92.129 Calculus 1B   Spring 2010   Exam I   Pg 1   Name ____________________   Section # ________

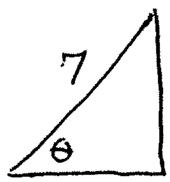
Instructions: Calculators and note sheets are **not allowed**. Answers with little or no supporting work will receive little or no credit. Work must be **neat**, **organized** and **easily interpreted**. Graders may deduct if your work is not expressed in complete mathematical sentence (i.e., using verbs such as $=, \in$, and $\leq$). ~~Use the boxes provided for your final answers~~. Your name and section number must be printed on ALL sheets.

1. (5 Points each) Determine the value of x:

a. $x = \log_5(1/25) + \log_5(125) + e^{\ln(2)}$

$$X = -2 + 3 + 2 = \boxed{3}$$

b. $x = \tan(\sin^{-1}(5/7))$



$$\sqrt{49-25} = \sqrt{24} = 2\sqrt{6}$$

$$X = \tan(\theta) = 5/2\sqrt{6}$$

$$\boxed{X = \frac{5}{2\sqrt{6}}}$$

2. (5 Points) Solve the following equation for x: $\ln(x^2+8) - \ln(x+1) = \ln(x+2)$

$$\ln\left[\frac{X^2+8}{X+1}\right] = \ln[X+2] \Rightarrow X^2+8 = (X+1)(X+2)$$

$$X^2+8 = X^2+3X+2 \Rightarrow 6 = 3X$$

$$\boxed{X = 2}$$

3. (10 Points) Use logarithmic differentiation to determine the derivative of $f(x) = x^{\cos(x)}$

$$\ln[f(x)] = \cos(x)\ln[x]$$

$$\frac{1}{f(x)}\frac{df}{dx} = -\sin(x)\ln(x) + \cos(x)\cdot\frac{1}{x}$$

$$\boxed{\frac{df}{dx} = X^{\cos(x)}\left[\frac{\cos(x)}{X} - \sin(x)\ln(x)\right]}$$

4. (20 Points) Using implicit differentiation, DERIVE the formula for the derivative of $y(\theta) = \cos^{-1}(3\theta)$

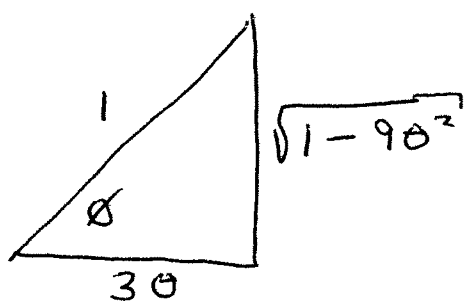
$$y(\theta) = \cos^{-1}(3\theta)$$

$$\cos(y(\theta)) = \cos(\cos^{-1}(3\theta)) = 3\theta$$

Differentiating:

$$-\sin(y(\theta))\frac{dy}{d\theta} = 3$$

$$\frac{dy}{d\theta} = -\frac{3}{\sin[\underbrace{\cos^{-1}(3\theta)}_{\phi}]}$$



$$\boxed{\frac{dy}{d\theta} = -\frac{3}{\sqrt{1-9\theta^2}}}$$

5. (5 Points each) Using the rules of differentiation, determine the derivatives of the following functions:

a. $f(x) = \ln(\sec(3x))$

$$\frac{df}{dx} = \frac{1}{\sec(3x)} \sec(3x)\tan(3x) \cdot 3$$

$$= \boxed{3\tan(3x)}$$

b. $g(\theta) = \theta \tan^{-1}(\sqrt{\theta})$ [Hint: $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$]

$$\frac{dg}{d\theta} = \tan^{-1}(\sqrt{\theta}) + \theta \cdot \frac{1}{1+\theta}\left(\frac{1}{2}\theta^{-1/2}\right)$$

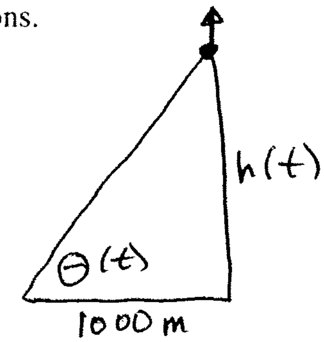
$$= \boxed{\tan^{-1}(\sqrt{\theta}) + \frac{1}{2}\frac{\sqrt{\theta}}{1+\theta}}$$

c. $h(x) = \ln\left(\frac{x(x^2+3)}{\sqrt{x+2}}\right) = \ln(x) + \ln(x^2+3) - \frac{1}{2}\ln(x+2)$

$$\boxed{\frac{dh}{dx} = \frac{1}{x} + \frac{2x}{x^2+3} - \frac{1}{2}\frac{1}{(x+2)}}$$

6. (20 Points total) A ground-based tracking camera is located 1000 meters away from where a rocket is launched straight up. The camera's elevation angle is continuously adjusted to keep the rocket in the center of the rocket's field of view.

a. (5 Points) Draw a diagram illustrating this problem. Use $\theta(t)$ to denote the elevation angle (angle above the horizon) of the camera when it is pointed at the rocket and $h(t)$ to denote the altitude of the rocket. Label all variables and dimensions.



b. (5 points) Determine a trigonometric formula relating $h(t)$ and $\theta(t)$.

$$\tan(\theta(t)) = \frac{h(t)}{1000}$$

c. (5 points) Differentiating the relationship developed in part (b), determine a relationship between the rate of change of the elevation angle, $\frac{d\theta}{dt}$, and the rate of change of the rocket's altitude, $\frac{dh}{dt}$.

$$\sec^2(\theta(t))\frac{d\theta}{dt} = \frac{1}{1000}\frac{dh}{dt}$$

d. (5 Points) When $\theta(t) = \pi/4$, its rate of change is measured to be 0.1 radian/second. At this instant, what is the rate of increase of the rocket's altitude, $\frac{dh}{dt}$? Provide units with your answer.

$\theta(t) = \frac{\pi}{4}$ radians

$\frac{d\theta}{dt} = 0.1$ rad/sec

$\sec\left(\frac{\pi}{4}\right) = \frac{1}{\cos(\pi/4)} = \sqrt{2}$

$$\frac{dh}{dt} = 1000\sec^2(\theta(t))\frac{d\theta}{dt}$$

$$= 1000\cdot(\sqrt{2})^2(0.1)$$

$$= 200 \text{ m/sec}$$

7. (20 Points total) Given that $y(x) = (x+1)^2 + e^{3x}$,

a. (8 Points) Find an expression for dy as a function of x and dx.

$$\frac{dy}{dx} = 2(x+1) + 3e^{3x}$$

Therefore $dy = \left[2x + 2 + 3e^{3x}\right] dx$

b. (7 Points) Using the expression in part (a), determine a linear approximation for $y(x)$ that is valid near $x = 0$.

$$y(0) = (0+1)^2 + e^{3\cdot 0} = 1 + 1 = 2$$

$$\left.\frac{dy}{dx}\right|_{x=0} = 2 + 3 = 5$$

$$y(0+dx) \approx y(0) + \left.\frac{dy}{dx}\right|_{x=0} dx$$

$$y(0+dx) \cong 2 + 5dx$$

c. (5 Points) Based upon this linear approximation, estimate the value of y(x) when $x = 0.1$.

$$y(0.1) = y(0+0.1) \cong 2 + 5(0.1)$$

$$= 2.5$$

$dx = 0.1$