Chapter 9: Alternating Current & Voltage

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The sinusoidal waveform (sine wave) is the fundamental alternating current (ac) and alternating voltage waveform.

Electrical sine waves are named from the mathematical function with the same shape.

Sine waves are characterized by the amplitude and period.
Polarity of a Sine Wave

Positive alternation

Negative alternation
The time required for a given sine wave to complete one full cycle is called the **period** \((T)\).

The Unit is the second \((s)\).
Frequency is the number of cycles that a sine wave completes in one second.

The Unit is the hertz (Hz)
Relationship of Frequency and Period

\[ f = \frac{1}{T} \quad \leftrightarrow \quad T = \frac{1}{f} \]

Question: Which sine wave has the higher frequency? Determine the frequency and the period of both waveforms.

\[ T = 333 \text{ ms, } f = 3 \text{ Hz} \quad \text{and} \quad T = 200 \text{ ms, } f = 5 \text{ Hz} \]
The instantaneous value is different at different points along the curve.
Peak Value

- The **peak value** of a sine wave is the value of voltage (or current) at the positive or the negative maximum (peaks) with respect to zero.
- Since positive and negative peak values are equal in magnitude, a sine wave is characterized by a single peak value.
The **peak-to-peak value** of a sine wave is the voltage (or current) from the positive peak to the negative peak.

\[
V_{pp} = 2V_p \\
I_{pp} = 2I_p
\]
The **rms value** (root mean square), also referred to as the **effective value**, of a sinusoidal voltage is actually a measure of the heating effect of the sine wave.

\[
V_{rms} = 0.707 \ V_p \\
I_{rms} = 0.707 \ I_p
\]
For some purposes, the **average value** (actually the halfwave average) is used to specify the voltage or current. By definition, the average value is as 0.637 times the peak value.

\[
V_{avg} = 0.637 \ V_p
\]

\[
I_{avg} = 0.637 \ I_p
\]
Angular Measurement

Angular measurements can be made in degrees (°) or radians. The radian (rad) is the angle that is formed when the arc is equal to the radius of a circle. There are 360° or $2\pi$ radians in one complete revolution.
Radian/Degree Conversion

\[
rad = \left(\frac{\pi \text{ rad}}{180^\circ}\right) \times \text{degrees} \quad \leftrightarrow \quad \text{degrees} = \left(\frac{180^\circ}{\pi \text{ rad}}\right) \times rad
\]
Sine Wave Angles

The angular measurement of a sine wave is based on $360^\circ$ or $2\pi$ rad for a complete cycle. A half-cycle is $180^\circ$ or $\pi$ rad; a quarter-cycle is $90^\circ$ or $\pi/2$ rad; and so on.
VOLTAGE AND CURRENT VALUES OF SINE WAVES

Sin Wave Equation

- Instantaneous values of a wave are shown as \(v\) or \(i\). The equation for the instantaneous voltage \(v\) of a sine wave is

\[ v = V_p \sin \theta \]

- where
- \(V_p = \text{Peak voltage}\)
- \(\theta = \text{Angle in rad or degrees}\)
The phase of a sine wave is an angular measurement that specifies the position of a sine wave relative to a reference. To show that a sine wave is shifted to the left or right of this reference, a term is added to the equation given previously.

\[ v = V_p \sin(\theta \pm \phi) \]

- where
- \( \phi = \text{Phase shift} \)
Phase of Sine Wave

The phase of a sine wave is an angular measurement that specifies the position of that sine wave relative to a reference.
Exercise

Determine $V_p$, $V_{pp}$, $V_{rms}$, and the half-cycle $V_{avg}$ for the sine wave

$V_p=4.5$ V, $V_{pp}=9$ V, $V_{rms}=3.18$ V, $V_{avg}=2.87$ V
Exercise

Determine the instantaneous value at $90^\circ$ on the horizontal axis for each voltage sine wave.

Sine wave $A$ is the reference. Sine wave $B$ is shifted left $20^\circ$ with respect to $A$, so $B$ leads. Sine wave $C$ is shifted right $45^\circ$ with respect to $A$, so $C$ lags.

\[
\begin{align*}
\nu_A &= V_p \sin \theta = (10 \text{ V}) \sin 90^\circ = 10 \text{ V} \\
\nu_B &= V_p \sin(\theta + \phi_B) = (5 \text{ V}) \sin(90^\circ + 20^\circ) = (5 \text{ V}) \sin 110^\circ = 4.70 \text{ V} \\
\nu_C &= V_p \sin(\theta - \phi_C) = (8 \text{ V}) \sin(90^\circ - 45^\circ) = (8 \text{ V}) \sin 45^\circ = 5.66 \text{ V}
\end{align*}
\]
VOLTAGE AND CURRENT VALUES OF SINE WAVES

The Phase Shift

- An important application of phase-shifted sine waves is in electrical power systems. Electrical utilities generate ac with three phases that are separated by 120° as illustrated.

