Chapter 10: Capacitor & Inductor

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Capacitors are one of the fundamental passive components. In its most basic form, it is composed of two conductive plates separated by an insulating dielectric.

The ability to store charge is the definition of capacitance.
Charging Process

Neutral (uncharged) capacitor

Electrons flow from A to B as the capacitor charges

After the capacitor charges to $V_s$, no electrons flow

Ideally, the capacitor retains charge when disconnected
Application

Wedding Camera’s Flash
Capacitance

- Capacitance is the ratio of charge to voltage
  \[ C = \frac{Q}{V} \]
- The amount of charge on a capacitor is determined by the size of the capacitor (C) and the voltage (V).
  \[ Q = CV \]
- The unit of Capacitance is the farad (F)
- Ex: A certain capacitor stores 75 \( \mu \)C when 5 V are applied across its plates. What is its capacitance?
  \[ C = \frac{Q}{F} = \frac{75}{5} = 15 \ \mu \text{F} \]
Current-voltage relationship

Remind that the current is the rate of charge

\[ i = \frac{dq}{dt} \]

We use derivative when units are small. This equation is the same we have seen in chapter 3.

The current-voltage relationship of the capacitor is

\[ i = C \frac{dv}{dt} \]

A capacitor is an open circuit to dc

The voltage on a capacitor cannot change abruptly
A capacitor stores energy in the form of an electric field that is established by the opposite charges on the two plates. The energy of a charged capacitor is given by the equation

\[ W = \frac{1}{2} CV^2 \]

Where
- \( W \) = the energy in joules
- \( C \) = the capacitance in farads
- \( V \) = the voltage in volts
The capacitance of a capacitor depends on three physical characteristics.

\[ C = (8.85 \times 10^{-12} \, F/m) \left( \frac{\varepsilon_r A}{d} \right) \]

- \(C\) is directly proportional to the relative dielectric constant and the plate area.
- \(C\) is inversely proportional to the distance between the plates.
Example:
Find the capacitance of a 4.0 cm diameter sensor immersed in oil if the plates are separated by 0.25 mm. ($\varepsilon_r = 4.0$ for oil)

$$C = (8.85 \times 10^{-12} \text{ } F/m) \left( \frac{\varepsilon_r A}{d} \right)$$

The plate area is $A = \pi r^2 = \pi (0.02 \text{ m}^2) = 1.26 \times 10^{-3} \text{ m}^2$

The distance between the plates is $0.25 \times 10^{-3} \text{ m}$

$$C = \left( 8.85 \times 10^{-12} \frac{F}{m} \right) \left( \frac{4.0 \times 1.26 \times 10^{-3}}{0.25 \times 10^{-3}} \right) = 178 \text{ pF}$$
### Capacitance

- Some common dielectric materials

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>TYPICAL $\varepsilon_r$ VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air (vacuum)</td>
<td>1.0</td>
</tr>
<tr>
<td>Teflon</td>
<td>2.0</td>
</tr>
<tr>
<td>Paper</td>
<td>2.5</td>
</tr>
<tr>
<td>Oil</td>
<td>4.0</td>
</tr>
<tr>
<td>Mica</td>
<td>5.0</td>
</tr>
<tr>
<td>Glass</td>
<td>7.5</td>
</tr>
<tr>
<td>Ceramic</td>
<td>1200</td>
</tr>
</tbody>
</table>
Mica capacitors are small with high working voltage. The **working voltage** is the voltage limit that cannot be exceeded.
Ceramic disks are small nonpolarized capacitors. They have relatively high capacitance due to high $\varepsilon_r$. 
Electrolytic capacitors have very high capacitance but they are not as precise as other types and tend to have more leakage current. Electrolytic types are polarized.
Variable capacitors typically have small capacitance values and are usually adjusted manually.

A solid-state device that is used as a variable capacitor is the varactor diode; it is adjusted with an electrical signal.
Capacitors use several labeling methods. Small capacitors values are frequently stamped on them such as .001 or .01, which have implied units of microfarads.

