Chapter 14:
RLC Circuits and Resonance

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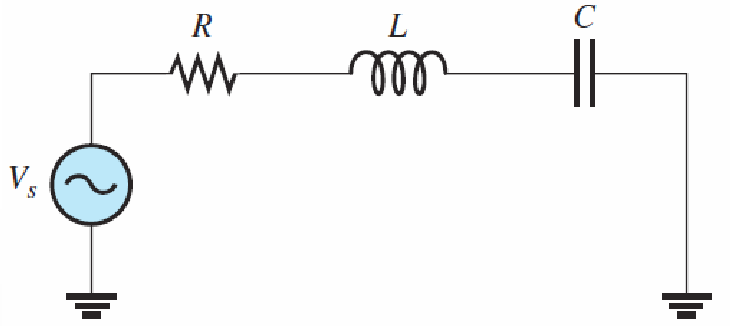
IMPEDANCE AND PHASE ANGLE OF SERIES *RLC* CIRCUITS



$$Z_{\text{tot}} = Z_R + Z_L + Z_C$$

$$Z_{\text{tot}} = R + j\omega L + \frac{1}{j\omega C}$$

$$Z_{\text{tot}} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$|Z_{\text{tot}}| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\theta = tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

$$X_L = 2\pi f L$$

$$X_C = \frac{1}{2\pi f C}$$

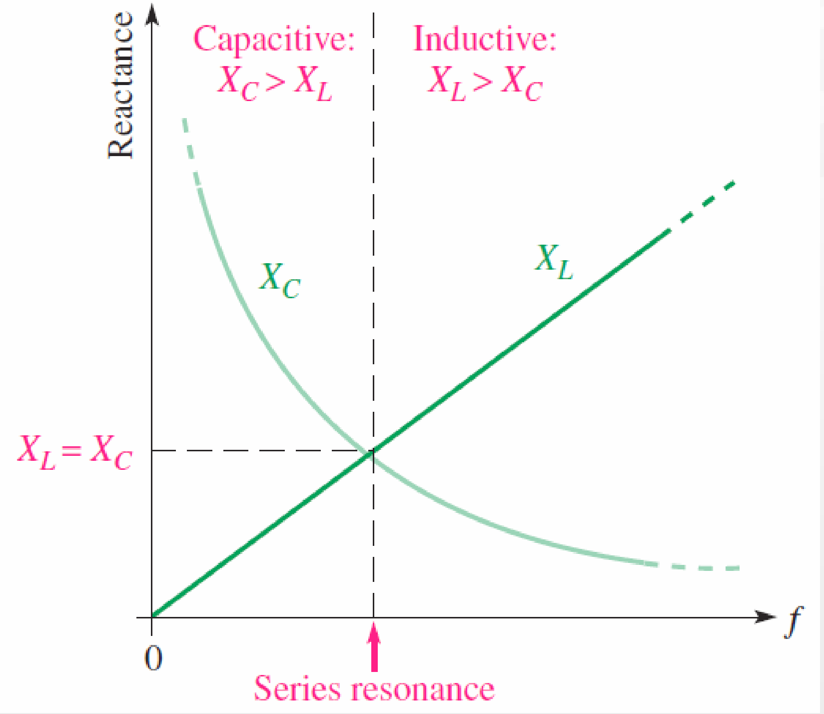
$$X_{tot} = |X_L - X_C|$$

$$Z_{tot} = \sqrt{R^2 + X_{tot}^2}$$

$$\theta = \tan^{-1}\left(\frac{X_{tot}}{R}\right)$$

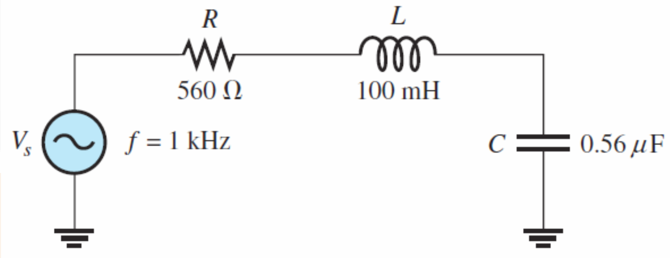
Variation of X_L and X_C with frequency

- In a series *RLC* circuit, the circuit can be capacitive or inductive, depending on the frequency.
- At the frequency where $X_C = X_L$, the circuit is at series resonance.
- Below the resonant frequency, the circuit is predominantly capacitive.
- Above the resonant frequency, the circuit is predominantly inductive.



