 Name SOLUTION Section # ________

Instructions: **Neither calculators nor notes are allowed. All answers must be supported by work on your exam sheets. Answers with little or no supporting work will receive little or no credit. Work must be neat, organized and easily interpreted. Graders may deduct if your work is not expressed in complete mathematical sentences (i.e., using verbs such as $=, \in$, and $\leq$). Your name and section number must be printed on ALL sheets.**

Memory Questions (4 Points each):

A. Factor: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

B. Expand $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

C. If a triangle has sides A, B, C and the angle opposite side C is denoted "c," provide an equation for C^2.

$C^2 = A^2 + B^2 - 2AB\cos(c)$

Analytic questions (Points as indicated):

1. (8 Points) **JUST PICK ONE:** Determine if either of the following limits exists. If it does, what is its value?

$$L_1 = \lim_{x \to \pi} \frac{\sin(x)}{\tan(x)}$$

OR

$$L_2 = \lim_{x \to -2} \frac{2x+4}{x^2 - 2x - 8}$$

$$= \lim_{x \to \pi} \frac{\sin(x)}{\left(\frac{\sin(x)}{\cos(x)}\right)}$$

$$= \lim_{x \to \pi} \frac{\cos(x)\cancel{\sin(x)}}{\cancel{\sin(x)}}$$

$$= \lim_{x \to \pi} \cos(x)$$

$$= -1$$

$$= \lim_{x \to -2} \frac{2\cancel{(x+2)}}{\cancel{(x+2)}(x-4)}$$

$$= \lim_{x \to -2} \frac{2}{x-4} = \frac{2}{-6} = -\frac{1}{3}$$

Does the limit exist?

If so, indicate what it equals:

$L_1 = -1$ $\quad L_2 = -\frac{1}{3}$

2. (8 Points each)

a. Determine the **domain** of the function $f(x) = \dfrac{\sqrt{x+5}}{x^5+1}$. Express your final answer using interval notation.

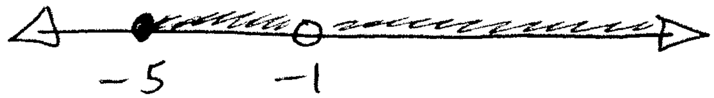
$x+5 \geq 0$

$\Rightarrow x \geq -5$

$x^5+1 \neq 0$

$x^5 \neq -1$

$x \neq -1$



$x \in [-5,-1) \cup (-1,\infty)$

$\boxed{x \in [-5,-1) \cup (-1,\infty)}$

b. Determine the **range** of the function $g(x) = 2x^2 + 3$. Express your final answer using interval notation.

Range of: x^2 is $[0,\infty)$

$2x^2$ is $[0,\infty)$

$g(x) = 2x^2+3$ is $[3,\infty)$

$\boxed{g(x) \in [3,\infty)}$

3. (8 Points) If the function $f(x) = x^2 + 1$ is shifted three units to the right, then flipped over the x-axis, and finally shifted 5 units up, what is the equation for the resulting function $g(x)$? You do <u>not</u> need to simplify your answer.

$$g(x) = -\big[\underbrace{(x-3)^2 + 1}_{\text{shifted 3 units right}}\big] + 5$$

flipped over x-axis

shifted 5 units up

$$\boxed{g(x) = -\left[(x-3)^2 + 1\right] + 5}$$

4. (8 Points total) Answer the following questions for $h(x) = 2x^2 - 3x - 1$

a. (4 points) What are the roots of $h(x)$, i.e., the values of x that make $h(x) = 0$?

$$x_{1,2} = \frac{3 \pm \sqrt{9+8}}{4}$$

$$= \frac{3}{4} \pm \frac{\sqrt{17}}{4}$$

$$\boxed{x = \frac{3}{4} \pm \frac{\sqrt{17}}{4}}$$

b. (4 Points) What is the x-coordinate of the vertex of $h(x)$?

$$x_v = -\frac{b}{2a} = \frac{3}{4}$$

(same as midpoint between two roots in part a)

$$\boxed{x_v = \frac{3}{4}}$$

5. a. (9 Points) Determine the amplitude, period, and frequency of the function $f(t) = -5\sin(3t - \pi) + 1$

