What is $\log_4(32)$ and Why?

Of all the fundamental functions used in introductory calculus, logarithms are often the most confusing to students. Hopefully this will help to clear things up.

You cannot understand logarithms if you do not first have a good understanding of exponential functions. So, let us start with one of the simplest exponential function, $f(x) = 2^x$, having base 2.

The value of this function, for any given value of x, is what you get if you multiply x-number of 2's together. For integer values of x this is easy. $f(3) = 2^3 = 2{\bullet}2{\bullet}2 = 8$ and $f(4) = 2^4 = 2{\bullet}2{\bullet}2{\bullet}2 = 16$. For negative exponents, the 2's go in the denominator so $f(-2) = 2^{-2} = \frac{1}{2^2} = \frac{1}{2{\bullet}2} = \frac{1}{4}$. If x takes on a non-integer value the calculation includes an extra factor that is a root of the base. So if the exponent is 1.5 we get:

$f(1.5) = 2^{1.5} = 2^1 2^{1/2} = 2\sqrt{2} \cong 2.8284$, and if $x = -1.25$ then $f(-1.25) = \dfrac{1}{2^1 2^{1/4}} = \dfrac{1}{2\sqrt[4]{2}} \cong \dfrac{1}{2.3784} \cong 0.4204$.

Graphing this function produces the familiar shape on page two (the points we calculated above are highlighted). Note that $f(0) = 1$, since any base to the power zero equals 1. If you go to your textbook you will find exponential plots with different bases; including bases that are less than one. If you spend a few minutes calculating the values of exponential functions with different bases and plotting them, they will become familiar pretty quickly. If they are not familiar you should do this now. Yes … right now!

Logarithms are merely functions that undo exponential functions; i.e., they are "the inverse" of exponential functions. But that may not clear up the confusion, so let's try a different tact.

If I tell you to calculate 3^2 you would compute $3{\bullet}3 = 9$ and instantly report the result. If I asked you how many 3s need to be multiplied together to get 9, you would quickly tell me 2.

You did it!! You just figured out the value of $\log_3(9)$ … namely 2. This is the correct value of $\log_3(9)$ since the expression $\log_3(9)$ merely asks the question: "How many 3s (the base of this logarithm) need to be multiplied together to get 9 (the argument of the logarithm in this case). The answer, again, is 2.

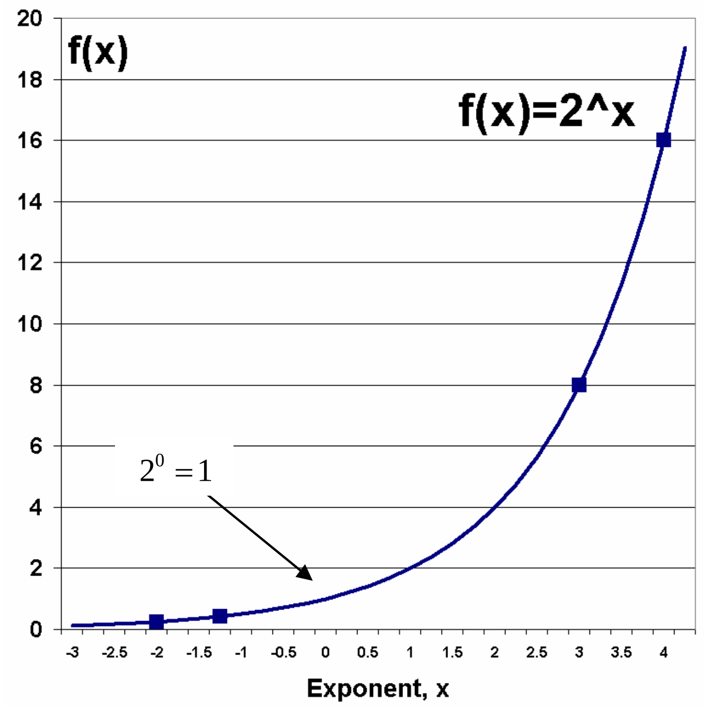
Do not let the notation confuse you. $\log_a(x)$ ALWAYS asks the same question, "How many of the base (how many a's) must be multiplied together to get the argument (x in this case)?" Said another way, "$\log_a(\otimes)$ is an exponent AND it is THE exponent you have to raise "a" to, to get $\otimes$." (You might want to memorize this.) Now, try to figure out the value of these logarithms (the answers are on the next page, but do not look until you re-read this paragraph and try a second, third or fourth time):

a) $\log_2(16)$ b) $\log_5(25)$ c) $\log_3(27)$ d) $\log_{1/2}(1/8)$ e) $\log_3(1/9)$ f) $\log_e(e^{17}) = \ln(e^{17}) = ?$

After you get these try: $\log_7(7^{(x^2 \tan(\sqrt{x}))})$. If this confuses you, read the preceding paragraph again and again until the light bulb comes on! Now try to figure out: $17^{\log_{17}(\sqrt{2})}$. First think about the meaning of the exponent (the number you must raise 17 to, to get $\sqrt{2}$) and what you will get if you raise 17 to that power!