Exercice

Determine the total impedance and the phase angle for the series *RLC* circuit



First, find X_C and X_L.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1\text{ kHz})(0.56\ \mu\text{F})} = 284\ \Omega$$

$$X_L = 2\pi fL = 2\pi(1\text{ kHz})(100\text{ mH}) = 628\ \Omega$$

In this case, X_L is greater than X_C, so the circuit is more inductive than capacitive. The magnitude of the total reactance is

$$X_{tot} = |X_L - X_C| = |628\ \Omega - 284\ \Omega| = 344\ \Omega \qquad \text{(inductive)}$$

The total circuit impedance is

$$Z_{tot} = \sqrt{R^2 + X_{tot}^2} = \sqrt{(560\ \Omega)^2 + (344\ \Omega)^2} = \mathbf{657\ \Omega}$$

The phase angle (between I and V_s) is

$$\theta = \tan^{-1}\left(\frac{X_{tot}}{R}\right) = \tan^{-1}\left(\frac{344\ \Omega}{560\ \Omega}\right) = \mathbf{31.6^\circ} \qquad \text{(current lagging } V_s\text{)}$$

Example

At $f = 1$ kHz, highly capacitive

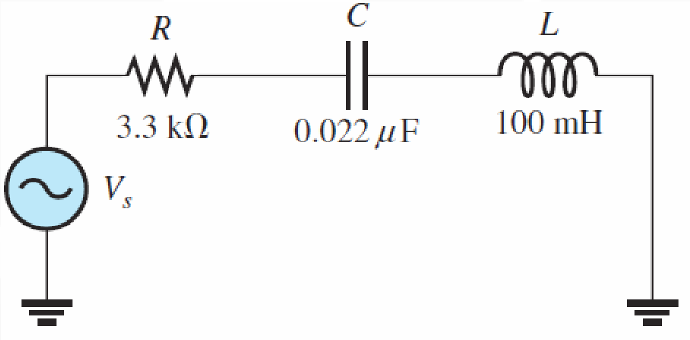
$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1\text{ kHz})(0.022\ \mu\text{F})} = 7.23\text{ k}\Omega$$

$$X_L = 2\pi fL = 2\pi(1\text{ kHz})(100\text{ mH}) = 628\ \Omega$$

$$X_{tot} = |X_L - X_C| = |628\ \Omega - 7.23\text{ k}\Omega| = 6.6\text{ k}\Omega$$

$$Z = \sqrt{R^2 + X_{tot}^2} = \sqrt{(3.3\text{ k}\Omega)^2 + (6.6\text{ k}\Omega)^2} = \mathbf{7.38\text{ k}\Omega}$$

$$\theta = \tan^{-1}\left(\frac{X_{tot}}{R}\right) = \tan^{-1}\left(\frac{6.60\text{ k}\Omega}{3.3\text{ k}\Omega}\right) = \mathbf{63.4^\circ}$$



At $f = 3.5$ kHz,

purely resistive but is slightly inductive

$$X_C = \frac{1}{2\pi(3.5\text{ kHz})(0.022\ \mu\text{F})} = 2.07\text{ k}\Omega$$

$$X_L = 2\pi(3.5\text{ kHz})(100\text{ mH}) = 2.20\text{ k}\Omega$$

$$X_{tot} = |2.20\text{ k}\Omega - 2.07\text{ k}\Omega| = 130\ \Omega$$

$$Z = \sqrt{(3.3\text{ k}\Omega)^2 + (130\ \Omega)^2} = \mathbf{3.30\text{ k}\Omega}$$

$$\theta = \tan^{-1}\left(\frac{130\ \Omega}{3.3\text{ k}\Omega}\right) = \mathbf{2.26^\circ}$$

