cylinder axis.

The charge distribution is not changed by the reflection, but the field is. This field doesn’t match the symmetry of the cylinder, so the cylinder’s field can’t look like this.
Electric Field of a Charged Cylinder

- Could the field look like the figure below? (Here we’re looking down the axis of the cylinder.)
- The next slide shows what the field would look like reflected in a plane containing the axis (left to right).

The charge distribution is not changed by reflecting it in a plane containing the axis.
Electric Field of a Charged Cylinder

- This reflection, which does not make any change in the charge distribution itself, *does* change the electric field.
- Therefore, the electric field of a cylindrically symmetric charge distribution **cannot have a component tangent to the circular cross section.**
Electric Field of a Charged Cylinder

- Based on symmetry arguments alone, an infinitely long charged cylinder must have a radial electric field, as shown below.
- This is the one electric field shape that matches the symmetry of the charge distribution.
Planar Symmetry

- There are three fundamental symmetries; the first is **planar symmetry**.

- Planar symmetry involves symmetry with respect to:
  - Translation parallel to the plane.
  - Rotation about any line perpendicular to the plane.
  - Reflection in the plane.
Cylindrical Symmetry

There are three fundamental symmetries; the second is **cylindrical symmetry**.

Cylindrical symmetry involves symmetry with respect to:

- Translation parallel to the axis.
- Rotation about the axis.
- Reflection in any plane containing or perpendicular to the axis.
Spherical Symmetry

- There are three fundamental symmetries; the third is **spherical symmetry**.
- Spherical symmetry involves symmetry with respect to
  - Rotation about any axis that passes through the center point.
  - Reflection in any plane containing the center point.
The Concept of Flux

- Consider a box surrounding a region of space.
- We can’t see into the box, but we know there is an outward-pointing electric field passing through every surface.
- Since electric fields point away from positive charges, we can conclude that the box must contain net positive electric charge.
The Concept of Flux

- Consider a box surrounding a region of space.
- We can’t see into the box, but we know there is an *inward-pointing* electric field passing through every surface.
- Since electric fields point toward negative charges, we can conclude that the box must contain net *negative* electric charge.
The Concept of Flux

- Consider a box surrounding a region of space.
- We can’t see into the box, but we know that the electric field points into the box on the left, and an equal electric field points out of the box on the right.
- Since this external electric field is not altered by the contents of the box, the box must contain zero net electric charge.

A field passing through the box implies there’s no net charge in the box.
A closed surface through which an electric field passes is called a **Gaussian surface**.

This is an imaginary, mathematical surface, not a physical surface.
Gaussian Surfaces

- A Gaussian surface is most useful when it matches the shape and symmetry of the field.
- Figure (a) below shows a **cylindrical** Gaussian surface.
- Figure (b) simplifies the drawing by showing two-dimensional end and side views.
- The electric field is everywhere **perpendicular** to the side wall and no field passes through the top and bottom surfaces.
Gaussian Surfaces

- Not every surface is useful for learning about charge.

- Consider the spherical surface in the figure.

- This is a Gaussian surface, and the protruding electric field tells us there’s a positive charge inside.

- It might be a point charge located on the left side, but we can’t really say.

- A Gaussian surface that doesn’t match the symmetry of the charge distribution isn’t terribly useful.
The Basic Definition of Flux

- Imagine holding a rectangular wire loop of area $A$ in front of a fan.

- The volume of air flowing through the loop each second depends on the angle between the loop and the direction of flow.

- The flow is *maximum* through a loop that is perpendicular to the airflow.

The air flowing through the loop is maximum when $\theta = 0^\circ$. 
The Basic Definition of Flux

- Imagine holding a rectangular wire loop of area $A$ in front of a fan.
- No air goes through the same loop if it lies parallel to the flow.

