Calculus 1B Exam II Spring 2010 Pg1 Name Solution Sec ________

Instructions: Calculators and note sheets are **not allowed**. Answers with little or no supporting work will receive little or no credit. Work must be **neat**, **organized** and **easily interpreted**. Graders may deduct if your work is not expressed in complete mathematical sentence (i.e., using verbs such as $=, \in$, and $\leq$). Your name and section number must be printed on ALL sheets.

1. (24 Points total) Answer the following question for the function $f(x) = x^4 - 2x^2 - 1$

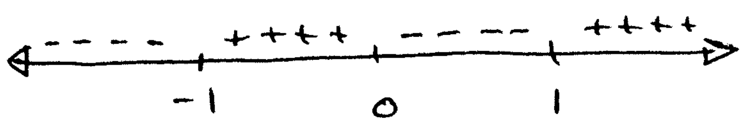
a. (8 points) What are the critical points of $f(x)$ (provide both x and y coordinates)?

$$\frac{df}{dx} = 4x^3 - 4x = 4x(x^2-1) = 0$$

$$x = 0, \quad x = \pm 1$$

$$\boxed{\begin{array}{c}\text{Critical Points} \\ (0,-1) \\ (-1,-2) \\ (1,-2)\end{array}}$$

b. (8 points) Find the regions where $f(x)$ is increasing and regions where $f(x)$ is decreasing. Illustrate you answer with a number line sketch **and** then express your final answer in interval notation.



$f'(-2) = -8(3) = -24 < 0$

$f'(-\frac{1}{2}) = -2(\frac{1}{4} - 1) > 0$

$f'(\frac{1}{2}) = 2(\frac{1}{4} - 1) < 0$

$f'(2) = 8(4-1) > 0$

$$\boxed{\begin{array}{l}f(x) \text{ decreasing} \\ \quad x \in (-\infty,-1) \cup (0,1) \\ f(x) \text{ increasing} \\ \quad x \in (-1,0) \cup (1,\infty)\end{array}}$$

c. (8 points) Identify any points where $f(x)$ achieves a local maximum or where it reaches a local minimum (provide both x and y coordinates). Justify why these points represent local extrema.

$f(x)$ has local minimums @ $(-1,-2)$ and $(1,-2)$ because $f'(x) < 0$ to the left of these points and $f'(x) > 0$ to the right

$f(x)$ has a local maximum @ $(0,-1)$ because $f'(x) > 0$ to the left and $f'(x) < 0$ to the right.

2. (18 points) Find the coordinates of the absolute maximum and the absolute minimum of the function $g(x) = 2x^3 - 3x^2 - 12x - 5$ on the closed interval $x \in [-2, 4]$.

$$g'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2)$$

$$= 6(x-2)(x+1) = 0 \text{ for critical points}$$

$\Rightarrow$ $x = 2$ and $x = -1$ are critical points

checking endpoints and critical points yields

x	$g(x)$
-2	$2(-8) - 3(4) - 12(-2) - 5 = -16 - 12 + 24 - 5 = -9$
-1	$2(-1) - 3(1) - 12(-1) - 5 = -2 - 3 + 12 - 5 = +2$
2	$2(8) - 3(4) - 12(2) - 5 = 16 - 12 - 24 - 5 = -25$
4	$2(64) - 3(16) - 12(4) - 5 = 128 - 48 - 48 - 5 = 27$

Absolute Minimum @ $(2, -25)$ Absolute Maximum @ $(4, 27)$

3. (15 Points) The velocity of a particle is given by $v(t) = 3t^2 + 6\cos(2t)$. Find the equation for the particle's position as a function of time, $x(t)$, if $x(0) = 2$. What is the value of $x(\pi)$?