At $f = 5$ kHz, predominantly inductive

$$X_C = \frac{1}{2\pi(5\text{ kHz})(0.022\ \mu\text{F})} = 1.45\text{ k}\Omega$$

$$X_L = 2\pi(5\text{ kHz})(100\text{ mH}) = 3.14\text{ k}\Omega$$

$$X_{tot} = |3.14\text{ k}\Omega - 1.45\text{ k}\Omega| = 1.69\text{ k}\Omega$$

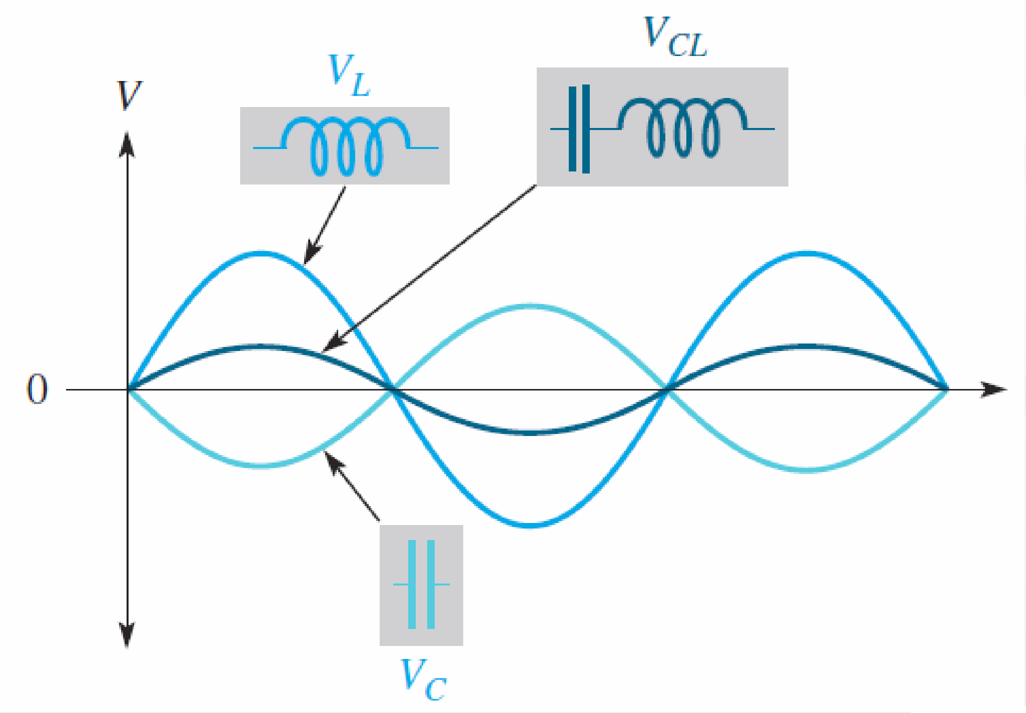
$$Z = \sqrt{(3.3\text{ k}\Omega)^2 + (1.69\text{ k}\Omega)^2} = \mathbf{3.71\text{ k}\Omega}$$

$$\theta = \tan^{-1}\left(\frac{1.69\text{ k}\Omega}{3.3\text{ k}\Omega}\right) = \mathbf{27.1^\circ}$$

Voltages in a series *RLC* circuits

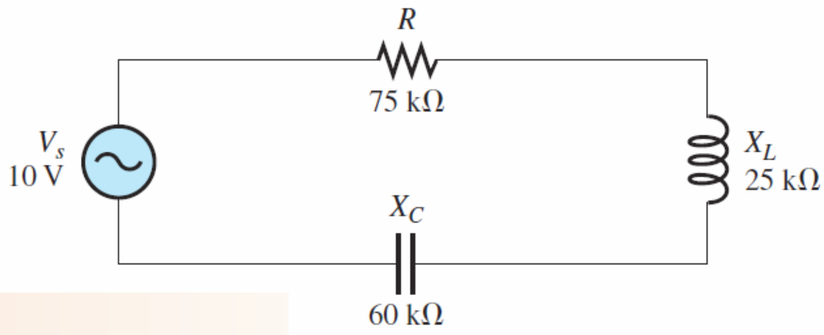
The voltages across the *RLC* components must add to the source voltage in accordance with KVL. Because of the opposite phase shift due to *L* and *C*, V_L and V_C effectively subtract.