Unit vector normal to loop

No air flows through the loop when $\theta = 90^\circ$. 
The Basic Definition of Flux

- Imagine holding a rectangular wire loop of area $A$ in front of a fan.
- The volume of air flowing through the loop each second depends on the angle $\theta$ between the loop normal and the velocity of the air:

\[ v_\perp = v \cos \theta \] is the component of the air velocity perpendicular to the loop.

\[ \text{volume of air per second (m}^3/\text{s}) = v_\perp A = vA \cos \theta \]
The electric flux $\Phi_e$ measures the amount of electric field passing through a surface of area $A$ whose normal to the surface is tilted at angle $\theta$ from the field.

$$\Phi_e = E_\perp A = EA \cos \theta$$

$E_\perp = E \cos \theta$ is the component of the electric field that passes through the surface.

$\theta$ is the angle between $\hat{n}$ and $\vec{E}$. Normal to surface

Surface of area $A$
The Area Vector

- Let’s define an area vector \( \vec{A} = A\hat{n} \) to be a vector in the direction of \( \hat{n} \), perpendicular to the surface, with a magnitude \( A \) equal to the area of the surface.
- Vector \( \vec{A} \) has units of \( m^2 \).

Area vector \( \vec{A} \) is perpendicular to the surface. The magnitude of \( \vec{A} \) is the surface area \( A \).
The Electric Flux

- An electric field passes through a surface of area $A$.
- The electric flux can be defined as the dot-product:

$$\Phi_e = \vec{E} \cdot \vec{A}$$

(electric flux of a constant electric field)

The electric flux through the surface is $\Phi_e = \vec{E} \cdot \vec{A}$.
Example 24.1 The Electric Flux Inside a Parallel-Plate Capacitor

**EXAMPLE 24.1**

The electric flux inside a parallel-plate capacitor

Two 100 cm² parallel electrodes are spaced 2.0 cm apart. One is charged to +5.0 nC, the other to −5.0 nC. A 1.0 cm × 1.0 cm surface between the electrodes is tilted to where its normal makes a 45° angle with the electric field. What is the electric flux through this surface?

**MODEL** Assume the surface is located near the center of the ca-
Example 24.1 The Electric Flux Inside a Parallel-Plate Capacitor

**Example 24.1** The electric flux inside a parallel-plate capacitor

The electric flux through the surface is \( \Phi_e = \vec{E} \cdot \vec{A} \).
Example 24.1 The Electric Flux Inside a Parallel-Plate Capacitor

\[
E = \frac{Q}{\epsilon_0 A_{\text{plates}}} = \frac{5.0 \times 10^{-9} \text{ C}}{(8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2)(1.0 \times 10^{-2} \text{ m}^2)}
\]

\[= 5.65 \times 10^4 \text{ N/C} \]

A 1.0 cm \( \times \) 1.0 cm surface has \( A = 1.0 \times 10^{-4} \text{ m}^2 \). The electric flux through this surface is

\[
\Phi_e = \vec{E} \cdot \vec{A} = EA \cos \theta
\]

\[= (5.65 \times 10^4 \text{ N/C})(1.0 \times 10^{-4} \text{ m}^2) \cos 45^\circ \]

\[= 4.0 \text{ N m}^2/\text{C} \]

**ASSESS** The units of electric flux are the product of electric field and area.
The Electric Flux of a Nonuniform Electric Field

- Consider a surface in a nonuniform electric field.
- Divide the surface into many small pieces of area $\delta A$.
- The electric flux through each small piece is
  \[ \delta \Phi_i = \vec{E}_i \cdot (\delta \vec{A})_i \]
- The electric flux through the whole surface is the surface integral:
  \[ \Phi_e = \int_{\text{surface}} \vec{E} \cdot d\vec{A} \]

The total area $A$ can be divided into many small pieces of area $\delta A$. \( \vec{E} \) may be different at each piece.
The Flux Through a Curved Surface

- Consider a curved surface in an electric field.
- Divide the surface into many small pieces of area $\delta A$.
- The electric flux through each small piece is $\delta \Phi_i = \vec{E}_i \cdot (\delta \vec{A})_i$.
- The electric flux through the whole surface is the **surface integral**:

$$\Phi_e = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$
Consider an electric field that is everywhere tangent, or parallel, to a curved surface.

