92.128 Calculus 1A Fall 2010 Exam II Name SOLUTION Section # ________

Instructions: Calculators and note sheets are **not allowed**. All answers must be supported by work on your exam sheets. Answers with little or no supporting work will receive little or no credit. Work must be **neat**, **organized** and **easily interpreted**. Graders will deduct points if your work is not expressed in complete mathematical sentences (e.g., $f'(x) = something$). Please box in your final answers. Your name and section number must be printed on ALL sheets.

MEMORY QUESTIONS (4 points each):

M.1. Expand and simplify (replace all trigonometric functions of standard angles with their values):

$$\sin(\theta+\pi/3) = \sin(\theta)\cos(\pi/3) + \sin(\pi/3)\cos(\theta)$$

$$= \frac{1}{2}\sin(\theta) + \frac{\sqrt{3}}{2}\cos(\theta)$$

M.2. A triangle has sides A, B, and C with opposite angles a, b, and c. Express the ratio $\frac{\sin(a)}{A}$ in terms of the remaining variables.

$$\frac{\sin(a)}{A} = \frac{\sin(b)}{B} = \frac{\sin(c)}{C}$$

Law of Sines

M.3. A cylinder of radius r and height h is topped with a hemi-sphere having the same radius. Its volume is given by:

$$V(r,h) = \pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$= \pi r^2 h + \frac{2}{3}\pi r^3$$

ANALYTIC QUESTIONS:

1. (15 points) Determine the derivative of $f(x)=\dfrac{1}{2x-1}$ using the limit of the difference quotient.
(Optional but advised: You should check your final answer using the rules of differentiation.)

$$f'(x) = \lim_{h\to 0} \frac{\dfrac{1}{2(x+h)-1} - \dfrac{1}{2x-1}}{h}$$

$$= \lim_{h\to 0} \frac{(2x-1)-(2(x+h)-1)}{h(2(x+h)-1)(2x-1)}$$

$$= \lim_{h\to 0} \frac{\cancel{2x} \cancel{-1} - \cancel{2x} - 2h \cancel{+1}}{h(2(x+h)-1)(2x-1)}$$

$$= \lim_{h\to 0} \frac{-2\cancel{h}}{\cancel{h}(2(x+h)-1)(2x-1)}$$

$$= \frac{-2}{(2x-1)^2} = -2(2x-1)^{-2}$$

Using the rules of differentiation

$$f'(x) = -(2x-1)^{-2}\cdot 2 = -2(2x-1)^{-2}$$

ok

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2. Find the derivative of the following functions using the rules of differentiation. **NOTE:** <u>YOU DO NOT NEED TO SIMPLIFY</u>. (Each problem is worth 6 points)

a. $h(x) = 2x^{2/5} - 3x^{-3} + 5\sqrt[3]{x} + 3$

$h'(x) = \frac{4}{5}x^{-3/5} + 9x^{-4} + \frac{5}{3}x^{-2/3}$

b. $s(x) = \frac{4x^2 + 3x - 1}{\sqrt{x}} = 4x^{3/2} + 3x^{1/2} - x^{-1/2}$

$s'(x) = 6x^{1/2} + \frac{3}{2}x^{-1/2} + \frac{1}{2}x^{-3/2}$

c. $f(t) = 3e^{-t}\sec(t)$

$f'(t) = -3e^{-t}\sec(t) + 3e^{-t}\sec(t)\tan(t)$

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3. Find the derivative of the following functions using the rules of differentiation. **NOTE:** YOU DO NOT NEED TO SIMPLIFY. (Each problem is worth 7 points)

a. $g(x) = \dfrac{2\tan(x)}{x^3 - \cos(x)}$

$$g'(x) = \frac{2\sec^2(x)(x^3 - \cos(x)) - 2\tan(x)(3x^2 + \sin(x))}{(x^3 - \cos(x))^2}$$

b. $r(\theta) = 2\cos(\sin(\sqrt{\theta - 3}))$

$$r'(\theta) = -2\sin(\sin(\sqrt{\theta - 3}))\cos(\sqrt{\theta - 3})\left(\frac{1}{2}(\theta - 3)^{-1/2}\right)$$

