Lecture 3

Chapter 23

Motion of a charged particle in an electric field.

Electric flux

Course website:
http://faculty.uml.edu/Andriy_Danylov/Teaching/PhysicsII
Today we are going to discuss:

Chapter 23, 24:

- Section 23.6 Motion of charged particle in an electric field
- Section 24.2 Idea of Flux
- Section 24.3 Electric Flux
Two protons, $A$ and $B$, are in an electric field. Which proton has the larger acceleration?

A) Proton $A$.
B) Proton $B$.
C) Both have the same acceleration.

Stronger field where field lines are closer together.

Weaker field where field lines are farther apart.
Now, let’s find an electric field of a large plate and a capacitor
The ring results can be extended to calculate the electric field of a uniformly charged disc.

Without derivation:

\[
E = \frac{\eta}{2\varepsilon_0} \left[ 1 - \frac{Z}{\sqrt{Z^2 + R^2}} \right]
\]

In the limit \( z \ll R \), it becomes

\[
E = \frac{\eta}{2\varepsilon_0}
\]

El. field created by an infinite charged plate
\( \eta \) - surface charge density

Interesting!!!
The field strength is independent of the distance \( z \)
The parallel-plate capacitor

Outside the plates
\[ E = \frac{\eta}{2\varepsilon_0} - \frac{\eta}{2\varepsilon_0} = 0 \]

Inside the plates
\[ E = \frac{\eta}{2\varepsilon_0} + \frac{\eta}{2\varepsilon_0} = \frac{\eta}{\varepsilon_0} \]

It is called a Parallel-plate capacitor

The field inside a parallel-plate capacitor is uniform

Let's bring them very close to each other
Motion of a charged particle in an electric field.
An electron is released from rest at the surface of the negative plate. How long does it take the electron to cross to the positive plate? **Example 23.8**


When it hits the positive plate \( x(t_f) = d \)

\[
x(t_f) = d = \frac{at_f^2}{2} \Rightarrow t_f = \sqrt{\frac{2d}{a}}
\]

\[
t_f = \sqrt{\frac{2d}{qE/\mu}} = \sqrt{\frac{2mu}{qE}} = \frac{m}{\mu E_a} = \sqrt{\frac{2mu}{qE}}
\]

It takes \( t_f \) seconds for an electron to travel between plates.

Plates of the capacitor are circular with radius \( R \)

\[
E = \frac{Q}{\varepsilon_0 A}
\]

\( \gamma = \frac{Q}{A} \) - surface charge density

The electron interacts with \( E \) and experiences force \( F_e \)

\[
F_e = qE \text{ and it is antiparallel with } E \quad (\text{since } q < 0)
\]

Apply New. 2nd law: \( F = ma \), so

\[
mu = qE \Rightarrow a = \frac{qE}{m} - \text{accel. of the electron.}
\]

\( a \) is constant so we can apply kinematic equations.

\[
x(t) = x_0 + v_0t + \frac{at^2}{2}
\]

The origin is at the initial electron's position \( (x_0 = 0) \)

\( v_0 = 0 \) - initial velocity is zero.
Chapter 24.

Gauss’s Law

Yankee Swap

How?
The idea behind Gauss’s law

It looks like number of lines passing through a closed surface are related to the amount of charge inside (Gauss’s Law)

But, first, we need to learn how to count lines

Flux
The Basic Idea of Flux

Imagine holding a rectangular wire loop of area $A$ in front of a fan.

- The volume of air flowing through the loop each second depends on the angle between the loop and the direction of flow.

- The flow is maximum through a loop that is perpendicular to the airflow.

- No air goes through the same loop if it lies parallel to the flow.
The Area Vector

Before defining the electric flux, we need to introduce the area vector.

- Let’s define an area vector $\vec{A} = A\hat{n}$ to be a vector in the direction of $\hat{n}$, perpendicular to the surface, with a magnitude $A$ equal to the area of the surface.

- Vector $\vec{A}$ has units of $m^2$.

Area vector $\vec{A}$ is perpendicular to the surface. The magnitude of $\vec{A}$ is the surface area $A$. 
The Electric Flux

Consider 1) a uniform electric field $E$
2) a flat surface

The electric flux through a surface of area $A$ can be defined as the dot-product:

$$\Phi_e = \vec{E} \cdot \vec{A}$$

$$\Phi_e = EA \cos \theta$$
ConcepTest

The electric flux through the shaded surface is

\[ \Phi_e = EACos\theta \]

Electric Flux

A) 0
B) 200 N m\(^2\)/C
C) 400 N m\(^2\)/C
D) Some other value
The electric flux through the shaded surface is

\[ \Phi_e = EAC\cos\theta \]

**ConcepTest**

**Electric Flux**

A) 0
B) 200 N m\(^2\)/C
C) 400 N m\(^2\)/C
D) Some other value
The electric flux through the shaded surface is

\[ \Phi_e = EA \cos \theta \]

A) 0
B) \(400 \cos 20^\circ \text{ N m}^2/\text{C}\)
C) \(400 \cos 70^\circ \text{ N m}^2/\text{C}\)
D) 400 N m\(^2/\text{C}\)
E) Some other value
The Electric Flux (general case)

Consider 1) a non-uniform electric field \( E \)
2) area is not flat (curved)

Divide the surface into many small pieces of area \( \delta A \).

- a) Each piece is small enough that it is essentially flat
- b) The field is nearly uniform over each piece
- c) Thus, the formula from the previous slide can be used

The electric flux through each small piece is:

\[
\delta \Phi_i = \vec{E}_i \cdot (\delta \vec{A})_i
\]

The electric flux through the whole surface is the surface integral:

\[
\Phi_e = \int \vec{E} \cdot d\vec{A}
\]
The Electric Flux through a closed surface

Consider 1) a non-uniform electric field $E$
2) a closed surface

NOTE: For a closed surface, we use the convention that the area vector $dA$ is defined to always point toward the outside.

The electric flux through a closed surface:

$$\Phi_e = \oint \vec{E} \cdot d\vec{A}$$

closed surface
Gauss’s Law

For any closed surface enclosing total charge $Q_{in}$, the net electric flux through the surface is:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

Gaussian surface

Properties:

1) It works for any closed surface called Gaussian (A Gaussian surface is an imaginary, mathematical surface)

2) $Q_{in}$ is the net charge enclosed by the Gaussian surface (charges outside must not be included) $Q_{in} = q_1 - q_2 + q_3$

3) Distribution of $Q_{in}$ doesn't matter

This result for the electric flux is known as Gauss’s Law.

Both, Gauss’s law and Coulomb’s law, help to find electric fields based on distribution of charges.
Gauss’s Law/Symmetry

\[ \Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{in}}}{\varepsilon_0} \]

Gaussian surface

Evaluation of this surface integral is often difficult. However, when the charge distribution has sufficient symmetry (spherical, cylindrical, planar), evaluation of the integral becomes simple.
Thank you

Bye Bye For Now