Amplitude = 5 × amplitude of sin function

$\boxed{\text{Amplitude} = 5}$

$3t_p$ = period of sin function

$= 2\pi$

$\boxed{t_p = 2\pi/3}$

$\boxed{\text{frequency} = 1/t_p = \frac{3}{2\pi}}$

For $f(t)$

Amplitude = 5

Period = $2\pi/3$

Frequency = $3/2\pi$

b. (5 Points) What is the exact value of $f(t)$ at $t = \pi/4$?

$$f(\pi/4) = -5\sin\left(\frac{3\pi}{4} - \pi\right) + 1$$

$$= -5\sin\left(-\frac{\pi}{4}\right) + 1$$

$$= +5\sin(\pi/4) + 1$$

$$= 5/\sqrt{2} + 1 = \frac{5\sqrt{2}+2}{2}$$

$f(\pi/4) = \frac{5\sqrt{2}+2}{2}$

6. (12 Points) Simplify the given expression for A using the Rules of Exponents. Eliminate any radicals and use only positive exponents in your answer.

$$A = \left(\frac{a^{4/3} b^{-3/4} \sqrt{c}}{c^{-3} a^{2}} \right)^{-2/3} = \left[a^{\frac{4}{3}-2}\, b^{-\frac{3}{4}}\, c^{\frac{1}{2}+3} \right]^{-2/3}$$

$$= \left[a^{-2/3}\, b^{-3/4}\, c^{7/2} \right]^{-2/3}$$

$$= a^{4/9}\, b^{1/2}\, c^{-7/3}$$

$$= \frac{a^{4/9}\, b^{1/2}}{c^{7/3}}$$

$$\boxed{A = \frac{a^{4/9}\, b^{1/2}}{c^{7/3}}}$$

7. a. (8 Points) If the position of a particle at time t is given by $P(t) = 3t^3 - 2t$ (where $P(t)$ is expressed in km and t in minutes), what is the **average rate of change** in the particle's position between time $t = -1$ and time $t = 2$?

Avg Rate of change $[a,b]$ $= \dfrac{P(b)-P(a)}{b-a}$

$\left\{\text{for } \begin{matrix} a=-1 \\ b=2 \end{matrix}\right\} = \dfrac{P(2)-P(-1)}{2-(-1)} = \dfrac{(24-4)-(-3+2)}{3}$

$= \dfrac{20-(-1)}{3} = \dfrac{21}{3} = 7$ km/min

The average rate of change of $P(t)$ between $t = -1$ and $t = 2$ is: 7 km/min
Don't forget to indicate units.

b. (4 Points) What is the average rate of change of $g(t) = 2\sin(t)$ between $t = 0$ and $t = \pi/2$? (Assume the same units as in part a of this question).

Avg Rate of change $[0, \pi/2]$ $= \dfrac{g(\pi/2) - g(0)}{\pi/2 - 0} = \dfrac{2-0}{\pi/2} = 4/\pi$ km/min

The average rate of change between $t = 0$ and $t = \pi/2$ is: $\dfrac{4}{\pi}$ km/min

8. (10 Points) Determine the vertical and horizontal asymptotes of $y(x) = \dfrac{\pi x^2 - 13x + 2}{x^2 - 7}$.

$$y(x) = \frac{\pi x^2 - 13x + 2}{(x+\sqrt{7})(x-\sqrt{7})}$$

Vertical asymptotes at $x = \pm\sqrt{7}$

The order of the numerator equals the order of the denominator so Horizontal asymptote is $y = \frac{\pi}{1} = \pi$

x-values of vertical asymptotes:

$x = \pm\sqrt{7}$

y-values of horizontal asymptotes:

$y = \pi$

Bonus: (5 Points each) For which of the following functions can you use the Intermediate Value Theorem to prove that a root exists between $x = 0$ and $x = \pi$? **NOTE:** You must explain WHY or WHY NOT for EACH function.

a. $f(x) = e^x - 2$

e^x and $e^x - 2$ are continuous functions for all values of x. $f(0) = 1 - 2 = -1$; $f(\pi) = e^\pi - 2 > 0$

Yes

$f(x)$ must take on all values between -1 an $e^\pi - 2 > 0$ between $x = 0$ and $x = \pi$, so a root must exist for $x \in [0, \pi]$

b. $g(x) = \sec(x)$

$\sec(x)$ is not continuous on $x \in [0, \pi]$

No

since it is discontinuous at $x = \pi/2$ where $\cos(x) = 0$

In any case, $\sec(x)$ is never $= 0$ since

$\sec(x) = \dfrac{1}{\cos(x)}$ and 1 is never $= 0$.