Section instructor ____________________ ***Section number*** __________

Last/First name Dawylo ____________________

Last 3 Digits of Student ID Number: __________

Show all work. Show all formulas used for each problem prior to substitution of numbers. Label diagrams and include appropriate units for your answers. *You may use an alphanumeric calculator during the exam as long as you do not program any formulas into memory. By using an alphanumeric calculator you agree to allow us to check its memory during the exam. Simple scientific calculators are always OK!*

A Formula Sheet Is Attached To The Back Of This Examination

Be Prepared to Show your Student ID Card

Score on each problem:

1. (30) ____

2. (20) ____

3. (20) ____

4. (20) ____

Total Score (out of 90 pts) ____

1. Conceptual Questions

(30 point) Put a circle around the letter that you think is the best answer.

1.1. (6pts) An electron is released from rest at x = 2 m in the potential shown. What does the electron do right after being released?

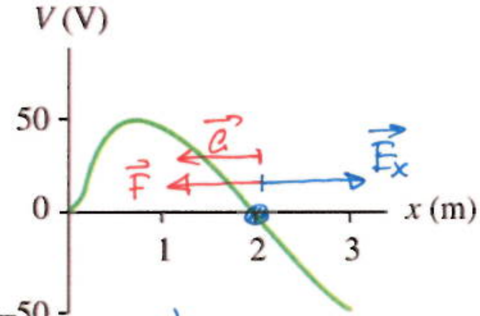
A) Stay at $x = 2$ m

B) Move to the right ($+x$) at steady speed

C) Move to the right with increasing speed

D) Move to the left ($-x$) at steady speed

E) Move to the left with increasing speed

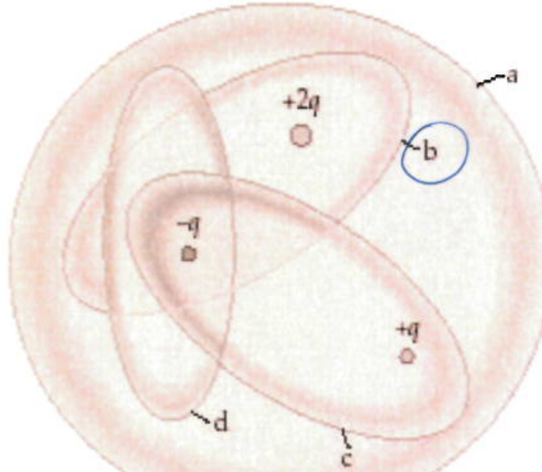


$E_x = -\frac{dV}{dx}$; since $\frac{dV}{dx} < 0 \Rightarrow E_x > 0$ (to the right)

Since $q<0$ (electron) and $\vec{F} = q\vec{E}$, $\Rightarrow \vec{F} < 0$, to the left.

Since $\vec{F} = m\vec{a} \Rightarrow \vec{a}$ is to the left. So, it moves to the left with increas. speed.

1.2.* (6pts) *Figure shows four Gaussian surfaces surrounding a distribution of charges. Which Gaussian surfaces have an electric flux of $+q/\varepsilon_0$ through them?



A) a

B) b

C) b and d

D) b and c

E) c

$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$

$Q_{in} = q$ only for a surface "b"!

1.3.(6pts) *This is a graph of the x-component of the electric field along the x-axis. The potential is zero at the origin. What is the potential at x=1m?*

