PHYS.1410 Exam Final Variant A Fall 2016

***Section instructor***____________________ ***Section number***__________

***Last/First name*** Dawyla

***Last 3 Digits of Student ID Number:*** __________

***Show all work. Show all formulas used for each problem prior to substitution of numbers. Label diagrams and include appropriate units for your answers.*** *You may use an alphanumeric calculator during the exam as long as you do not program any formulas into memory. By using an alphanumeric calculator you agree to allow us to check its memory during the exam. Simple scientific calculators are always OK!*

***A formula sheet is attached to the Back of this examination***

***Be Prepared to Show your Student ID Card***

***Score on each problem:***

*1. (30)* ____

*2. (30)* ____

*3. (30)* ____

*4. (30)* ____

*5. (20)* ____

*6. (20)* ____

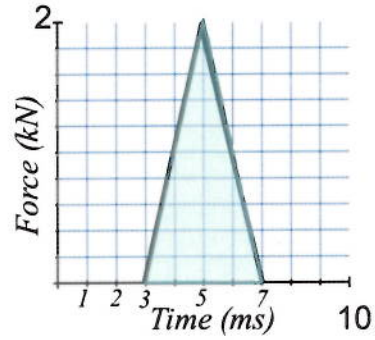
*7. (20)* ____

***Total Score (out of 180 pts)*** ____

## *1. Conceptual Questions*

**(30 point) Put a circle around the letter that you think is the best answer.**

**1.1.** (6pts) The force exerted by a tennis racket on the ball during a serve can be approximated by the F vs time plot below. What is the impulse on the ball?




*A) 8 N s*

*B) 4 N s*

*C) 2 N s*

*D) 0 N s*

*E) None of the above*

$J = \text{Area under F-vs-t curve} = 2\cdot\left(\frac{1}{2}\cdot 2kN\cdot 4ms\right) = 4 N\cdot s$

**1.2.** (6pts) When you ride a bicycle, in what direction is the angular velocity of the wheels?

*A)* to your left

*B)* to your right

*C)* forwards

*D)* backwards

*E)* up


**1.3.** (6pts) There are two dumbbells connected with massless rods? $I_A$ and $I_B$ are their moments of inertia about the midpoint of the rods. What is the ratio $I_B/I_A$?

*A)* 1

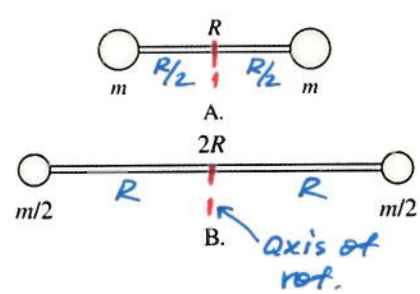
*B)* 2

*C)* 3

*D)* 4

*E)* 5

*F)* *None of the above*




$$\left.\begin{aligned} I_B &= 2\cdot\left(\frac{m}{2}\right)(R)^2 = mR^2 \\ I_A &= 2\cdot m\cdot\left(\frac{R}{2}\right)^2 = \frac{mR^2}{2} \end{aligned}\right\} \Rightarrow \frac{I_B}{I_A} = 2$$

**1.4.** (6pts) A rod attached to the wall with a hinge is hold horizontally as shown in the figure. The mass of the rod is 1.0 kg. Its length is 2.0 m. Assume the acceleration due to gravity is 10 m/s$^2$. What is the torque due to gravity relative to the hinge?

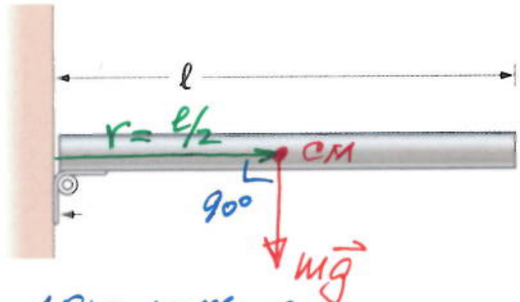
*A) 10 N m.*

*B) 20 N m*

*C) 30 N m.*

*D) 40 N m.*

*E) None of the above.*




$$\vec{\tau} = \vec{r} \times m\vec{g} = \frac{\ell}{2} mg \cdot \sin 90° = \frac{mg\ell}{2} = \frac{1.0\,kg \cdot 10\,m/s^2 \cdot 2\,m}{2} = 10\ N \cdot m$$