Notice that V_C is out of phase with V_L. When they are algebraically added, the result is…



Exercice

Find the voltage across each component



First, find the total reactance.

$$X_{tot} = |X_L - X_C| = |25\text{ k}\Omega - 60\text{ k}\Omega| = 35\text{ k}\Omega$$

The total impedance is

$$Z_{tot} = \sqrt{R^2 + X_{tot}^2} = \sqrt{(75\text{ k}\Omega)^2 + (35\text{ k}\Omega)^2} = 82.8\text{ k}\Omega$$

Apply Ohm's law to find the current.

$$I = \frac{V_s}{Z_{tot}} = \frac{10\text{ V}}{82.8\text{ k}\Omega} = 121\ \mu\text{A}$$

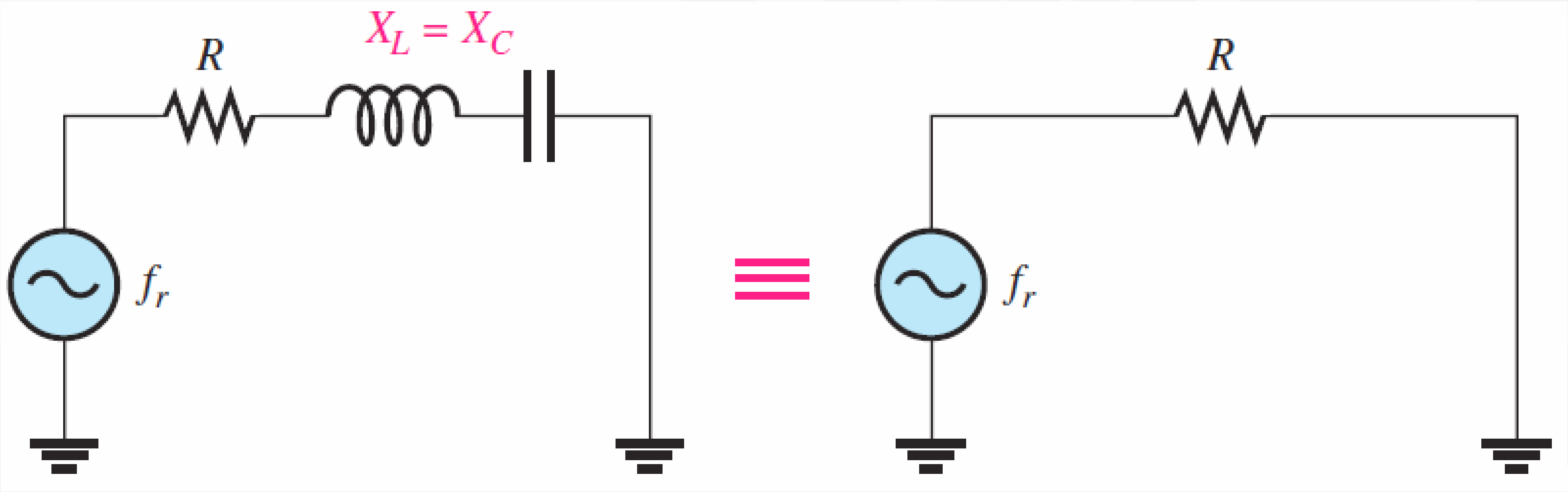
Now, apply Ohm's law to find the voltages across R, L, and C.

$$V_R = IR = (121\ \mu\text{A})(75\text{ k}\Omega) = \mathbf{9.08\ V}$$
$$V_L = IX_L = (121\ \mu\text{A})(25\text{ k}\Omega) = \mathbf{3.03\ V}$$
$$V_C = IX_C = (121\ \mu\text{A})(60\text{ k}\Omega) = \mathbf{7.26\ V}$$