$\vec{E} \cdot d\vec{A}$ is zero at every point on the surface, because $\vec{E}$ is perpendicular to $d\vec{A}$ at every point.

Thus $\Phi_e = 0$. 

$\vec{E}$ is everywhere tangent to the surface. The flux is zero.
Electric Fields Perpendicular to a Surface

- Consider an electric field that is everywhere perpendicular to the surface and has the same magnitude $E$ at every point.
- In this case,

$$\Phi_e = \int_{\text{surface}} \vec{E} \cdot d\vec{A} = \int_{\text{surface}} E\, dA = E \int_{\text{surface}} dA = EA$$
TACTICS BOX 24.1

Evaluating surface integrals

1. If the electric field is everywhere tangent to a surface, the electric flux through the surface is $\Phi_e = 0$.

2. If the electric field is everywhere perpendicular to a surface and has the same magnitude $E$ at every point, the electric flux through the surface is $\Phi_e = EA$. 
The electric flux through a closed surface is

\[ \Phi_e = \oint \vec{E} \cdot d\vec{A} \]

The electric flux is still the summation of the fluxes through a vast number of tiny pieces, pieces that now cover a closed surface.

**NOTE:** For a closed surface, we use the convention that the area vector \( d\vec{A} \) is defined to always point \textit{toward the outside}. 
Tactics: Finding the Flux Through a Closed Surface

TACTICS BOX 24.2

Finding the flux through a closed surface

1. Choose a Gaussian surface made up of pieces that are everywhere tangent to the electric field or everywhere perpendicular to the electric field.
2. Use Tactics Box 24.1 to evaluate the surface integrals over these surfaces, then add the results.

Exercise 10
Electric Flux of a Point Charge

- The flux integral through a spherical Gaussian surface centered on a single point charge is
  \[ \Phi_e = \oint \vec{E} \cdot d\vec{A} = EA_{\text{sphere}} \]

- The surface area of a sphere is \( A_{\text{sphere}} = 4\pi r^2 \).

- Using Coulomb’s law for \( E \), we find
  \[ \Phi_e = \frac{q}{4\pi\varepsilon_0 r^2} \cdot 4\pi r^2 = \frac{q}{\varepsilon_0} \]
The electric flux through a spherical surface centered on a single positive point charge is \( \Phi_e = \frac{q}{\epsilon_0} \).

This depends on the amount of charge, but not on the radius of the sphere.

For a point charge, electric flux is independent of \( r \).

Every field line passes through the smaller and the larger sphere. The flux through the two spheres is the same.
Electric Flux of a Point Charge

- The electric flux through any arbitrary closed surface surrounding a point charge $q$ may be broken up into spherical and radial pieces.

- The total flux through the spherical pieces must be the same as through a single sphere:

$$
\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}
$$
Electric Flux of a Point Charge

- The electric flux through any arbitrary closed surface entirely outside a point charge $q$ may also be broken up into spherical and radial pieces.

- The total flux through the concave and convex spherical pieces must cancel each other.

- The net electric flux is zero through a closed surface that does not contain any net charge.
Electric Flux of Multiple Charges

- Consider an arbitrary Gaussian surface and a group of charges $q_1, q_2, q_3, \ldots$

- The contribution to the total flux for any charge $q_i$ inside the surface is $q_i / \epsilon_0$.

- The contribution for any charge outside the surface is zero.

- Defining $Q_{in}$ to be the sum of all the charge inside the surface, we find $\Phi_e = Q_{in} / \epsilon_0$. 

The fluxes due to charges outside the surface are all zero.

Two-dimensional cross section of a Gaussian surface

Total charge inside is $Q_{in}$.

The fluxes due to charges inside the surface add.
Gauss’s Law

- For any *closed* surface enclosing total charge $Q_{\text{in}}$, the net electric flux through the surface is

\[ \Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0} \]

- This result for the electric flux is known as **Gauss’s Law**.
1. Gauss’s law applies only to a *closed* surface, called a Gaussian surface.