**1.5.** (6pts) Calculate the vector product of the vectors in the figure if A=4 and B=2.

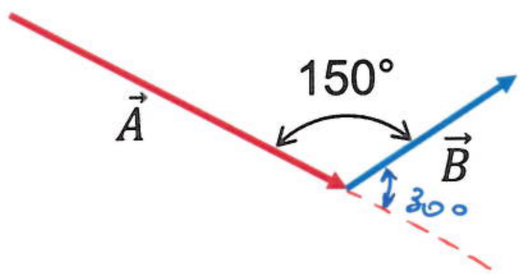
*A) 8 and the direction is out of the page*

*B) 8 and the direction is into the page*

*C) 4 and the direction is out of the page*

*D) 4 and the direction is into the page*

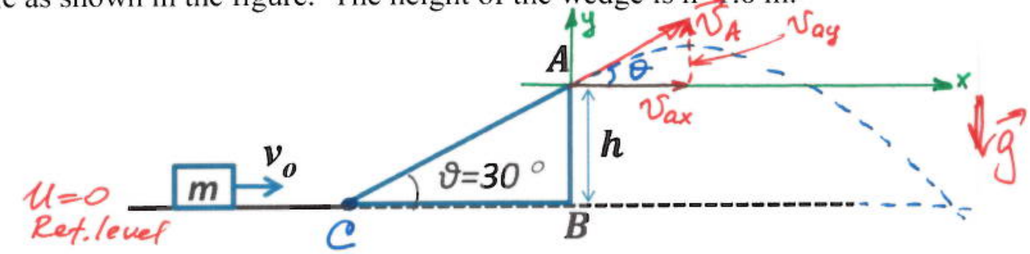
*E) 8 and the direction is along the vector A*




$$\vec{A} \times \vec{B} = AB \cdot \sin\theta \cdot (\text{out of the page direction}) = 4 \cdot 2 \cdot \underbrace{\sin 30°}_{=1/2} = 4$$

***Problem 2. (30 pts)***

A block of slippery cheese slides on a horizontal table at 10 m/s. It then slides up the wedge rigidly attached to the table as shown in the figure. The height of the wedge is h=1.8 m. Friction is negligible




a) (15 pts) What will be the value of velocity at the top of the incline, point A (use conservation of energy).

$$E_C = E_A \Rightarrow K_C + \cancel{U_C}^{\,0} = K_A + U_A$$

$$\frac{m v_0^2}{2} = \frac{m v_A^2}{2} + U_A = \frac{m v_A^2}{2} + mgh$$

$$v_A = \sqrt{v_0^2 - 2gh} = \sqrt{(10\,m/s)^2 - 2\cdot 9.8\,m/s^2\cdot 1.8\,m} \approx 8.045\,m/s \approx 8\,m/s$$

b) (5pts) Introduce a coordinate system and find x and y components of the velocity at point A.

$$v_{ax} = v_A\cdot\cos\theta = 8\,m/s\cdot\cos 30^\circ = 6.93\,m/s$$
$$v_{ay} = v_A\cdot\sin\theta = 8\,m/s\cdot\sin 30^\circ = 4.0\,m/s$$

c) (5 pts) Find a total flight time before it hits the ground. (Kinematics)

$$\overset{=-h}{y} = \cancel{y_0}^{\,0} + v_{ay}\cdot t - \frac{g\cdot t^2}{2} \Rightarrow t^2 - \left(\frac{2v_{ay}}{g}\right)t - \frac{2h}{g} = 0 \Rightarrow t^2 - 0.816t - 0.367 = 0$$

$t_1 = 1.14\,s \approx 1.1\,s$

$t_2 = -0.32$ (no need)

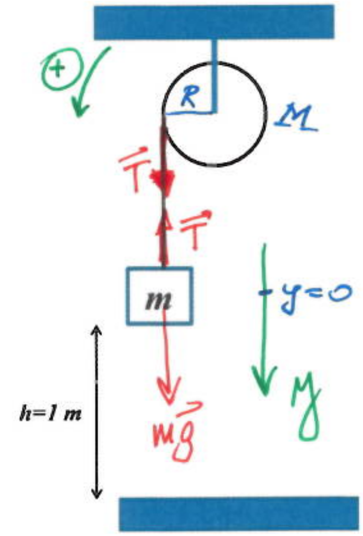
d) (5 pts) How far from the right edge of the wedge will the block travel before hitting the table? (Kinematics)

$$x(t_1) = v_{ax}\cdot t_1 = v_A\cdot\cos\theta\cdot t_1 = 7.94\,m \approx 7.9\,m$$

