Last/First name A. Danylov

1. 30 pts ______
2. 20 pts ______
3. 20 pts ______
4. 10 pts ______

1. Conceptual Questions

(30 point) Put a circle around the letter that you think is the best answer.

1.1. (6pts) *The line integral of B around the loop shown in Fig.3 is $\mu_0 \cdot 5.0$ A. Current I_4 is*

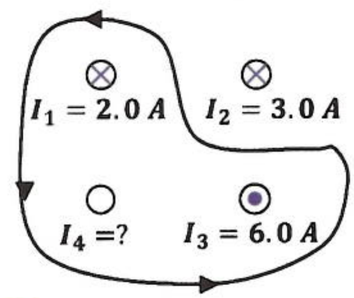
A) 0 A

B) 3.0 A into the page

C) 3.0 A out of the page

D) 1.0 A into the page

E) 1.0 A out of the page



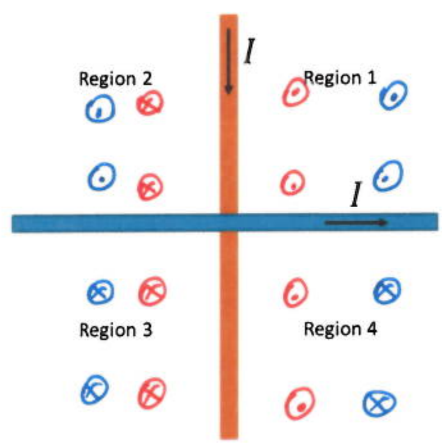
$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{in}$

$\mu_0 \cdot 5A = \mu_0(-I_1 + I_3 + I_4)$

$5A = -2A + 6A + I_4 \Rightarrow I_4 = 1A$ (out of the page)

1.2. (6pts). Two fixed wires cross each other perpendicularly so that they do not actually touch but are very close to each other, as shown in the figure. Equal currents I exist in each wire in the directions indicated. In what region(s) will there be some points of zero net magnetic field?

Regions 2 and 4.



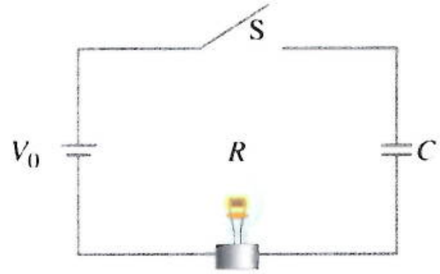
1.3. (6pts) In the circuit shown, the capacitor is originally uncharged. Describe the behavior of the lightbulb from the instant switch S is closed until a long time later.

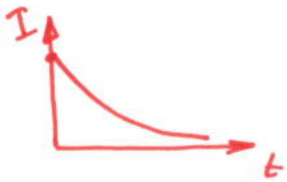
A) No light.

B) First, it is bright, then dim.

C) First, it is dim, then bright.

D) Steady bright.





1.4. (6pts) A negatively charged particle moves along a very long wire with a current as it is shown in the figure. What is the direction of the force acting on the particle

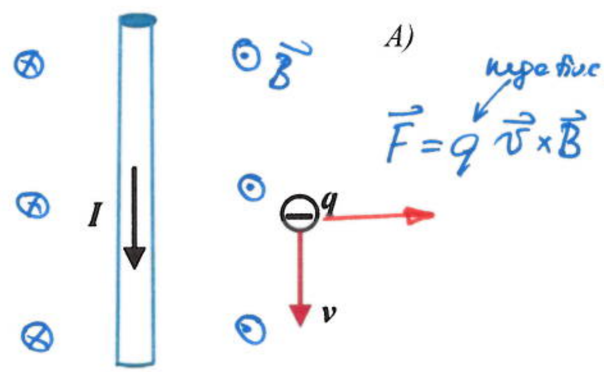
A)there is no force

B) to the left

C) to the right

D) into the page

E) out of the page



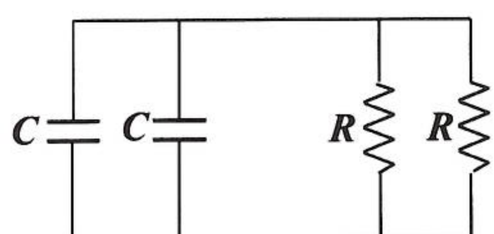
1.5. (6pts) What is the time constant for the discharge of the capacitors shown in the figure?