4. (15 Points) A particle's position as a function of time for $t \geq 0$ is given by $g(t) = t + \sin(2t)$. Find an equation for the particle's velocity, $v(t)$, and its acceleration, $a(t)$. What is the first value of t for which $v(t) = 0$?

$$V(t) = 1 + 2\cos(2t)$$

$$a(t) = -4\sin(2t)$$

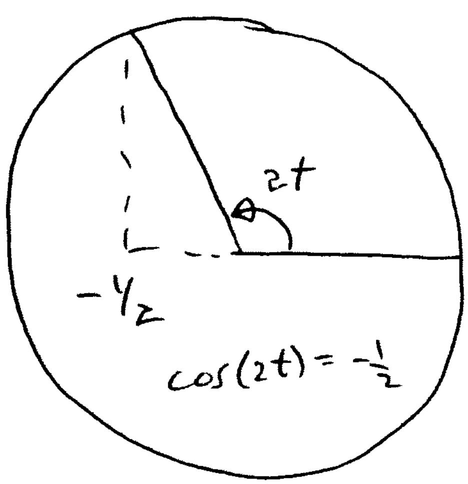
Setting $V(t) = 0$

$$1 + 2\cos(2t) = 0$$

$$\cos(2t) = -1/2$$

$$2t = \frac{2}{3}\pi \quad (120^\circ)$$

$$\boxed{t = \pi/3}$$



$$\Rightarrow 2t = \frac{2}{3}\pi$$

5. (7 points) Find the equation of the line (in slope-intercept form) that is tangent to $y = 3\sqrt{x} + \frac{1}{x}$ at $x = 1$.

$$\frac{dy}{dx} = \frac{3}{2} x^{-1/2} - x^{-2} \qquad y = 3x^{1/2} + x^{-1}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{3}{2} - 1 = 1/2 \qquad y(1) = 3 + 1 = 4$$

Must pass through $(1,4)$

$$y_T = \frac{1}{2}x + b \qquad 4 = \frac{1}{2}(1) + b$$

$$\Rightarrow b = 7/2$$

$$\boxed{y_T = \frac{1}{2}x + \frac{7}{2}}$$

6. (7 points) Find the values of t where $f(t) = 3t^4 + 8t^3 - 18t^2 + 11$ has a horizontal tangent.

$$\frac{df}{dt} = 12t^3 + 24t^2 - 36t = 0$$

$$12t(t^2 + 2t - 3) = 0$$

$$12t(t+3)(t-1) = 0$$

$$\Rightarrow \boxed{t = 0,\ t = -3 \text{ or } t = 1}$$

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7. (12 points) The trajectory of a particle in the x-y plane is described by the parametric equations $x(t) = \sin(\pi t)$ and $y(t) = e^{2t} + e^{t}$. Determine $\frac{dy}{dx}$ as a function of t. What is the slope of the line tangent to the particle's trajectory at $t = 0$? (Note: You do <u>NOT</u> need to find the equation of the tangent line.)

$$\left.\begin{aligned} \frac{dy}{dt} &= 2e^{2t} + e^{t} \\ \frac{dx}{dt} &= \pi\cos(\pi t) \end{aligned}\right\} \quad \frac{dy}{dx} = \frac{2e^{2t} + e^{t}}{\pi\cos(\pi t)}$$

$$\left.\frac{dy}{dx}\right|_{t=0} = \frac{2\cdot 1 + 1}{\pi(1)} = \frac{3}{\pi}$$

BONUS: (10 Points) Determine the exact value of the second derivative of $f(x) = e^{x}\sin(x)$ at $x = \pi$.

$$\frac{df}{dx} = e^{x}\sin(x) + e^{x}\cos(x)$$

$$\frac{d^2f}{dx^2} = \cancel{e^{x}\sin(x)} + e^{x}\cos(x) + e^{x}\cos(x) - \cancel{e^{x}\sin(x)}$$

$$= 2e^{x}\cos(x)$$

$$\left.\frac{d^2f}{dx^2}\right|_{x=\pi} = 2e^{\pi}\cos(\pi) = -2e^{\pi}$$