Series Resonance

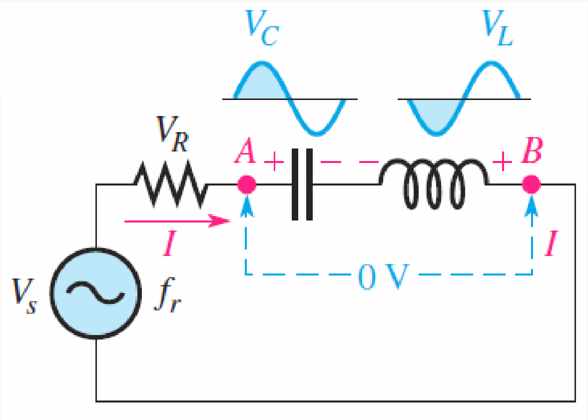
- At series resonance, X_C and X_L cancel. V_C and V_L also cancel because the voltages are equal and opposite. The circuit is purely resistive at resonance.

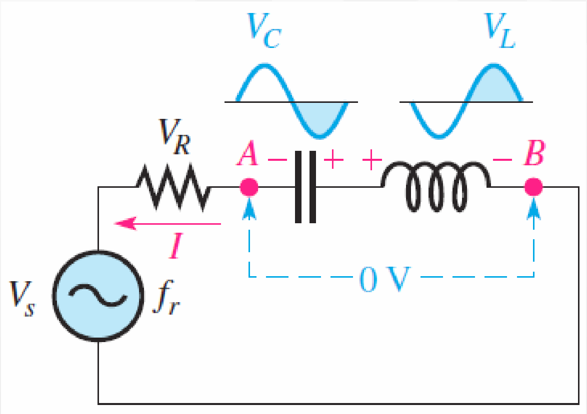
$$X_L = X_C$$
$$Z_r = R$$

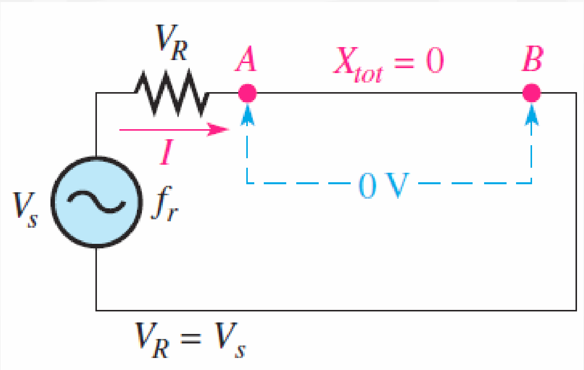


Series Resonance

- At the resonant frequency, f_r, the voltages across *C* and *L* are equal in magnitude.
- Since they are 180^o out of phase with each other, they cancel, leaving 0 V across the *CL* combination (point *A* to point *B*).
- The section of the circuit from *A* to *B* effectively looks like a short at resonance (neglecting winding resistance).







Series Resonance

- For a given series *RLC* circuit, resonance occurs at only one specific frequency. A formula for this resonant frequency is developed as follows:

$$X_L = X_C$$

- Substitute the reactance formulas, and solve for the resonant frequency, f_r

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

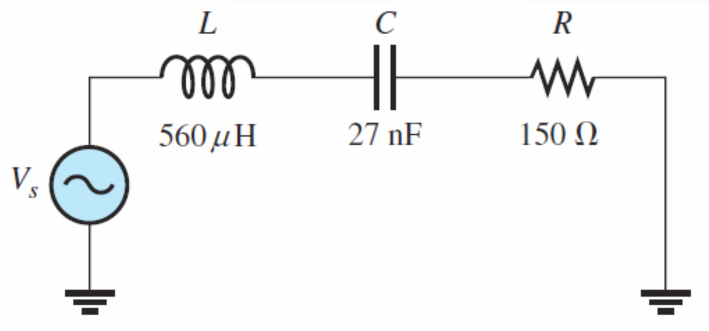
$$(2\pi f_r L)(2\pi f_r C) = 4\pi^2 f_r^2 LC = 1$$

