Instructions: Calculators and note sheets are not allowed. Answers with little or no supporting work will receive little or no credit. Work must be neat, organized and easily interpreted. Graders may deduct if your work is not expressed in complete mathematical sentence (i.e., using verbs such as =, ∈, and ≤). Use the boxes provided for your final answers. Your name and section number must be printed on ALL sheets.

1. (5 Points each) Determine the value of x:

   a. \( x = \log_5 \left( \frac{1}{25} \right) + \log_5 (125) + e^{\ln(2)} \)

   b. \( x = \tan(\sin^{-1}(5/7)) \)

2. (5 Points) Solve the following equation for x: \( \ln(x^2 + 8) - \ln(x + 1) = \ln(x + 2) \)

3. (10 Points) Use logarithmic differentiation to determine the derivative of \( f(x) = x^{\cos(x)} \)
4. (20 Points) Using implicit differentiation, DERIVE the formula for the derivative of $y(\theta) = \cos^{-1}(3\theta)$
5. (5 Points each) Using the rules of differentiation, determine the derivatives of the following functions:

a. \( f(x) = \ln(\sec(3x)) \)

b. \( g(\theta) = \theta \tan^{-1}(\sqrt{\theta}) \)  
   [Hint: \( \frac{d}{dx} \tan^{-1}(x) = \frac{1}{1 + x^2} \)]

c. \( h(x) = \ln \left( \frac{x(x^2 + 3)}{\sqrt{x + 2}} \right) \)
6. (20 Points total) A ground-based tracking camera is located 1000 meters away from where a rocket is launched straight up. The camera’s elevation angle is continuously adjusted to keep the rocket in the center of the rocket’s field of view.

a. (5 Points) Draw a diagram illustrating this problem. Use \( \theta(t) \) to denote the elevation angle (angle above the horizon) of the camera when it is pointed at the rocket and \( h(t) \) to denote the altitude of the rocket. Label all variables and dimensions.

b. (5 points) Determine a trigonometric formula relating \( h(t) \) and \( \theta(t) \).

c. (5 points) Differentiating the relationship developed in part (b), determine a relationship between the rate of change of the elevation angle, \( \frac{d\theta}{dt} \), and the rate of change of the rocket’s altitude, \( \frac{dh}{dt} \).

d. (5 Points) When \( \theta(t) = \frac{\pi}{4} \), its rate of change is measured to be 0.1 radian/second. At this instant, what is the rate of increase of the rocket’s altitude, \( \frac{dh}{dt} \)? Provide units with your answer.
7. (20 Points total) Given that \( y(x) = (x + 1)^2 + e^{3x} \),

a. (8 Points) Find an expression for \( dy \) as a function of \( x \) and \( dx \).

b. (7 Points) Using the expression in part (a), determine a linear approximation for \( y(x) \) that is valid near \( x = 0 \).

c. (5 Points) Based upon this linear approximation, estimate the value of \( y(x) \) when \( x = 0.1 \).