2. A Gaussian surface is not a physical surface. It need not coincide with the boundary of any physical object (although it could if we wished). It is an imaginary, mathematical surface in the space surrounding one or more charges.

3. We can’t find the electric field from Gauss’s law alone. We need to apply Gauss’s law in situations where, from symmetry and superposition, we already can guess the *shape* of the field.
PROBLEM-SOLVING STRATEGY 24.1

Gauss’s law

MODEL  Model the charge distribution as a distribution with symmetry.

VISUALIZE  Draw a picture of the charge distribution.
- Determine the symmetry of its electric field.
- Choose and draw a Gaussian surface with the same symmetry.
- You need not enclose all the charge within the Gaussian surface.
- Be sure every part of the Gaussian surface is either tangent to or perpendicular to the electric field.

SOLVE  The mathematical representation is based on Gauss’s law

\[ \Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0} \]

- Use Tactics Boxes 24.1 and 24.2 to evaluate the surface integral.

ASSESS  Check that your result has correct units and significant figures, is reasonable, and answers the question.

Exercise 19
Example 24.3 Outside a Sphere of Charge

**EXAMPLE 24.3** Outside a sphere of charge

In Chapter 23 we asserted, without proof, that the electric field outside a sphere of total charge $Q$ is the same as the field of a point charge $Q$ at the center. Use Gauss’s law to prove this result.

**MODEL** The charge distribution within the sphere need not be uniform (i.e., the charge density might increase or decrease with $r$), but it must have spherical symmetry in order for us to use Gauss’s law. We will assume that it does.
Example 24.3 Outside a Sphere of Charge

EXAMPLE 24.3

Outside a sphere of charge
Example 24.3 Outside a Sphere of Charge

A Gaussian surface is placed outside a sphere of total charge $Q$. The electric field $\vec{E}$ is everywhere perpendicular to the surface of the sphere. The flux through the Gaussian surface is given by $\Phi = \int \vec{E} \cdot d\vec{A}$. The magnitude of the electric field $E$ is determined by the charge enclosed within the Gaussian surface and the radius of the sphere. The electric field $\vec{E}$ is constant and radially outward from the center of the sphere.
EXAMPLE 24.3 Outside a sphere of charge

SOLVE Gauss’s law is

\[ \Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0} = \frac{Q}{\epsilon_0} \]
EXAMPLE 24.3  Outside a sphere of charge

SOLVE  Thus we have the simple result that the net flux through the Gaussian surface is

$$\Phi_e = EA_{\text{sphere}} = 4\pi r^2 E$$
Example 24.3 Outside a Sphere of Charge

**Example 24.3** Outside a sphere of charge

$\vec{E}$ is everywhere perpendicular to the surface.

Gaussian surface

Sphere of total charge $Q$
### Example 24.6 The Electric Field of a Plane of Charge

**Example 24.6**  The electric field of a plane of charge

Use Gauss’s law to find the electric field of an infinite plane of charge with surface charge density \( \eta \) (C/m\(^2\)).

**Model** A uniformly charged flat electrode can be modeled as an infinite plane of charge.
Example 24.6 The Electric Field of a Plane of Charge

**EXAMPLE 24.6** The electric field of a plane of charge

Symmetry allows the electric field to point only straight toward or away from the plane. With this in mind, choose as a Gaussian surface a cylinder with length $L$ and faces of area $A$ centered on
Example 24.6 The Electric Field of a Plane of Charge

Infinite plane of charge

Gaussian surface

$\vec{E}$

$d\vec{A}$

Area $A$

$L$
Example 24.6 The Electric Field of a Plane of Charge

EXAMPLE 24.6

The electric field of a plane of charge

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Example 24.6 The Electric Field of a Plane of Charge

Example 24.6  The electric field of a plane of charge

Solve  The charge inside the cylinder is the charge contained in area $A$ of the plane. This is

\[ Q_{\text{in}} = \eta A \]

With these expressions for $Q_{\text{in}}$ and $\Phi_e$, Gauss’s law is

\[ \Phi_e = 2EA = \frac{Q_{\text{in}}}{\varepsilon_0} = \frac{\eta A}{\varepsilon_0} \]

Thus the electric field of an infinite charged plane is

\[ \eta \]
Example 24.6 The Electric Field of a Plane of Charge

EXAMPLE 24.6  The electric field of a plane of charge
Conductors in Electrostatic Equilibrium

- The figure shows a Gaussian surface just inside a conductor’s surface.