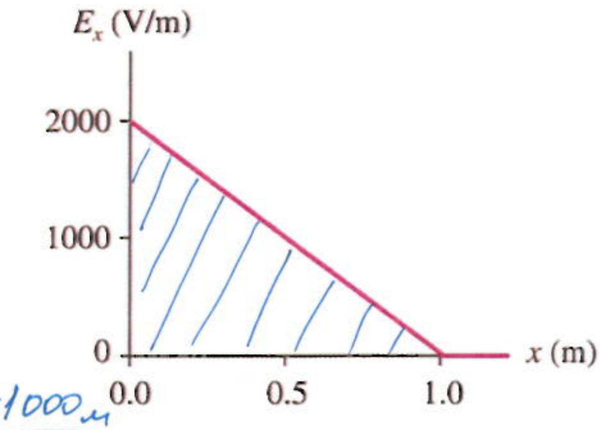
A) 2000 V

B) 1000 V

C) 0 V

D) -1000 V

E) -2000 V

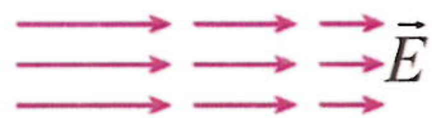


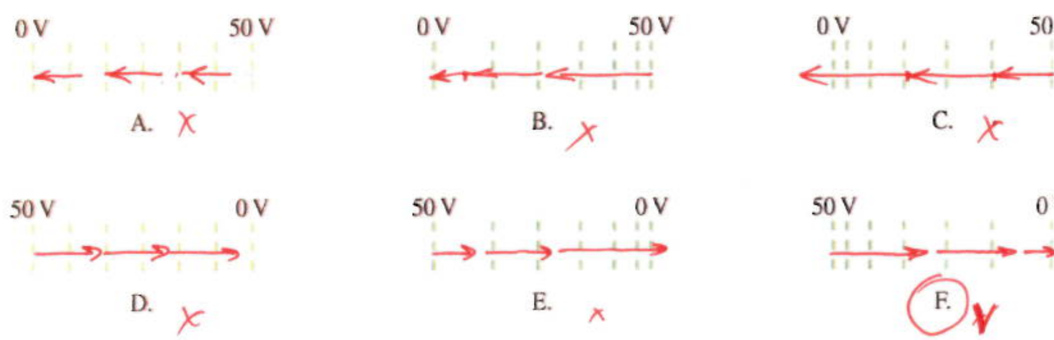
$V \leftarrow \vec{E}$

$\Delta V = V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$ $i \to x = 0$, $f = x = 1m$

$V(x=1m) = -\text{Area} = -\frac{1}{2} \cdot 1m \cdot 2000\,V/m = -1000\,V$

1.4. *(6pts) Which set of equipotential surfaces matches this electric field?*

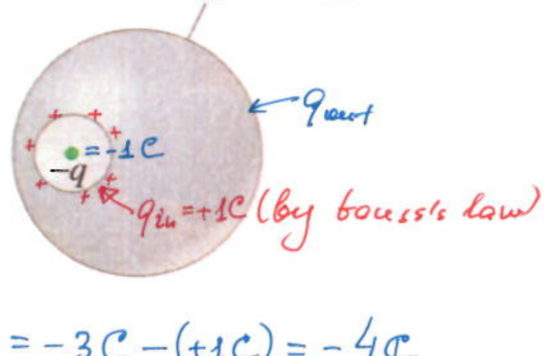




1.5. **(6pts)** ***A solid conductor carries a net ~~positive~~ negative charge Q = -3 C. There is a hollow cavity within the conductor, at whose center is a negative point charge -1 C. What is the charge on***

(a) the inner surface of the conductor's cavity

q_{inner}= +1 C



(b) the outer surface of the conductor?

q_{outer}= −4 C

conserv. of charge

$Q_{total} = q_{in} + q_{out} \Rightarrow q_{out} = Q_{total} - q_{in} = -3C - (+1C) = -4C$

Problem 2. (20 pts)

An electric charge +Q is distributed uniformly throughout a nonconducting sphere of radius R.

(a) (2pts) Draw a Gaussian surface in the figure;

(b) (2pts) Draw electric field and area vectors;

(c) (6pts) Determine the electric field outside the sphere (r > R)

(d) (10pts) Determine the electric field inside the sphere (r < R)

(show how you handle a linear integral)

c) Gauss's law ($r \geq R$, outside)

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

Gauss. surface.

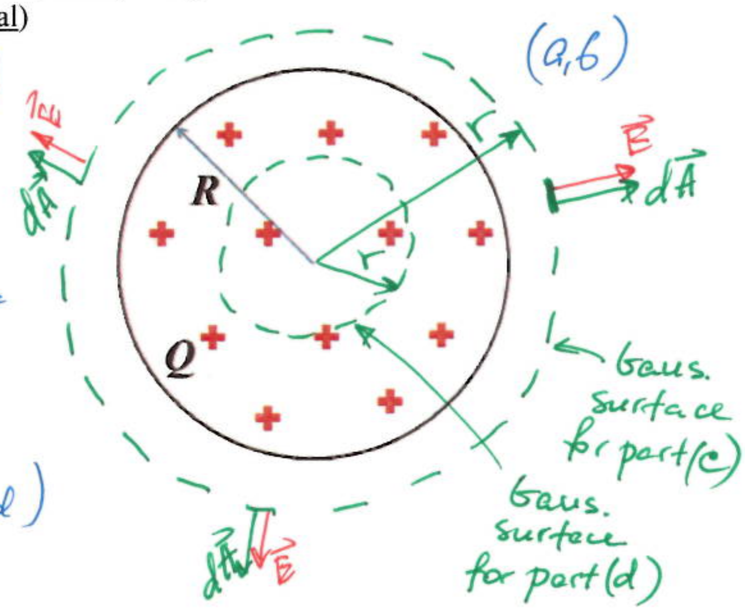
$$\oint \vec{E} \cdot d\vec{A} = \left\| \vec{E} \uparrow\uparrow d\vec{A} \right\| = \oint E\, dA = \left\| E = \text{const} \right\| =$$

$$= E \oint dA = \left\| \oint dA = 4\pi r^2 \right\| = 4\pi r^2 \cdot E = \frac{Q_{in}}{\varepsilon_0}$$