A) 4RC

B) RC/4

C) RC

D) RC/2



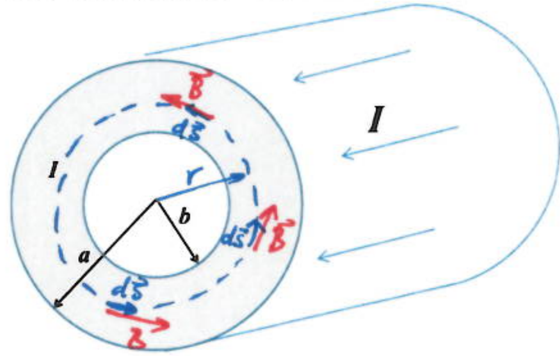
$C_{eq} = C + C = 2C$

$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R} \Rightarrow R_{eq} = \frac{R}{2}$

$\tau = R_{eq} \cdot C_{eq} = \frac{R}{2} \cdot 2C = RC$

Problem 2. (20 pts)

The figure shows a cross section of a hollow cylindrical conductor of radii ***a*** and ***b***, carrying a uniformly distributed current ***I***. Determine the magnetic field due to this current at $b \le r \le a$ (inside the wire).



a) Write the Ampere's Law (1pts)

b) Draw an Amperian loop on the picture (2pts).

c) Draw the magnetic field and ds vectors on the picture (4pts).

d) Find a current enclosed within an Amperian loop (6pts). *Show your work!*

e) Find the magnetic field at a distance r from the axis at $b \le r \le a$ *(7pts).*

$$\left\| \oint \vec{B} \cdot d\vec{s} = \mu_0 I_{in} \right\|$$

$$\oint \vec{B} \cdot d\vec{s} = \left\| \vec{B} \uparrow\uparrow d\vec{s} \right\| = \oint B\, ds = \left\| \begin{matrix} B = \text{const} \\ \text{on the Amp. loop} \end{matrix} \right\| = B \oint ds = B \cdot 2\pi r = \mu_0 I_{inc}$$

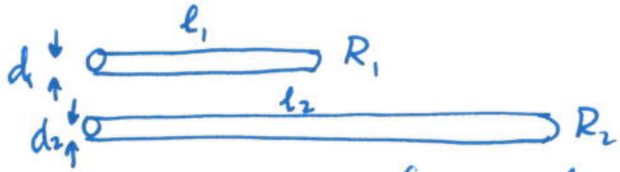
$$I_{inc} = J \cdot (A_r - A_b) = \left(\frac{I}{A_a - A_b}\right)(A_r - A_b) = I \frac{\pi(r^2 - b^2)}{\pi(a^2 - b^2)} = I \frac{r^2 - b^2}{a^2 - b^2}$$

$$B \cdot 2\pi r = \mu_0 \left(\frac{r^2 - b^2}{a^2 - b^2}\right)$$

$$\left\| B(r) = \frac{\mu_0}{2\pi r} \left(\frac{r^2 - b^2}{a^2 - b^2}\right) \right\|$$

Problem 3. (20 pts)

a) *(10pts)* Two aluminum wires have the same resistance. If one has twice the length of the other, what is the ratio of the diameter of the longer wire to the diameter of the shorter wire?