- The electric field must be zero at all points within the conductor, or else the electric field would cause the charge carriers to move and it wouldn’t be in equilibrium.

- By Gauss’s Law, \( Q_{\text{in}} = 0 \)
Conductors in Electrostatic Equilibrium

- The external electric field right at the surface of a conductor must be perpendicular to that surface.

- If it were to have a tangential component, it would exert a force on the surface charges and cause a surface current, and the conductor would not be in electrostatic equilibrium.
Electric Field at the Surface of a Conductor

- A Gaussian surface extending through the surface of a conductor has a flux only through the outer face.
- The net flux is 
  \[ \Phi_e = AE_{\text{surface}} = \frac{Q_{\text{in}}}{\epsilon_0}. \]
- Here \( Q_{\text{in}} = \eta A \), so the electric field at the surface of a conductor is
  \[ \vec{E}_{\text{surface}} = \left( \frac{\eta}{\epsilon_0}, \text{perpendicular to surface} \right) \]
  where \( \eta \) is the surface charge density of the conductor.
Conductors in Electrostatic Equilibrium

- The figure shows a charged conductor with a hole inside.
- Since the electric field is zero inside the conductor, we must conclude that $Q_{\text{in}} = 0$ for the interior surface.
- Furthermore, since there’s no electric field inside the conductor and no charge inside the hole, the electric field in the hole must be zero.

A hollow completely enclosed by the conductor

The flux through the Gaussian surface is zero. There’s no net charge inside, hence no charge on this interior surface.
Faraday Cages

- The use of a conducting box, or *Faraday cage*, to exclude electric fields from a region of space is called **screening**.

(a) Parallel-plate capacitor

\[ \vec{E} \]

We want to exclude the electric field from this region.

(b) The conducting box has been polarized and has induced surface charges.

\[ \vec{E} = 0 \]

The electric field is perpendicular to all conducting surfaces.
Conductors in Electrostatic Equilibrium

- The figure shows a charge $q$ inside a hole within a neutral conductor.
- Net charge $-q$ moves to the inner surface and net charge $+q$ is left behind on the exterior surface.

The flux through the Gaussian surface is zero, hence there’s no net charge inside this surface. There must be charge $-q$ on the inside surface to balance point charge $q$.

The outer surface must have charge $+q$ so that the conductor remains neutral.
Tactics: Finding the Electric Field of a Conductor in Electrostatic Equilibrium

TACTICS BOX 24.3

Finding the electric field of a conductor in electrostatic equilibrium

1. The electric field is zero at all points within the volume of the conductor.
2. Any excess charge resides entirely on the exterior surface.
3. The external electric field at the surface of a charged conductor is perpendicular to the surface and of magnitude $\eta/\varepsilon_0$, where $\eta$ is the surface charge density at that point.
4. The electric field is zero inside any hole within a conductor unless there is a charge in the hole.

Exercises 20–24
Example 24.7 The Electric Field at the Surface of a Charged Metal Sphere

**EXAMPLE 24.7**  
The electric field at the surface of a charged metal sphere

A 2.0-cm-diameter brass sphere has been given a charge of 2.0 nC. What is the electric field strength at the surface?

