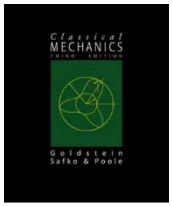


Classical Mechanics

Chapter 3. *Homework 3*
(Due to February 23, 2017).
Central Forces

Problem 3A. **(10 points)**

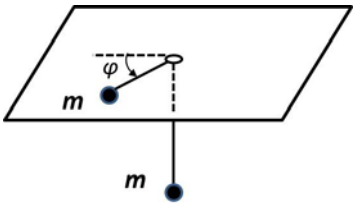
(a) Determine the trajectory $r(\theta)$ for a particle of mass ***m*** moving in an oscillator potential $V(r) = kr^2/2$, where ***k*** is a constant.
(b) Demonstrate by converting to Cartesian coordinates that the trajectory is an ellipse with semi-major axis $a = r_0/\sqrt{1-\varepsilon}$, where

$$\varepsilon = \sqrt{1-(kl^2/mE^2)}; \quad r_0 = l/\sqrt{mE}$$

(make a convenient choice of a constant of integration $\theta_0 = \pi/4$)
(c) Sketch the trajectory. Show parameters (semi-major/semi-minor axis, the force center, and r_0)

Problem 3B. **(10 points)**

An inextensible massless string of length ***l*** passes through a hole in a frictionless table. A point mass ***m*** at one end moves on the table and a point mass ***m*** hangs from the other end. Assume that hanging ***m*** moves in a vertical line only.



a) Clearly define the generalized coordinates;
b) Write down the potential energy of the system. Is the force central?
c) Write the Lagrangian of the system;
d) Find the equations of motion;
e) Integrate the equations of motion to get the two first integrals of motion;
f) Write the effective one-dimentional potential energy and plot it with respect to a distance from m, which is on the table, to the hole in the table;
g) If mass m, which is on the table, moves in a circle, what is the radius of the circle?
h) If mass on the table undergoes *small* radial oscillations around a stable circular orbit, determine the effective spring constant and angular frequency for small oscillations;
i) Is this perturbed circular orbit closed?
j)

Scattering

Problem 3C. **(10 points)**

Determine the differential cross section and the total cross section for scattering of particles from a perfectly rigid sphere of radius a. Assume the particles obey "the law of reflection" and the potential is

$$V(r) = 0,\ r > a$$
$$V(r) = \infty,\ r \le a$$

Problem 3D. **(10 points)**

A rocket with velocity $\boldsymbol{v_0}$ (at r equals infinity) and impact parameter $\boldsymbol{s}$ approach a planet of radius $\boldsymbol{R}$ and mass $\boldsymbol{M}$. What is the condition that the rocket will strike the planet?

Problem 3E. **(10 points)**

A uniform beam of particles with energy $\boldsymbol{E}$ is scattered by a repulsive central potential $(r) = {}^{\gamma}/_{r^2}$. Derive the differential elastic cross section

$$\frac{\gamma\pi^2}{E\,\theta^2 \sin\theta}\frac{\pi-\theta}{(2\pi-\theta)^2}$$

where θ is a scattering angle. Sketch carefully the angular dependence. Discuss the total cross section. What happens if the potential is attractive (γ<0)?