***Problem 3. (30 pts)***

An $m=2.0\ kg$ block is attached to a massless string that is wrapped around a $M=1.0\ kg$, $R=0.2\ m$ radius cylinder, as shown in the figure. The cylinder rotates on an axel through the center. The block is released from the rest $h=1.0\ m$ above the floor. The moment of inertia of the cylinder is $I=\frac{1}{2}MR^2$

a) *(5 pts) Draw a free-body diagram (show forces, coordinate system(s), positive direction for the rotation and a torque).*




b) *(15 pts) Find linear acceleration of the block and the tension in the wire* (Translational and rotational N. 2nd laws).

$$\left.\begin{array}{l} \text{for } m \Rightarrow mg - T = ma \quad (1) \\ \text{for } M \Rightarrow \sum \tau = I\alpha \Rightarrow T\cdot R = I\cdot\alpha \quad (2) \\ a = \alpha\cdot R \quad (3) \end{array}\right\}$$

$$\begin{array}{l} (1) \\ (2) \end{array}\left\{\begin{array}{l} mg - T = ma \\ RT = I\cdot\frac{a}{R} \end{array}\right. \Rightarrow T = I\cdot\frac{a}{R^2} \quad \text{put it in } (1)$$

$$mg - I\frac{a}{R^2} = ma \Rightarrow a\left(m + I/R^2\right) = mg \Rightarrow \boxed{a = \frac{mg}{m + I/R^2}}$$

$$a = \frac{g}{1+\frac{I}{mR^2}} = \frac{g}{1+\frac{\frac{1}{2}MR^2}{mR^2}} = \frac{g}{1+\frac{M}{2m}} = \frac{9.8\ m/s^2}{1+\frac{1.0\,kg}{2\cdot 2.0\,kg}} = 7.84\ m/s^2$$

$$(1) \Rightarrow T = m(g-a) = 2.0\,kg\cdot(9.8\ m/s^2 - 7.84\ m/s^2) = 3.92\ N$$

c) *(10 pts) How long does it take for the block to reach the floor? (Kinematics)*

kinem. eq-ns: $\overset{=h}{y} = \cancel{y_0}^{\,0} + \cancel{v_{0y}}^{\,0,\text{ released}}\, t + \frac{at^2}{2}$

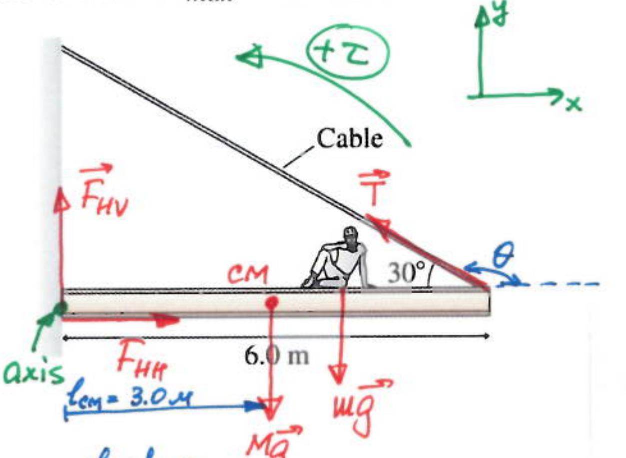
$$h = \frac{at^2}{2}$$

$$t = \sqrt{\frac{2h}{a}} = \sqrt{\frac{2\cdot 1.0\,m}{7.84\,m/s^2}} = 0.5\ s$$

***Problem 4 (30 pts).***

An $m=80\ kg$ worker sits down $2.0\ m$ from the end of an $M=1450\ kg$ steel beam of length $l = 6.0\ m$ to eat his lunch. The cable supporting the beam is rated at $T_{max} = 15000N$.

a) (5 pts) *Draw a free-body diagram of the beam*




b) (10 pts) *Find the tension in the cord, T. Should the worker be worried (larger or smaller $T_{max}$)?*

1) $\sum_i \tau = 0$ 2) $\sum_i F_x = 0$ 3) $\sum_i F_y = 0$

Torques are calculated with respect to A.