$$f_r^2 = \frac{1}{4\pi^2 LC}$$

- Take the square root of both sides. The formula for resonant frequency is

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Exercice

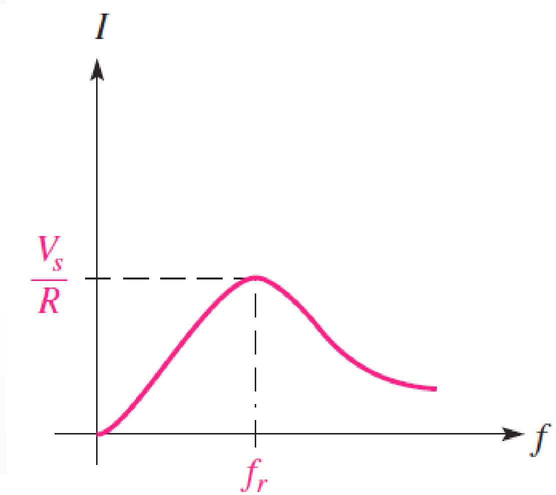


Find the series resonant frequency for the circuit

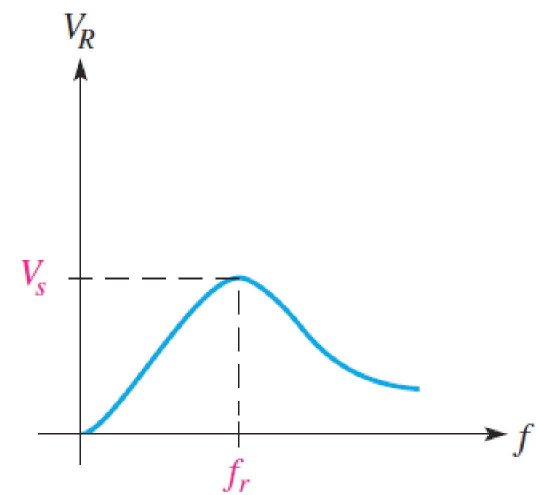
The resonant frequency is

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(560\ \mu\text{H})(27\ \text{nF})}} = \mathbf{40.9\ kHz}$$