- Normally, 3-phase power is delivered to the user with three hot lines plus neutral. The voltage of each phase, with respect to neutral is 120 V.
Power in resistive AC circuits

- The power relationships developed for dc circuits apply to ac circuits except you must use rms values in ac circuits when calculating power.

- Power formulas are:
  - \[ P = V_{rms} I_{rms} \]
  - \[ P = \frac{V_{rms}^2}{R} \]
  - \[ P = I_{rms}^2 R \]

- For example, the dc and the ac sources produce the same power to the bulb.
Example:
Assume a sine wave with a peak value of 40 V is applied to a 100 W resistive load. What power is dissipated?

Solution: $V_{rms} = 0.707 \times V_p = 0.707 \times 40 \text{ V} = 28.3 \text{ V}$

$P = \frac{V_{rms}^2}{R} = \frac{28.3}{100} = 8 \text{ W}$
Superimposed dc and ac voltages

Frequently dc and ac voltages are together in a waveform. They can be added algebraically, to produce a composite waveform of an ac voltage “riding” on a dc level.

\[ V_{DC} + V_p \]  
Nonalternating

\[ V_{DC} < V_p \]  
Alternating
Alternators are ac generators. Utility companies use 3-phase alternators and deliver all three phases to industrial customers.

The rotor shown is a permanent magnet that produces a strong magnetic field. As it sweeps by each stator winding, a sine wave is produced across that winding. The neutral is the reference.
In vehicles, alternators generate ac, which is converted to dc for operating electrical devices and charging the battery. AC is more efficient to produce and can be easily regulated, hence it is generated and converted to dc by diodes.

The output is taken from the rotor through the slip rings.
A pulse can be described as a very rapid transition (leading edge) from one voltage or current level (baseline) to another level; and then, after an interval of time, a very rapid transition (trailing edge) back to the original baseline level.
Nonsinusoidal Waveforms

Pulse Waveform

Actual pulses are never ideal

Rise and fall times

Rise and fall times are measured between the 10% and 90% levels.

Pulse width

Pulse width is measured at the 50% level.
Repetitive Pulses: Any waveform that repeats itself at fixed intervals is periodic.

The **duty cycle** is the ratio of the pulse width ($t_W$) to the period ($T$) and is usually expressed as a percentage.

Percent duty cycle $= \left( \frac{t_W}{T} \right) \times 100\%$
Exercise

- Determine the period, frequency, and duty cycle for the pulse waveform.

- $T = 10 \mu s \quad f = \frac{1}{T} = \frac{1}{10} = 100 \, kHz$

- Percent duty cycle $= \left( \frac{t_W}{T} \right) 100\% = \left( \frac{1\mu s}{10\mu s} \right) 100\% = 10\%$
**Nonsinusoidal Waveforms**

**Pulse Waveform**

A square wave is a pulse waveform with a duty cycle of 50%.

The average value of a pulse waveform is equal to its baseline value plus the product of its duty cycle and its amplitude

$$V_{avg} = \text{baseline} + (\text{duty cycle})(\text{amplitude})$$

**Example:** Determine the average voltage of the positive-going waveforms

$$V_{avg} = 1 \text{ V} + (50)(5 \text{ V}) = 1 + 2.5 \text{ V} = 3.5 \text{ V}$$
Triangular and sawtooth waveforms are formed by voltage or current ramps (linear increase/decrease).

Triangular waveforms have positive-going and negative-going ramps of equal duration.

The sawtooth waveform consists of two ramps, one of much longer duration than the other.
Harmonics

- All repetitive non-sinusoidal waveforms are composed of a **fundamental frequency** (repetition rate of the waveform) and **harmonic frequencies**.
- **Odd harmonics** are frequencies that are odd multiples of the fundamental frequency.
- **Even harmonics** are frequencies that are even multiples of the fundamental frequency.
Oscilloscope

Typical oscilloscopes. Copyright © Tektronix
A sinusoidal voltage is applied to the resistive circuit.

Determine: $I_p$, $I_{pp}$, $I_{rms}$, $I_{avg}$ & $i$ at the positive peak

- $I_p = \frac{V_p}{R} = 10 \text{ mA} / I_{pp} = 20 \text{ mA}$
- $I_{rms} = 7.07 \text{ mA} / I_{avg} = 0 \text{ A}$
- $i$ at the positive peak $= 10 \text{ mA}$