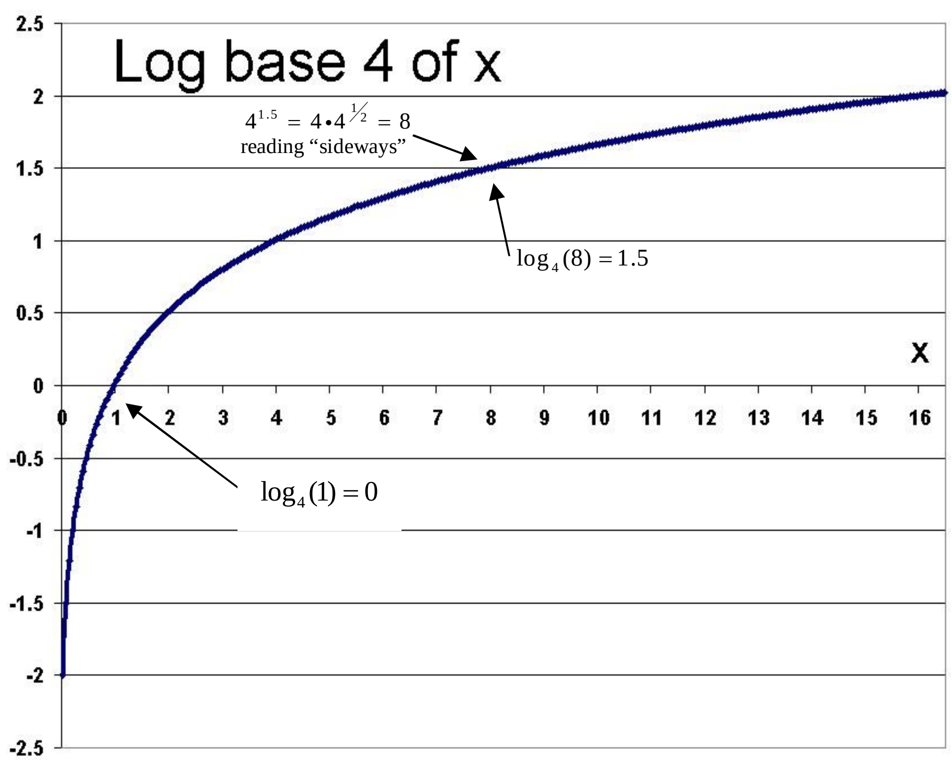
Are you ready? What is $\log_4(32)$? Let's see … $32 = 4 \bullet 4 \bullet 2 = 4^2 \bullet \sqrt{4} = 4^2 4^{1/2} = 4^{2+1/2} = 4^{5/2}$ so, since $\log_4(32)$ is equal to the number you have to raise 4 to to get 32, it must equal $2\,{}^1\!/_2$ or ${}^5\!/_2$. Wasn't that easy?

Another way to home in on the value of a logarithm is to bracket its value using just integer exponents. In the case of $\log_4(32)$ we know that $4^2 = 16$ and $4^3 = 64$ so, since 32 is between 16 and 64, the answer must be between 2 and 3.



Here is the plot of $f(x) = 2^x$ promised earlier. It is also a plot of $\log_2(x)$ but "**on its side**." If you want to know $\log_2(4)$ you locate 4 on the y-axis, go over to the curve, and then move down to the answer on the x-axis (in this case, the value 2). Of course 2 is the correct answer since it is "the exponent you have to raise 2 to, to get 4." Try some other questions such as, what is $\log_2(2)$? Of course the answer is 1 which is always the value of $\log_q(q)$ since $q^1 = q$. Speaking of a question that always has the same answer regardless of the base, what is $\log_P(1)$? Since $P^0 = 1$ the answer is zero.

The next graph is a plot of $g(x) = \log_4(x)$. Note that it is also the plot of $h(x) = 4^x$ but "**on its side**." Note that $\log_4(8) = 1.5$ represents the same point as $4^{1.5} = 8$ on the "sideways" plot. Find $\log_4(16)$ on this plot. Then see if you can read the "sideways" plot of 4^2. Again, you get to the same point on the graph. What is $\log_4(1)$? Did you forget already? Remember, $Log_B(1) = 0$ no matter what the value of the base, B.



Finally, make sure you understand why a plot of $\log_a(x)$ is always a reflection of the plot of a^x across the line $y = x$ when plotted on the same graph (see your instructor to discuss).

Here are the answers to the questions on the first page:

a) 4 b) 2 c) 3

d) 3 e) -2 f) 17

and

$$\log_7(7^{(x^2 \tan(\sqrt{x}))}) = x^2 \tan(\sqrt{x})$$

$$17^{\log_{17}(\sqrt{2})} = \sqrt{2}$$