1) $\sum_i \tau = 0$ lever arms = 0 $(180-30°)$

$$\cancel{\tau_{HH}}^{0} + \cancel{\tau_{HV}}^{0} - Mg\cdot\frac{l}{2} - mg\,l_1 + T\cdot l\cdot \sin\theta = 0$$

$$T = \frac{Mg\frac{l}{2} + mg\,l_1}{l\cdot\sin 150°} = \left\| \begin{matrix} l_1 = l - 2.0m = \\ = 4.0m \end{matrix} \right\| = \frac{1450kg\cdot 3.0m + 80kg\cdot 9.8\,m/s^2\cdot 4.0m}{(6.0m)\cdot \sin 150°} = 15255N$$

which is larger than $T_{max} = 15000N$. So, he should be worried.

c) (7 pts) *Find the horizontal force exerted by the wall on the beam.*

(2) $\sum_i F_x = 0 \Rightarrow F_{HH} - T\cdot\cos 30° = 0$

$$F_{HH} = T\cdot\cos 30° = 15255N\cdot\frac{\sqrt{3}}{2} = 13211\,N$$

d) *(8 pts) Find the vertical force exerted by the wall on the beam.*

(3) $\sum_i F_y = 0 \Rightarrow F_{HV} - Mg - mg + T\cdot\sin 30° = 0$

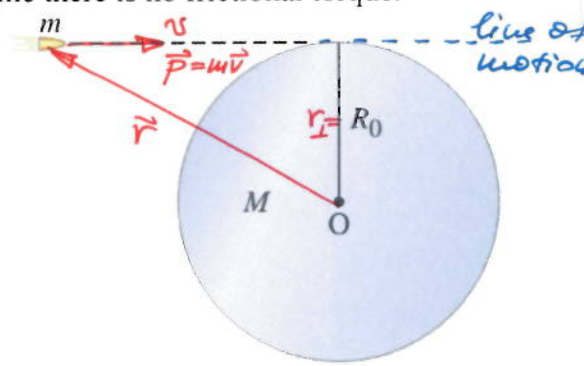
$$F_{HV} = (M+m)g - T\cdot\sin 30°$$

$$F_{HV} = (1450kg + 80kg)\cdot 9.8\,m/s^2 - 15255N\cdot\frac{1}{2} = 7366.5N$$

***Problem 5 (20 pts)***

A bullet of mass $m$ moving with velocity $v$ strikes and becomes embedded at the edge of a cylinder of mass $M$ and radius $R_0$. The cylinder, initially at rest, begins to rotate about its symmetry axis, which remains fixed in position. Assume there is no frictional torque.

$I_{cylinder} = \frac{1}{2} M R^2$




*a) (5 pts)What is conserved during this collision?*

Angular momentum is conserved.

$$\boxed{L_i = L_f}$$

*b) (15 pts) What is the angular velocity of the cylinder after this collision?*

$$L_i^{bul.} + \cancel{L_i^{cyl.}}^{\,0} = L_f^{bul+cyl.}$$

$$L_i^{bul} = |\vec{r} \times \vec{p}| = r_\perp \cdot p = R_0 \cdot m \cdot V$$

$$L_i^{cyl.} = I_{cyl} \cdot \cancel{\omega_0}^{\,0} = 0$$

$$L_f^{bul+cyl} = I_f \cdot \omega_f = (\underbrace{mR_0^2}_{I_{bullet}} + \underbrace{\tfrac{1}{2} M R_0^2}_{I_{cyl.}})\omega_f$$

