Lecture 6

Chapter 25

The Electric Potential Energy

Course website:
http://faculty.uml.edu/Andriy_Danylov/Teaching/PhysicsII
Today we are going to discuss:

Chapter 25:

- Section 25.1 Electric Potential Energy
- Section 25.2 The potential energy of a point charge
New Idea

So far, we used vector quantities:

1. Electric Force (F)
   \[ F_{2 \text{on} 1} = \frac{K|q_1||q_2|}{r^2} \]

2. Electric Field (E)
   \[ \vec{E} = \frac{\vec{F}_{\text{on} q}}{q} \]

But, as you know, it is not easy to deal with vectors

Let’s introduce scalar quantities instead of a FORCE and FIELD
Let’s define a conservative force and give an idea of PE

Consider different paths between two points in a field

Recall for Physics I, a work done by a force

\[ W = \int \vec{F} \cdot d\vec{s} \]

PE idea: That is what we have in our example. So, you can “attach” this number to the final point and give this number 5 a fancy name – the potential energy (PE) (U_f=5J) with respect to the initial point, where we can assume PE to be zero (the reference point).

For every conservative force a potential energy can be introduced.

Since the gravitational and electrical forces are conservative forces, corresponding potential energies can be introduced.
Since the gravitational force is a conservative force, a gravitational potential energy (scalar quantity) can be introduced.

\[ U_{\text{grav}} = mg \]

This fall can be described using a gravitational force (vector quantity), which describes an interaction between the Earth and a cat.

\[ F_{\text{grav}} = mg \quad \Rightarrow \quad U_{\text{grav}} = mgy \]

Since the gravitational force is a conservative force, a gravitational potential energy (scalar quantity) can be introduced.

**The case of two point masses**

\[ F_G = G \frac{m_1 m_2}{r^2} \quad \Rightarrow \quad U_G = G \frac{m_1 m_2}{r} \]

**The case of two point charges**

\[ F_e = k \frac{q_1 q_2}{r^2} \quad \Rightarrow \quad U(r) = \frac{k q_1 q_2}{r} \]
Potential energy is an energy of interaction, so there must be at least two interacting electric objects.
Potential energy of $q$ in a uniform electric field (in a capacitor)
Potential energy of $q$ in a uniform electric field

Consider a charge $q$ inside a capacitor.

It moves from an initial point to a final point.

There is a constant force $F = qE$

The work done on $q$ is:

$$W = \int_{i}^{f} \vec{F} \cdot d\vec{s} = q \int_{i}^{f} \vec{E} \cdot d\vec{s} = -q \int_{i}^{f} E ds = -qE \int_{i}^{f} ds = -qE(s_f - s_i)$$

Recall from Physics I

$$-\Delta U = W = \Delta K$$

$$-\Delta U = -(U_f - U_i) = -qE(s_f - s_i)$$

$$U_f - U_i = qEs_f - qEs_i$$

To get the most general expression, let’s introduce $U_0$, which is a potential energy at the reference point $s=0$

$$U_f - U_i = qEs_f + U_0 - U_0 - qEs_i$$

Electric potential energy of charge $q$ and a charged capacitor

$$U = U_0 + qEs$$

It is convenient to choose the potential energy at the reference point $U_0 = 0$

$$U = qEs$$
ConcepTest

Two positive charges are equal. Which has more electric potential energy?

A) Charge A
B) Charge B
C) They have the same potential energy
D) Both have zero potential energy

\[ U = qEs \]
Two negative charges are equal. Which has more electric potential energy?

A) Charge A
B) Charge B
C) They have the same potential energy
D) Both have zero potential energy

The potential energy of a negative charge decreases in the direction opposite to $\vec{E}$. 

$$U = -|q|E_s$$
ConcepTest

A positive charge moves as shown. Its kinetic energy

A) Increases.
B) Remains constant.
C) Decreases.

The potential energy of a positive charge decreases in the direction of $\vec{E}$.

$U$ increases  $\Delta U > 0$

Total energy is constant  $-\Delta U = \Delta K$

$\Delta K < 0$

$K$ decreases

$U = qEs$

$S$
Potential energy of two point charges
The potential energy of two point charges

Electrostatic potential energy.

\[ W = \int F_e \cdot dr = \int \frac{kqQ}{r^2} \, dr = kqQ \log \frac{r_0}{r} \]

We know that \( W = -\Delta U \); \( U \) - pot. energy, so

\[ -[U(r) - U_0] = -\left[ \frac{kqQ}{r} - \frac{kqQ}{r_0} \right] \]

As you remember, only differences in \( U \) have physical meaning. So we are free to choose any reference point. It's common to choose \( U(r) \) to be zero at \( r = \infty \).

Ref. point \( r_0 \to \infty \); \( U_0 = 0 \), so

\[ U(r) - U_0 = \frac{kqQ}{r} - \frac{kqQ}{r_0} \]

Finally,

\[ U(r) = \frac{kqQ}{r} \]

pot. energy of the system of two charges \( Q, q \).

This is explicitly the energy of the system, not the energy of just \( q \) or \( Q \).

Note that the potential energy of two charged particles approaches zero as \( r \to \infty \).
ConcepTest Potential energy

A positive and a negative charge are released from rest in vacuum. They move toward each other. As they do:

A) A positive potential energy becomes more positive.
B) A positive potential energy becomes less positive.
C) A negative potential energy becomes more negative.
D) A negative potential energy becomes less negative.
E) A positive potential energy becomes a negative potential energy.

\[ U(r) = \frac{kqQ}{r} \]

Opposite signs, so \( U \) is Negative.

\( U \) increases in magnitude as \( r \) decreases.
A proton is fired from far away at a 1.0-mm-diameter glass sphere that has been charged to +100 nC. What initial speed must the proton have to just reach the surface of the glass?

**Example 25.2**

\[
\begin{align*}
U_i &= \frac{kqQ}{r} = 0 \\
K_i &= \frac{1}{2}mv_0^2; \\
U_f &= \frac{kqQ}{R} \\
K_f &= 0 \\
\text{it reaches with } U_f = 0
\end{align*}
\]

Total energy is conserved, i.e.

\[
K_i + U_i = K_f + U_f
\]

\[
\frac{1}{2}mv_0^2 = \frac{kqQ}{R} \quad \Rightarrow \quad v_0 = \sqrt{\frac{2kqQ}{mR}} = 1.86 \times 10^7 \text{ m/s}
\]
Thank you

Bye Bye For Now
Ok, now we know the PE expression of a charge inside of a capacitor. Let’s play with that expression to see where PE is larger or smaller, etc.

**Potential energy of a positive charge, +q**

\[ U = qEs \]

If +q moves in the direction of E, then \( \Delta U < 0 \)

If we use Conservation of energy

\[ -\Delta U = W = \Delta K \]

We will get \( \Delta K > 0 \) \( \Rightarrow K_{-\text{plate}} > K_{+\text{plate}} \)

The charge gains kinetic energy as it moves toward the negative plate.
Potential energy of a positive charge, \(-q\)

Since, the charge is negative, let’s rewrite \(U\) in this form:

\[
U = -|q|Es
\]

If \(-q\) moves in the direction opposite of \(E\), then \(\Delta U < 0\)

If we use conservation of energy:

\[
-\Delta U = W = \Delta K
\]

We will get \(\Delta K > 0\)  \(\Rightarrow\) \(K_{+\text{plate}} > K_{-\text{plate}}\)

The charge gains kinetic energy as it moves away from the negative plate.

From these two examples, you see that PE can be used to analyze motion instead of force.