Instructions: Calculators and note sheets are **not allowed**. **No electronic devices may be used for any reason.** Answers with little or no supporting work will receive little or no credit. Work must be **neat**, **organized** and **easily interpreted**. Graders may deduct if your work is not expressed in complete mathematical sentence (i.e., using verbs such as $=, \in$, and $\leq$).

1. (4 points each) Determine the values of x and y.

a. $x = \log_2(4^{\log_2(8)})$

b. $y = \sin[\cos^{-1}(4/5)]$

2. (22 points total) **Differentiate each function**. Simplification is not required. Where indicated, state the domain of the original function. Each derivative is 4 Points. Each requested domain is 1 Point.

a. $f(x) = \tan^{-1}\left(e^{5x}\right)$ Domain of $f(x)$: _____

b. $g(x) = \cos(2x)\sinh(x^2)$

Problem #2 [Continued]

c.　$s(x) = \dfrac{x \ln(x)}{x + e^x}$

d. $h(x) = \ln(x^2 - 7x + 12)$　　　　　　　Domain of $h(x)$: _____

e. $s(t) = t^{\sin(t)}$

3. (15 Points total) Answer the following questions for the function $f(x) = \dfrac{x}{x^2 + 4}$

a. (1 point) Does $f(x)$ have any vertical asymptotes? _____ If so, for what x-value(s)? _____

b. (2 points) Does $f(x)$ have a horizontal asymptote? _____ If so, what value does $f(x)$ approach as x approaches $-\infty$?: _____ As x approaches $+\infty$?: _____

c. (4 points) What are the critical points of $f(x)$ (only the x-values are required)?

d. (6 Points) Find the regions where $f(x)$ is increasing and regions where $f(x)$ is decreasing. Illustrate you answer with a number line sketch **and** then express your final answer in interval notation.

e. (2 Points) Identify any points where $f(x)$ achieves a local maximum or where it reaches a local minimum (provide <u>both</u> x and y coordinates).

(See Over)

4. (12 Points) Using implicit differentiation, DERIVE the formula for the derivative of $y(x) = \cos^{-1}(\ln(x))$

5. (10 Points) The height of the cone in this problem is always equal to 4 times its radius. At time t_0 the cone's height is $40cm$ and the cone's volume is increasing at a rate of $40\pi cm^3$ per minute. What is the rate of change of the cone's radius, $\dfrac{dr}{dt}$? Start by drawing a neat diagram showing all the given information. Make sure your answer includes units.

6. (14 Points total) A rectangular banner has a red border and a rectangular white center. The width of the border at the top and bottom is 1 foot. The border on each side is $\frac{1}{2}$ foot. The total area of the banner is $32\,ft^2$. Your job is to determine the dimensions of the banner, x and y, that will maximize the area of the interior white section. See figure.

 a. (3 Points) What is the constraint equation?

b. (3 Points) What is the objective function?

c. (8 Points) Determine the dimensions x and y that maximize the area of the white section. Be sure to demonstrate that your solution provides a maximum. What is the maximum area obtained?

7. (12 Points) The velocity of a particle is given by $v(t) = 4\sinh(2t) + 5t^2$ for $t \in [0,10]$.

a. (8 points) Given that the particle's initial position at $t = 0$ is $x(0) = 3$, find an equation for $x(t)$ for $t \in [0,10]$.

b. (4 points) What is the particle's position at $t = 3$ seconds? Express $x(3)$, if possible, using exponential functions. Can you estimate the numeric value of $x(3)$?

8. (7 Points) Approximate the area under the function $g(x) = 2x^2 - 7x + 3$ over the interval $x \in [-2, 6]$ using **4 equal subdivisions** (n=4) and the **midpoint** rule. Please use the table provided to organize your calculation of the estimated area. Use of the provided number line may be helpful but is optional.

$$\Delta x =$$
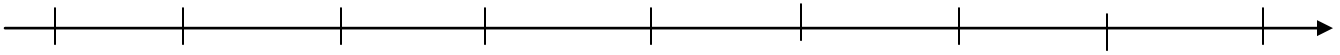
i	x_i	$g(x_i)$	ΔA_i

Approximate Total Area=_____

Bonus Questions [5 points each] (Note: You may do **ONLY TWO** problems. Provide your answers on page 9.)

B.1 (5 Points)   If a and b are both greater than zero and $a^2 + b^2 = 8$, what values of a and b maximize $Q = a^2 b$?

B.2 (5 Points)   Determine the value of $\lim\limits_{x \to 0} \dfrac{\tan(x)}{\sinh(x)}$.

B.3 (5 Points)   Using implicit differentiation, derive the derivative of $y(x) = \sinh^{-1}(x)$

B.4 (5 Points) Use the definite integral $\int\limits_{-2}^{6} (2x^2 - 7x + 3)dx$ to determine the exact area for problem 8.

Provide your solution to your selected bonus problems here:

Solution to First Chosen Bonus Problem [B. ___]

Solution to Second Chosen Bonus Problem [B. ___]

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO!
Note: You may use this page as a scratch area … It will NOT be graded.