Electrolytic capacitors have larger values, so are read as $\mu$F. The unit is usually stamped as $\mu$F, but some older ones may be shown as MF or MMF).
When capacitors are connected in series, the total capacitance is smaller than the smallest one.

The general equation for capacitors in series is

\[
\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_N}
\]

Example: 3 capacitors in series: \( C_1 = 100 \ \mu\text{F} \), \( C_2 = 150 \ \mu\text{F} \) and \( C_3 = 250 \ \mu\text{F} \). What is the total capacitance?

\[
\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_1 C_2 + C_2 C_3 + C_1 C_3}{C_1 C_2 C_3} = \frac{1}{100} + \frac{1}{150} + \frac{1}{250}
\]

\[ C_T = 20 \ \mu\text{F} \]
When capacitors are connected in parallel, the total capacitance is the sum of the individual capacitors. The general equation for capacitors in parallel is

\[ C_T = C_1 + C_2 + C_3 + \cdots + C_N \]
Exercise

Find the total capacitance. All $C = 100 \ \mu F$

$C_T = 146 \ \mu F$
Next: Inductor
When a length of wire is formed into a coil, it becomes a basic inductor. When there is current in the inductor, a three-dimensional magnetic field is created.

A change in current causes the magnetic field to change. This in turn induces a voltage across the inductor that opposes the original change in current.
The current-voltage relationship of the inductor is

\[ di = \frac{1}{L} vdt \quad i = \frac{1}{L} \int_{t_0}^{t} v(t) dt + i(t_0) \]

An inductor acts like a short circuit to dc.

The current through an inductor cannot change instantaneously.
Inductance is a measure of a coil’s ability to establish an induced voltage as a result of a change in its current, and that induced voltage is in a direction to oppose that change in current.

The unit is the **Henry**, symbolized by H.

An inductor stores **energy** in the magnetic field created by the current.

\[ W = \frac{1}{2} LI^2 \]
**Inductance**

- Physical Parameters
  \[ L = \frac{N^2 \mu A}{l} \]
  - \( L \) is the inductance (H)
  - \( N \) is the number of turns
  - \( \mu \) is the permeability (H/m)
  - \( A \) is the cross-sectional area (m\(^2\))
  - \( l \) is the core length (m)
The amount of voltage induced in a coil is directly proportional to the rate of change of the magnetic field with respect to the coil.
When the current through a coil changes and an induced voltage is created as a result of the changing magnetic field, the direction of the induced voltage is such that it always opposes the change in the current.

Switch open: Constant current and constant magnetic field; no induced voltage.

At instant of switch closure: Expanding magnetic field induces voltage, which opposes an increase in total current.

The total current remains the same at this instant.
Lenz’s Laws

- Right after switch closure: The rate of expansion of the magnetic field decreases, allowing the current to increase exponentially as induced voltage decreases.
- Switch remains closed: Current and magnetic field reach constant value.
Lenz’s Laws

- At instant of switch opening: Magnetic field begins to collapse, creating an induced voltage, which opposes a decrease in current.

- After switch opening: Rate of collapse of magnetic field decreases, allowing current to decrease exponentially back to original value.
Types of Inductors

- Fixed
- Variable
- Air core
- Iron core
- Ferrite core
Series & Parallel Inductors

- **Total Series Inductance**

\[ L_T = L_1 + L_2 + L_3 + \cdots + L_n \]

- **Total Parallel Inductance**

\[ L_T = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_n}} \]
The wire has resistance distributed along the length.

Equivalent circuit:

In many applications, the winding resistance can be ignored and the coil can be considered an ideal inductor.

In other cases, the resistance must be considered.
Energy storage and heat conversion

\[ P = I^2 R_W \]

Conversion of electrical energy to heat due to winding resistance

Energy stored in magnetic field

\[ W = \frac{1}{2} LI^2 \]