so, $$R_0 m V = (m R_0^2 + \tfrac{1}{2} M R_0^2)\omega_f$$

$$\omega_f = \frac{mV}{(m + \frac{M}{2})R_0}$$

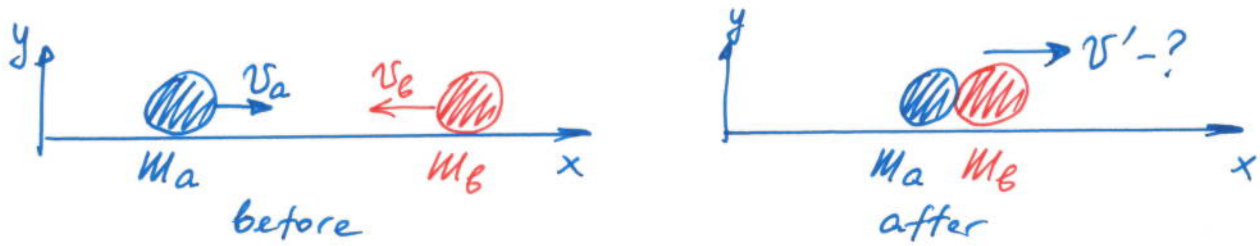
***Problem 6 (20 pts)***

A 100 g ball moving to the right at 4.0 m/s collides head-on with a 200 g ball that is moving to the left at 3.0 m/s.

a) (2 pts) What is conserved during this collision?

since $\sum_i \vec{F}^{ext} = 0 \Rightarrow$ linear momentum is conserved.

b) (4 pts) Draw a diagram and a coordinate system; show velocities.




c) (14 pts) If the collision is *perfectly inelastic*, what is the speed and direction of the combined balls after the collision?

$$\boxed{P_i = P_f}$$

?

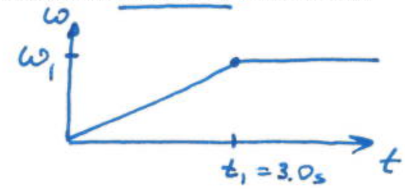
$$m_a v_a - m_b v_b = (m_a + m_b) v'$$ assume $v'$ is positive.

$$v' = \frac{m_a v_a - m_b v_b}{m_a + m_b} = \frac{0.1\,kg \cdot 4.0\,m/s - 0.2\,kg \cdot 3.0\,m/s}{(0.1+0.2)\,kg} = -0.667\,m/s$$

So, they will move to the left (meaning of the minus)

***Problem 7 (20 pts)***

Starting from rest, a 12-cm-diameter compact disk takes 3.0 s to reach its operating angular velocity of 2000 rpm. Assume that the angular acceleration is constant. The disk's moment of inertia is $2.5\times10^{-5}$ kg m$^2$.




*a) (6 pts) Find the angular acceleration of the disc.*

$$\omega_1 = 2000\,\frac{\text{rev}}{\text{min}}\cdot\left(\frac{2\pi\,\text{rad}}{1\,\text{rev}}\cdot\frac{1\,\text{min}}{60\,\text{s}}\right) = \frac{200\pi}{3}\,\frac{\text{rad}}{\text{s}}$$

rot. kin. eq-n $\omega_1 = \cancelto{0}{\omega_0} + \alpha\cdot t_1$

$$\alpha = \frac{\omega_1}{t_1} = \frac{200\pi/3}{3.0\,\text{s}} = \frac{200\pi}{9}\,\frac{\text{rad}}{\text{s}^2} = 69.81\ \text{rad/s}^2$$

*b) (6 pts) How much torque is applied to the disk during this initial 3.0 s?*

Rot. N. 2nd low $\Rightarrow$ $\tau = \alpha\cdot I = 2.5\cdot10^{-5}\,\text{kg}\cdot\text{m}^2\cdot 69.81\,\frac{\text{rad}}{\text{s}^2} = 1.75\cdot10^{-3}\ \text{N}\cdot\text{m}$

*c) (2 pts) How much torque is applied to the disk after 3.0 s?*

since at $t \geq t_1 = 3.0\,\text{s}$, $\omega = \text{const}$, it means $\alpha = 0 \Rightarrow \tau = I\cancelto{0}{\alpha} = 0$

*d) (6 pts) How many revolutions does it make before reaching full speed?*

rot. kinem. eq-n: $\theta_1 = \cancelto{0}{\theta_0} + \cancelto{0}{\omega_0}t_1 + \frac{\alpha\cdot t_1^2}{2} =$

$$\theta_1 = \frac{\left(\frac{200\pi}{9}\,\frac{\text{rad}}{\text{s}}\right)\cdot(3\,\text{s})^2}{2} = \frac{200\pi}{2} = 100\pi\,\text{rad}\cdot\frac{1\,\text{rev}}{2\pi\,\text{rad}} = 50\ \text{rev}$$