$$V(t) = 3t^2 + 6\cos(2t)$$

$$X(t) = t^3 + 3\sin(2t) + C$$

$$X(0) = 0 + 0 + C = 2 \Rightarrow C = 2$$

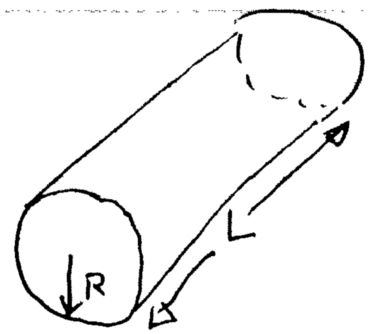
Therefore $\boxed{X(t) = t^3 + 3\sin(2t) + 2}$

$$\boxed{X(\pi) = \pi^3 + 0 + 2 = 2 + \pi^3}$$

4. (25 points) You want to ship a cylindrical container of maximum volume via the U.S. Post Office. The post office limits the size of your container by insisting that the length of the cylinder, L, plus its girth (the circumference of the circular cross-section having radius R) be limited to a total of 84 inches. Find the length, L, radius, R, and the volume, V, of the cylinder having the maximum possible volume.

a. (5 points) What is the Constraint Equation for this problem?

$$L + 2\pi R = 84$$



b. (5 points) What is the Objective Function for this problem?

$$V = \pi R^2 L$$

c. (15 points) Determine the optimum values of the dimensions L and R and for the container's volume, V. Reminder: you must demonstrate that your solution yields a maximum and not a minimum volume. [Note: 10 points will be based on process and 5 points for your final values.]

$$L = 84 - 2\pi R \Rightarrow V(R) = \pi R^2 (84 - 2\pi R)$$

$$= 84\pi R^2 - 2\pi^2 R^3$$

$$\frac{dV(R)}{dR} = 168\pi R - 6\pi^2 R^2 = 6\pi R(28 - \pi R)$$

$$\frac{dV}{dR} = 0 \Rightarrow R = 0 \left\{ \begin{matrix} \text{Not a} \\ \text{Maximum} \\ \text{Volume} \end{matrix} \right\} \text{ or } \boxed{R = 28/\pi}$$

$$\frac{d^2V}{dR^2} = 168\pi - 12\pi^2 R \qquad \left.\frac{d^2V}{dR^2}\right|_{R=28/\pi} = 168\pi - 12\cdot 28\pi = 168\pi - 336\pi < 0$$

$\Rightarrow$ Concave Down & Maximum

Therefore: $R^* = 28/\pi$ inches

$$L^* = 84 - 2\pi\left(\frac{28}{\pi}\right) = 28 \text{ inches} \qquad V^* = \pi\left(\frac{28}{\pi}\right)^2 28 = \frac{28^3}{\pi} \text{ in}^3$$

You may use the back of this sheet if you need additional room.

5. (18 points) The function $h(x) = x^3 - 7$ has a root near $x = 2$. With this as your first estimate (i.e., $x_0 = 2$), use Newton's method to determine the value of x_1. Finally, set up an equation for x_2. You do NOT need to simplify the expression for x_2 or calculate a value for x_2.

$h(x) = x^3 - 7$

$$x_{n+1} = x_n - \frac{h(x_n)}{h'(x_n)} \qquad h'(x) = 3x^2$$

$$x_1 = 2 - \frac{8-7}{3(2^2)} = 2 - \frac{1}{12} = \frac{23}{12}$$

$$x_2 = \frac{23}{12} - \frac{\left(\frac{23}{12}\right)^3 - 7}{3\left(\frac{23}{12}\right)^2}$$

Bonus (10 Points) Determine the value of the following limit: $\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$. Provide your solution on the back of this sheet.

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO!

Provide your solution to the bonus problem here:

$$\lim_{x \to 0} \frac{e^x - 1 - x}{x^2} \overset{0/0}{=} \lim_{x \to 0} \frac{e^x - 1}{2x}$$

$$\overset{0/0}{=} \lim_{x \to 0} \frac{e^x}{2} = 1/2$$