FINITE ELEMENT MODELING OF THE ARC REATTACHMENT PROCESS IN DC PLASMA TORCHES

Juan Pablo Trelles, Emil Pfender, and Joachim V. R. Heberlein

Department of Mechanical Engineering, University of Minnesota

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DC Arc Plasma Torches

- arc length $\propto$ voltage drop
- force balance: drag vs. magnetic
- arc dynamics $\rightarrow$ jet forcing
- enhanced cold flow entrainment
Approaches for Arc Modeling

1. Thermal **equilibrium** \((\text{LTE})\) + **high \(\sigma\)** at electrodes:

2. **LTE** + **high \(\sigma\)** at electrodes + **arc reattachment model**:

3. Thermal **non-equilibrium** \((\text{NLTE})\):
Numerical Approach: Stabilized FEM

- **System of transient – advective – diffusive – reactive equations:**

\[
A_0 \frac{\partial Y}{\partial t} + (A \cdot \nabla)Y - \nabla \cdot (K \nabla Y) - (S_1 Y + S_0) = R(Y) = 0
\]
Numerical Approach: Stabilized FEM

- **System of transient – advective – diffusive – reactive equations:**

  \[
  A_0 \frac{\partial Y}{\partial t} + \left( A \cdot \nabla \right) Y - \nabla \cdot (K \nabla Y) - (S_1 Y + S_0) = R(Y) = 0
  \]

- **Stabilized and Multi-scale Methods:** \( Y = Y + Y' \)

  \[
  \int_{\Omega} W \cdot R(Y) d\Omega + \int_{\Omega'} P(W) \tau \cdot R(Y) d\Omega' = 0
  \]

  total = large + small

- **Solution:** \( \alpha \)-method, Globalized Inexact-Newton, Pre-Cond. GMRES
Arc Dynamics: Approach 1 (LTE)

1. old attachment
2. new attachment forms
3. new attachment remains

➢ Too large voltage drop !!!
Reattachment Process

1. attachment movement

2. new attachment appears

3. new attachment remains

experiments, simplified geometry

Arc Reattachment Model

- **Specify** “breakdown” electric field $E_b$ (parameter)
- **If** $\max E_n > E_b \Rightarrow$ reattachment occurs:

\[ \sigma_b(X) \text{ profile:} \]
Arc Dynamics: Approach 2 (LTE + Reattachment)

✓ reattach. cond.

attachment growth

new attachment

➤ More realistic voltage drop
Comparison with Experiments

\[ E_b = 5 \cdot 10^4 \text{ V/m} \]

\[ E_b = 2 \cdot 10^4 \text{ V/m} \]

- Adjusting \( E_b \) can match Voltage Drop Frequency OR Magnitude BUT NO BOTH

\[ f_p \sim 3.30 \]

\[ E_b = 5 \cdot 10^4 \text{ V/m} \]

\[ f_p \sim 3.23 \]

\[ E_b = 2 \cdot 10^4 \text{ V/m} \]

\[ f_p \sim 8.98 \]

\[ E_b = 2 \cdot 10^4 \text{ V/m} \]
Improved Approach: Non-Equilibrium Model

- Thermal non-equilibrium \((T_e \neq T_h)\) (NLTE):

<table>
<thead>
<tr>
<th>(i)</th>
<th>(Y_i)</th>
<th>transient</th>
<th>advective</th>
<th>diffusive</th>
<th>reactive</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(p)</td>
<td>(\frac{\partial p}{\partial t})</td>
<td>(\bar{u} \cdot \nabla p + \rho \nabla \cdot \bar{u})</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>(\bar{u})</td>
<td>(\rho \frac{\partial \bar{u}}{\partial t})</td>
<td>(\rho \bar{u} \cdot \nabla \bar{u} - \nabla p)</td>
<td>(-\nabla \cdot \vec{r})</td>
<td>(\vec{J}_q \times \vec{B})</td>
</tr>
<tr>
<td>3</td>
<td>(T_h)</td>
<td>(\rho \frac{\partial h_h}{\partial t})</td>
<td>(\rho \bar{u} \cdot \nabla h_h)</td>
<td>(-\nabla \cdot \vec{q}_h)</td>
<td>(\frac{Dp_h}{Dt} + \dot{Q}_{eh})</td>
</tr>
<tr>
<td>4</td>
<td>(T_e)</td>
<td>(\rho \frac{\partial h_e}{\partial t})</td>
<td>(\rho \bar{u} \cdot \nabla h_e)</td>
<td>(-\nabla \cdot \vec{q}_e)</td>
<td>(\frac{Dp_e}{Dt} + \dot{Q}_j - \dot{Q}<em>r - \dot{Q}</em>{eh})</td>
</tr>
<tr>
<td>5</td>
<td>(\phi_p)</td>
<td>0</td>
<td>0</td>
<td>(-\nabla \cdot \vec{J}_q)</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>(\vec{A})</td>
<td>(\frac{\partial \vec{A}}{\partial t})</td>
<td>(\nabla \phi_p - \bar{u} \times \nabla \times \vec{A})</td>
<td>(\eta \nabla^2 \vec{A})</td>
<td>0</td>
</tr>
</tbody>
</table>

If \(T_e = T_h\) \(\Rightarrow\) LTE model
Arc Dynamics: Approach 3 (NLTE)
Comparison with Experiments

- Voltage frequencies NLTE & LTE can match
- BUT … more realistic voltage drops with NLTE model
- Wide spectra in exp. data due to pure Ar & new anode
Conclusions & Further Work

1) Development of Reattachment Model for LTE simulations

2) Experimental voltage magnitude OR frequency can be matched by model … but NOT BOTH

3) A Thermal Non-Equilibrium Model gives better agreement with experimental voltage magnitudes AND frequencies

4) Non-equilibrium description essential for realistic arc modeling

Future Work: NLTE model to gas mixtures (Ar-He, Ar-H₂) + chemical non-equilibrium