 Name ____________________ Section # ________

Instructions: Calculators and note sheets are **not allowed**. Answers with little or no supporting work will receive little or no credit. Work must be **neat**, **organized** and **easily interpreted**. Graders may deduct if your work is not expressed in complete mathematical sentence (i.e., using verbs such as $=, \in$, and $\leq$). Use the boxes provided for your final answers. Your name and section number must be printed on ALL sheets.

1. (5 Points each) Determine the value of x:

a. $x = \log_5(1/25) + \log_5(125) + e^{\ln(2)}$

b. $x = \tan(\sin^{-1}(5/7))$

2. (5 Points) Solve the following equation for x: $\ln(x^2+8) - \ln(x+1) = \ln(x+2)$

3. (10 Points) Use logarithmic differentiation to determine the derivative of $f(x) = x^{\cos(x)}$

4. (20 Points) Using implicit differentiation, DERIVE the formula for the derivative of $y(\theta) = \cos^{-1}(3\theta)$

5. (5 Points each) Using the rules of differentiation, determine the derivatives of the following functions:

a. $f(x) = \ln(\sec(3x))$

b. $g(\theta) = \theta \tan^{-1}(\sqrt{\theta})$ [Hint: $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$]

c. $h(x) = \ln\left(\frac{x(x^2+3)}{\sqrt{x+2}}\right)$

6. (20 Points total) A ground-based tracking camera is located 1000 meters away from where a rocket is launched straight up. The camera's elevation angle is continuously adjusted to keep the rocket in the center of the rocket's field of view.

a. (5 Points) Draw a diagram illustrating this problem. Use $\theta(t)$ to denote the elevation angle (angle above the horizon) of the camera when it is pointed at the rocket and $h(t)$ to denote the altitude of the rocket. Label all variables and dimensions.

b. (5 points) Determine a trigonometric formula relating $h(t)$ and $\theta(t)$.

c. (5 points) Differentiating the relationship developed in part (b), determine a relationship between the rate of change of the elevation angle, $\frac{d\theta}{dt}$, and the rate of change of the rocket's altitude, $\frac{dh}{dt}$.

d. (5 Points) When $\theta(t) = \pi/4$, its rate of change is measured to be 0.1 radian/second. At this instant, what is the rate of increase of the rocket's altitude, $\frac{dh}{dt}$? Provide units with your answer.

7. (20 Points total) Given that $y(x) = (x+1)^2 + e^{3x}$,

a. (8 Points) Find an expression for dy as a function of x and dx.

b. (7 Points) Using the expression in part (a), determine a linear approximation for $y(x)$ that is valid near $x = 0$.

c. (5 Points) Based upon this linear approximation, estimate the value of y(x) when $x = 0.1$.