(area sphere with r)

$Q_{in} = Q$ (the whole charge is enclosed)

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}, \quad r \geq R$$



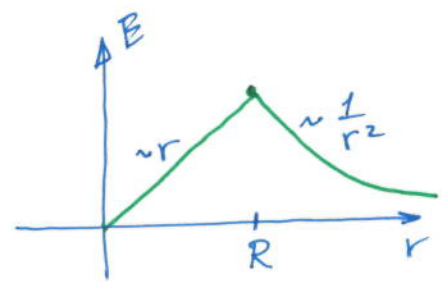
d) $r \leq R$ (inside)

similar $E \cdot 4\pi r^2 = \frac{Q_{in}}{\varepsilon_0}$

$$Q_{in} = \rho \cdot V_r = \left\| \rho = \frac{Q}{V_R} = \frac{Q}{\frac{4\pi}{3}R^3} \right\| = \left(\frac{Q}{\frac{4\pi}{3}R^3}\right) \cdot \frac{4\pi}{3} r^3 = Q \cdot \frac{r^3}{R^3}, \text{ so}$$

(volume charge density)

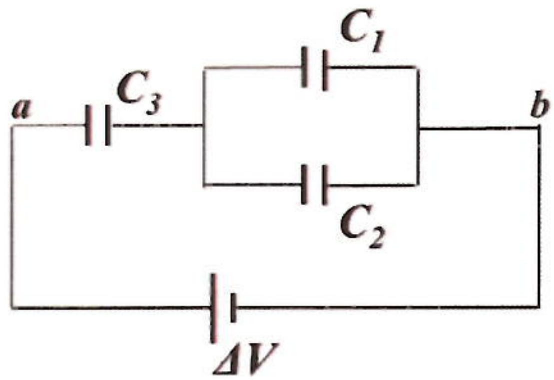
$$E \cdot 4\pi r^2 = \frac{Q \frac{r^3}{R^3}}{\varepsilon_0} \Rightarrow E(r) = \frac{Q \cdot r}{4\pi\varepsilon_0}$$



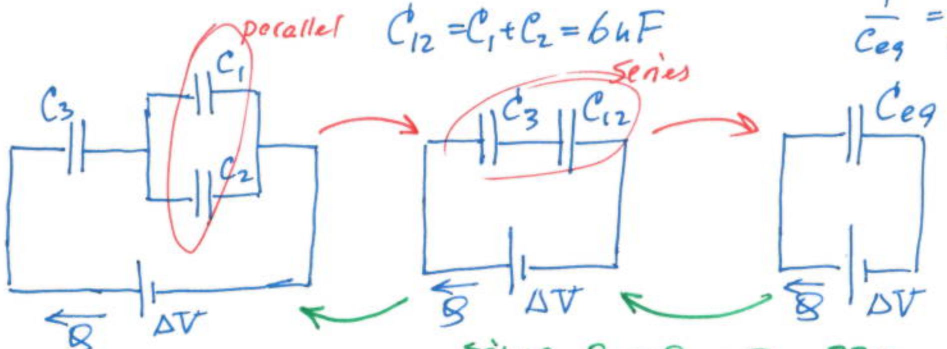
Problem 3. (20 pts)

(a) ***Determine the equivalent capacitance between points a and b. Assume that $C_1=2\ nF$, $C_2=4\ nF$, $C_3=8\ nF$, $\Delta V=3V$.***

(b) ***What are the charge on and the potential difference across each capacitor in the figure.***



$C = \frac{Q}{\Delta V}$



$C_{12} = C_1 + C_2 = 6\,uF$

$\frac{1}{C_{eq}} = \frac{1}{C_3} + \frac{1}{C_{12}} \Rightarrow C_{eq} = \frac{C_3 \cdot C_{12}}{C_3 + C_{12}} = \frac{8 \cdot 6}{8+6}$

$C_{eq} = \frac{48}{14}\,uF \approx 3.428\,uF$

$Q = C_{eq} \cdot \Delta V = \frac{48}{14}\,uF \cdot 3V = \frac{72}{7}\,uC \approx 10.286\,uC$

since $Q_3 = Q_{12} = Q = 72/7\ uC$

$\Delta V_3 = \frac{Q_3}{C_3} = \frac{72/7\ uC}{8\,uF} = \frac{9}{7}V \approx 1.285V$