$$R = \rho \frac{\ell}{A}$$

$$\ell_2 = 2\ell_1$$

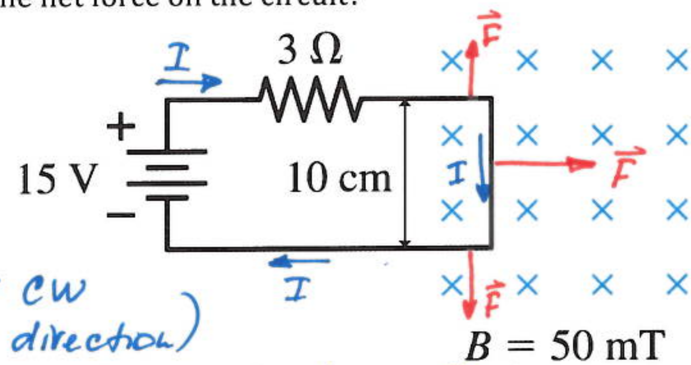
$$R_1 = R_2 \Rightarrow \rho \frac{\ell_1}{A_1} = \rho \frac{\ell_2}{A_2} \Rightarrow \frac{\ell_1}{\pi(d_1^2/4)} = \frac{2\cdot\ell_1}{\pi(d_2^2/4)}$$

$$\frac{1}{d_1^2} = \frac{2}{d_2^2} \Rightarrow \frac{d_2}{d_1} = \sqrt{2}$$

b) *(10pts)* The right edge of the circuit in the figure extends into a 50 mT uniform magnetic field.

i. What is the magnitude of the net force on the circuit?

ii. What is the direction of the net force on the circuit?



$$I = \frac{\mathcal{E}}{R} = \frac{15\,V}{3\,\Omega} = 5A \ (\text{cw direction})$$

The force on the wire $\vec{F} = I(\vec{\ell} \times \vec{B})$, $\vec{\ell} \perp \vec{B}$

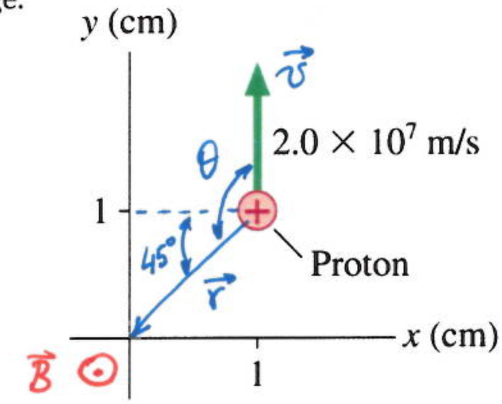
$$F = I\ell B = 5A \cdot 0.1m \cdot 50\cdot 10^{-3}T = 0.025N$$ +7

$\vec{F}$ direction: RHR; $\vec{F}$ is to the right. +3

Forces on top and bottom pieces cancel each other.

Problem 4. (10 pts)

What is the magnetic field at the origin in the figure?
Give your answer as a vector. Z axis is out of the page.



$$\left\| \vec{B} = \frac{\mu_0}{4\pi} q \frac{\vec{v} \times \hat{r}}{r^2} \right\| =$$

$$= \frac{\mu_0}{4\pi} \cdot q \cdot \frac{v \cdot \sin\theta}{r^2} \cdot (+\hat{z})$$

$$r = \sqrt{(0.01\,m)^2 + (0.01\,m)^2} = \sqrt{2 \cdot 10^{-4}\,m^2}$$

$$r = \sqrt{2} \cdot 10^{-2}\,m$$

$$\theta = 90° + 45° = 135°$$

$$\vec{B} = \frac{4\pi \cdot 10^{-7} \frac{T \cdot m}{A} \cdot (1.6 \cdot 10^{-19} C) \cdot 2 \cdot 10^{7}\, m/s \cdot \sin 135°}{4\pi \cdot 2 \cdot 10^{-4}\, m^2} (+\hat{z}) = 1.13 \cdot 10^{-15} (+\hat{z})\ T$$