Summary of the Basic Rules of Algebraic Manipulation

Commutative properties (re-arrangement)

Addition: $a+b+c=a+c+b=b+a+c=c+b+a$ **etc.**

Multiplication: $abc=bac=cba=bca=acb$ **etc.**

Associative properties (order of computation)

Addition: $(a+b)+c=a+(b+c)$ **Multiplication:** $(ab)c=a(bc)$

Distributive properties

$a(b+c+d)=ab+ac+ad$ **and/or** $(a+b+c)d=ad+bd+cd$

Pulling out common factors: $x\Delta+x\Phi+x\Gamma+x\Theta=x(\Delta+\Phi+\Gamma+\Theta)$

Binomial mult: $(a+b)(c+d)=a(c+d)+b(c+d)=ac+ad+bc+bd$

Multinomial mult: $(a+b+c)(d+e+f)=ad+ae+af+bd+be+bf+cd+ce+cf$

Identity properties and Additive inverse property

$a=a\cdot 1=1\cdot a=a/1=a\pm 0=a$, $1=\frac{\Omega}{\Omega}$ **for** $\Omega\neq 0$ **and** $\Theta+(-\Theta)=0$

Treat leading "-" sign as multiplication by (-1)

$-a=(-1)a$ $-abc=(-1)abc=(-a)bc=a(-b)c=ab(-c)$ $(-a)(-b)=(-1)(-1)ab=ab$

$-(-a)=(-1)(-1)a=a$ $-(\Delta+\Theta)=(-1)(\Delta+\Theta)=(-1)\Delta+(-1)\Theta=-\Delta-\Theta$

Properties of equality

Add same to both sides: **If** $a=b$ **then** $a+\Theta=b+\Theta$

Multiply both sides by same: **If** $a=b$ **then** $a\Theta=b\Theta$ **(avoid using if** $\Theta=0$ **)**

Dividing both sides by same: **If** $a\Theta=b\Theta$ **and** $\Theta\neq 0$ **then** $a=b$

Operations with fractions

Equivalent fractions: $\frac{ab}{c}=a\frac{b}{c}=\frac{b}{c}a=b\frac{a}{c}=\frac{a}{c}b=ab\frac{1}{c}$ $\frac{-a}{b}=-\frac{a}{b}=\frac{a}{-b}=(-1)\frac{a}{b}$

Addition/subtraction: $\frac{a}{\Theta}\pm\frac{b}{\Theta}=\frac{a\pm b}{\Theta}$ **(same denom)** $\frac{a}{\Theta}\pm\frac{b}{\Delta}=\frac{a\Delta\pm b\Theta}{\Theta\Delta}$ **(diff denom)**

Multiplication: $\frac{x}{y}\cdot\frac{\Lambda}{\Theta}=\frac{x\Lambda}{y\Theta}$

Division: $\frac{\left(x/y\right)}{\left(\Lambda/\Theta\right)}=\left(\frac{x}{y}\right)\cdot\left(\frac{\Theta}{\Lambda}\right)=\frac{x\Theta}{y\Lambda}$ **("flip and multiply")** $\frac{\left(x/y\right)}{\Theta}=\frac{x/y}{\Theta/1}=\frac{x}{y}\frac{1}{\Theta}=\frac{x}{y\Theta}$

Cross multiplying: **If** $\frac{a}{b}=\frac{\Lambda}{\Theta}$ **then** $\frac{a}{b}b\Theta=\frac{\Lambda}{\Theta}b\Theta$ **yielding** $a\Theta=\Lambda b$

Key identities: $(x-y)(x+y)=x^2-y^2$ $a^3\pm b^3=(a\pm b)(a^2\mp ab+b^2)$

$(p\pm q)=p^2\pm 2pq+q^2$ $\sqrt{x^2}=|x|$

Absolute values: $|x|=a\Rightarrow x=\pm a$ $|x|\geq a\Rightarrow x\geq a$ **or** $x\leq -a$ $|x|\leq a\Rightarrow -a\leq x\leq a$

Distance form: $|x-c|<d$ **can be interpreted as "The distance between x and c is less than d."**