$\Delta V_{12} = \frac{Q_{12}}{C_{12}} = \frac{72/7\ uC}{6\,uF} = \frac{12}{7}V \approx 1.714V$

since $\Delta V_1 = \Delta V_2 = \Delta V_{12} = \frac{12}{7}V$

$Q_1 = C_1 \cdot \Delta V_1 = 2\,uF \cdot \frac{12}{7}V = \frac{24}{7}\,uC$

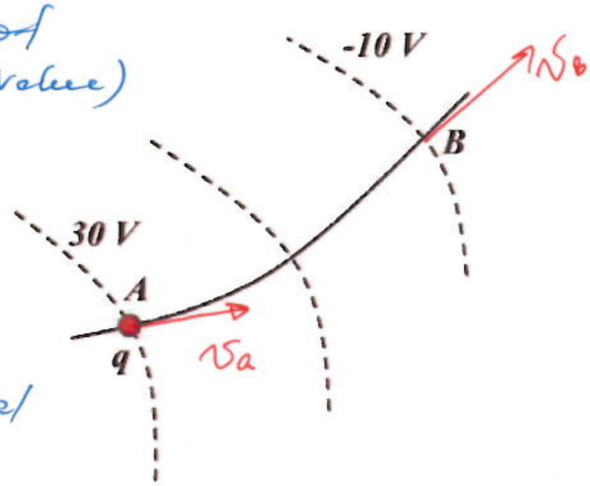
$Q_1 \approx 3.428\,uC$

$Q_2 = C_2 \cdot \Delta V_2 = 4\,uF \cdot \frac{12}{7}V = \frac{48}{7}\,uC$

$Q_2 \approx 6.857\,uC$

Problem 4 (20 pts).

A proton's speed as it passes point A is 200,000 m/s. It follows the trajectory shown in the figure with a solid line. The dashed lines in the figure are equipotential lines. What is the proton's speed at point B?



There is not enough information to use the "force approach". We can not find the el. field (only the averaged value). So, "the energy approach"

Conservation of Energy:

$$E_a = E_b$$

$$K_a + U_a = K_b + U_b$$

since $U = qV$, V - el. potential

$$K_b = K_a + q(V_a - V_b)$$

$$\frac{1}{2} m v_b^2 = \frac{1}{2} m v_a^2 + q(V_a - V_b)$$

$$v_b = \sqrt{v_a^2 + \frac{2q}{m}(V_a - V_b)} =$$

$$v_b = \sqrt{(200{,}000\ m/s)^2 + \frac{2(1.6\cdot 10^{-19} C)}{1.67\cdot 10^{-27} kg}\cdot(30V - (-10V))} = 218.3\ m/s$$

Formula Sheet: Electricity and Magnetism

Coulomb's law

$$F = k\frac{qQ}{r^2}$$

Electric Field

$$\vec{E} = \frac{\vec{F}}{q}$$

Field of a point charge

$$E = k\frac{Q}{r^2}$$

Electric field inside a capacitor

$$E = \frac{\eta}{\varepsilon_0}$$

Principle of superposition

$$\vec{E}_{net} = \sum_{i=1}^{N} \vec{E}_i$$

Electric flux

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

Gauss's law

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

Electric potential

$$V = \frac{U}{q}$$

$$\Delta V = V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

For a point charge $V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

For a paralle-plate capacitor

$$V = Es$$

Potential Energy

$$U = qV$$

Two point charges

$$U = k\frac{qQ}{r}$$

Capacitors

$$C = \frac{Q}{\Delta V}$$

Parallel-plate $C = \varepsilon_0\frac{A}{d}$

Capacitors connected in parallel

$$C_{eq} = C_1 + C_2 + \cdots$$

Capacitors connected in series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$$

Energy stored in a capacitor

$U = \frac{Q^2}{2C}$

Constants

Charge on electron

$e = 1.60 \cdot 10^{-19}\, C$

Electron mass $m = 9.11 \cdot 10^{-31}\, kg$

Proton mass $m = 1.67 \cdot 10^{-27}\, kg$

Permittivity of free space

$$\varepsilon_0 = 8.85 \cdot 10^{-12}\, C^2/Nm^2$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.99 \cdot 10^9\, Nm^2/C^2$$

Kinematic eq-ns with const. accel:

$v(t) = v_{0x} + at$

$x(t) = x_0 + v_{0x}t + (1/2)\, at^2$

$v^2 = v_{0x}^2 + 2a(x - x_0)$

$L = 2\pi R$

$A = \pi R^2$

$V = (4/3)\pi R^3$