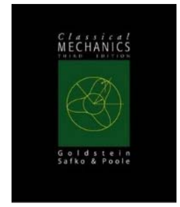

Classical Mechanics

Chapter 8. The Hamilton Equations of Motion

Homework 4

(Due to March 28, 2017).

Problem 4A. **(10 points)**

A spherical pendulum consists of a particle of mass $\boldsymbol{m}$ in a gravitational field constrained to move on a surface of a sphere of radius $\boldsymbol{l}$. Use the polar angle $\boldsymbol{\theta}$ (measured from the downward vertical) and the azimuthal angle $\boldsymbol{\varphi}$ to obtain the equations of motion in the Hamiltonian formulation. Expand the Hamiltonian to second order about uniform circular motion with $\boldsymbol{\theta = \theta_0}$ and show that the resulting expression is just that for a simple harmonic oscillator with

$$\omega^2 = \left[{}^{g}\!/_{l\cos\theta_0}\right](1 + 3\cos^2\theta_0)$$

Problem 4B. **(10 points)**

A dynamical system has the Lagrangian

$$L = \frac{1}{2}\left(\dot{q}_1^2 + \frac{\dot{q}_2^2}{a + bq_1^2}\right) - \frac{1}{2}(k_1 q_1^2 + k_2)$$

Where $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{k_1}$, $\boldsymbol{k_2}$ are constants.

a) Find a Hamiltonian corresponding to this Lagrangian.
b) What quantities are conserved?
c) Find the equations of motion in the Hamiltonian formulation and solve them.

Problem 4C. **(10 points)**

Consider the motion of a particle P of mass m moving in the plane under the influence of a force of magnitude $\alpha m/r^2$ directed towards a fixed point O, where r is the distance from O to P. Where α *is* a constant. Assume that the potential energy is zero as $r \to \infty$.

a) Find a Lagrangian.
b) Find a Hamiltonian corresponding to this Lagrangian.
c) What quantities are conserved?
d) Find the equations of motion in the Hamiltonian formulation.
e) Write down the equation for r