**MODEL**  Brass is a conductor. The excess charge resides on the surface.

**VISUALIZE**  The charge distribution has spherical symmetry. The electric field points radially outward from the surface.
Example 24.7 The Electric Field at the Surface of a Charged Metal Sphere

**Example 24.7** The electric field at the surface of a charged metal sphere

**Solve** We can solve this problem in two ways. One uses the fact that a sphere, because of its complete symmetry, is the one shape for which any excess charge will spread out to a uniform surface charge density. Thus

\[
\eta = \frac{q}{A_{\text{sphere}}} = \frac{q}{4\pi R^2} = \frac{2.0 \times 10^{-9} \text{C}}{4\pi (0.010 \text{ m})^2} = 1.59 \times 10^{-6} \text{ C/m}^2
\]

From Equation 24.20, we know the electric field at the surface has strength

\[
E_{\text{surface}} = \frac{\eta}{\varepsilon_0} = \frac{1.59 \times 10^{-6} \text{ C/m}^2}{8.85 \times 10^{-12} \text{ F/m}} = 1.8 \times 10^5 \text{ N/C}
\]
**Example 24.7 The Electric Field at the Surface of a Charged Metal Sphere**

**Solution** Alternatively, we could have used the result, obtained earlier in the chapter, that the electric field strength outside a sphere of charge \( Q \) is

\[
E_{\text{outside}} = \frac{Q_{\text{in}}}{4\pi\varepsilon_0 r^2}.
\]

But \( Q_{\text{in}} = q \) and, at the surface, \( r = R \).

Thus

\[
E_{\text{surface}} = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2} = \left(9.0 \times 10^9 \text{ N m}^2/\text{C}^2\right) \frac{2.0 \times 10^{-9} \text{ C}}{(0.010 \text{ m})^2}
\]
Chapter 24 Summary Slides
**Gauss’s Law**

For any *closed* surface enclosing net charge \( Q_{\text{in}} \), the net electric flux through the surface is

\[
\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}
\]

The electric flux \( \Phi_e \) is the same for *any* closed surface enclosing charge \( Q_{\text{in}} \).

To solve electric field problems with Gauss’s law:

**MODEL** Model the charge distribution as one with symmetry.

**VISUALIZE** Draw a picture of the charge distribution. Draw a Gaussian surface with the same symmetry as the electric field, every part of which is either tangent to or perpendicular to the electric field.

**SOLVE** Apply Gauss’s law and Tactics Boxes 24.1 and 24.2 to evaluate the surface integral.

**ASSESS** Is the result reasonable?
Symmetry

The symmetry of the electric field must match the symmetry of the charge distribution.

In practice, $\Phi_e$ is computable only if the symmetry of the Gaussian surface matches the symmetry of the charge distribution.
**Important Concepts**

**Charge** creates the electric field that is responsible for the electric flux.

\[ \Phi = \int \mathbf{E} \cdot d\mathbf{A} \]  

- \( Q_{in} \) is the sum of all enclosed charges. This charge contributes to the flux.
- Gaussian surface
- Charges outside the surface contribute to the electric field, but they don’t contribute to the flux.
Important Concepts

**Flux** is the amount of electric field passing through a surface of area $A$:

$$\Phi_e = \vec{E} \cdot \vec{A}$$

where $\vec{A}$ is the area vector.

For closed surfaces:
A net flux in or out indicates that the surface encloses a net charge.

Field lines through but with no *net* flux mean that the surface encloses no *net* charge.
**Important Concepts**

**Surface integrals** calculate the flux by summing the fluxes through many small pieces of the surface:

\[ \Phi_e = \sum \vec{E} \cdot \delta\vec{A} \]

\[ \rightarrow \int \vec{E} \cdot d\vec{A} \]

**Two important situations:**

If the electric field is everywhere tangent to the surface, then

\[ \Phi_e = 0 \]

If the electric field is everywhere perpendicular to the surface *and* has the same strength \( E \) at all points, then

\[ \Phi_e = EA \]
Applications

**Conductors in electrostatic equilibrium**

- The electric field is zero at all points within the conductor.
- Any excess charge resides entirely on the exterior surface.
- The external electric field is perpendicular to the surface and of magnitude $\eta/\varepsilon_0$, where $\eta$ is the surface charge density.
- The electric field is zero inside any hole within a conductor unless there is a charge in the hole.