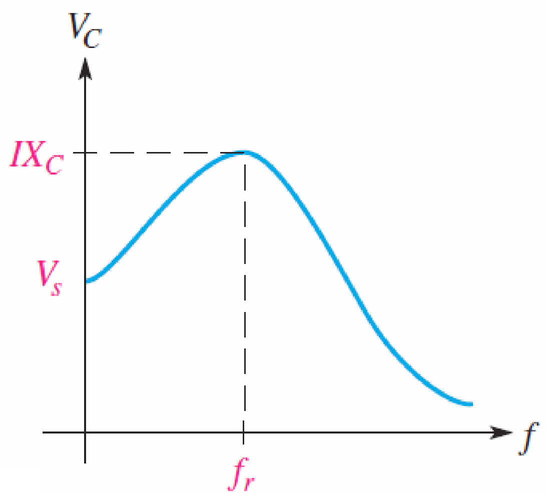
Frequency Response



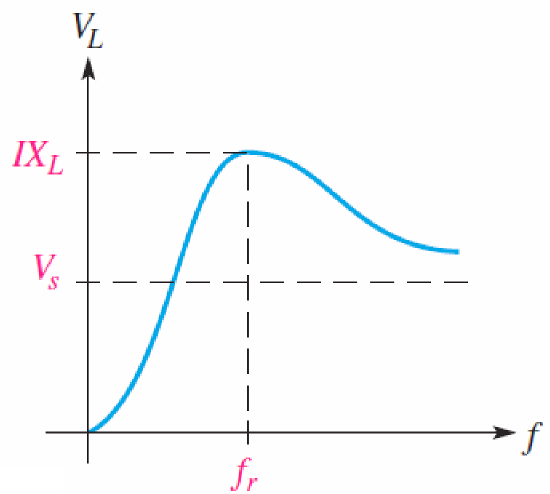
Current



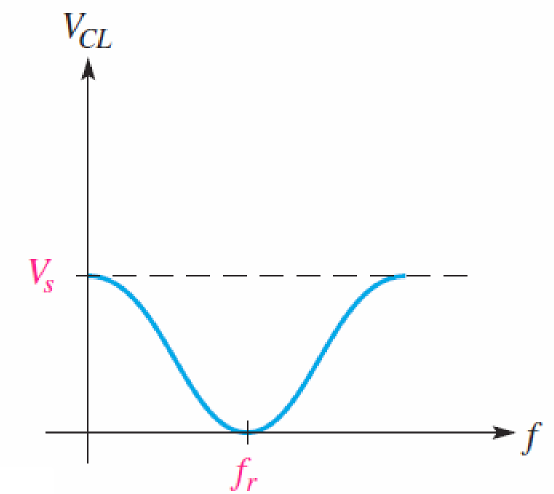
Resistor voltage



Capacitor voltage



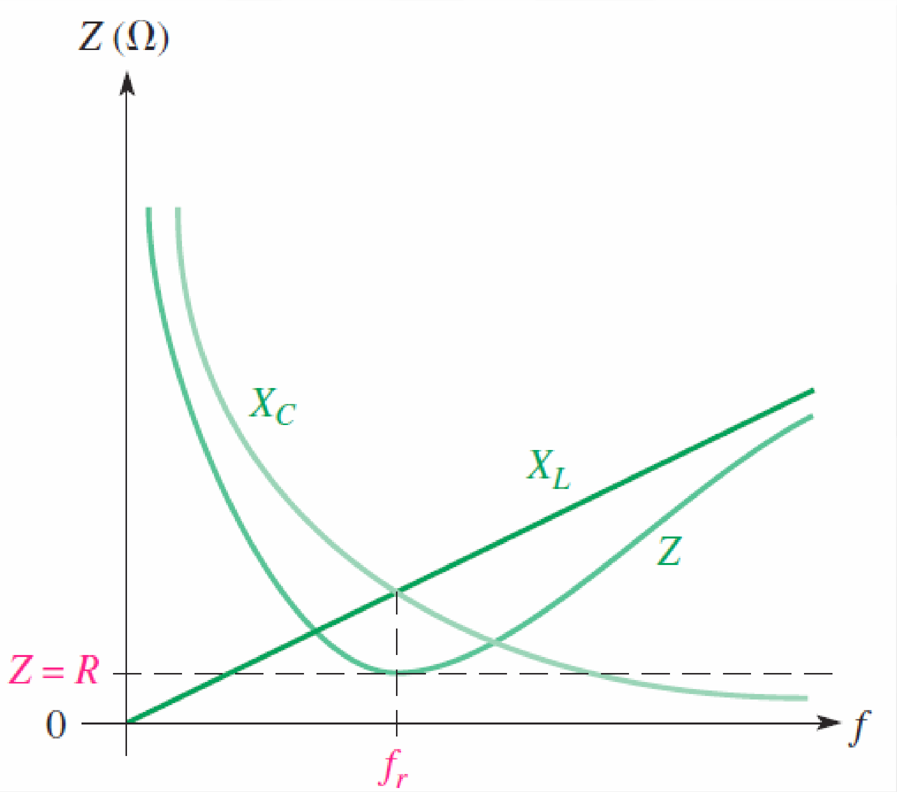
Inductor voltage



Voltage across C and L combined

Impedance of series *RLC* circuits

- The general shape of the impedance versus frequency for a series *RLC* circuit is superimposed on the curves for X_L and X_C. Notice that at the resonant frequency, the circuit is resistive, and $Z = R$



Series Resonance

Summary

- Summary of important concepts for series resonance:
  - Capacitive and inductive reactances are equal.
  - Total impedance is a minimum and is resistive.
  - The current is maximum.
  - The phase angle between V_S and I_S is zero.
  - f_r is given by $f_r = \frac{1}{2\pi\sqrt{LC}}$