Theorem 1: Cauchy-Riemann Equations:

Let \( f(z) = u(x,y) + i \; v(x,y) \) be defined and continuous in some neighborhood of a point \( z = x+iy \) and differentiable at \( z \) itself. Then, at that point, the first order partial derivative of \( u \) and \( v \) exist and satisfy the Cauchy-Riemann equations.

\[
\frac{du}{dx} = \frac{dv}{dy} \quad \text{and} \quad \frac{dv}{dx} = -\frac{du}{dy}
\]

Hence, if \( f(z) \) is analytic in a domain \( D \), those partial derivatives exist and satisfy Cauchy-Riemann Equations at all points of \( D \).
Theorem 2: Cauchy-Riemann Equations:

If two real-valued continuous functions \(u(x,y)\) and \(v(x,y)\) of two real variables \(x,y\) have continuous first partial derivatives that satisfied the Cauchy-Riemann equations in some domain \(D\), then the complex function \(f(z) = u(x,y) + i \ v(x,y)\) is analytic in \(D\).

Theorem 3: Laplace’s Equations:

If \(f(z) = u(x,y) + i \ v(x,y)\) is analytic in a domain \(D\), then both \(u\) and \(v\) satisfy Laplace’s equation:

\[
\nabla^2 u(x, y) = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0
\]

\[
\nabla^2 v(x, y) = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0
\]

In \(D\) and have continuous second partial derivatives in \(D\).