Lecture 11

Chapter 8

Centripetal Force

Course website:
http://faculty.uml.edu/Andriy_Danylov/Teaching/PhysicsI

Today we are going to discuss:

Chapter 8:

- ***Uniform Circular Motion:*** *Section 8.2*
- ***Circular Orbits:*** *Section 8.3*
- ***Reasoning about Circular Motion:*** *Section 8.4*

Let’s recall circular motion (accelerations)

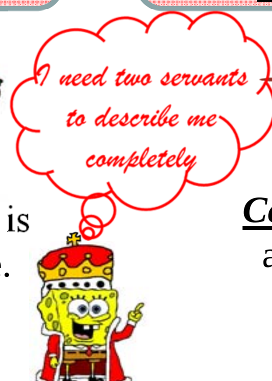
(two servants of a king)

$\vec{v}_{\tan} = (magnitude; direction)$

a_{tan} results from a change in the *magnitude* of v_{tan}

a_R *(centripetal acceleration)* results from a change in the *direction* of v_{tan}

$$a_{tan} = \frac{dv_{tan}}{dt}$$



$$a_R = \frac{v_{tan}^2}{r}$$

Tangential acceleration a_{tan} is always *tangent* to the circle.

Centripetal acceleration a_R always points *toward the center* of the circle.

Finally, any object that is undergoing circular motion experiences two accelerations: centripetal and tangential.

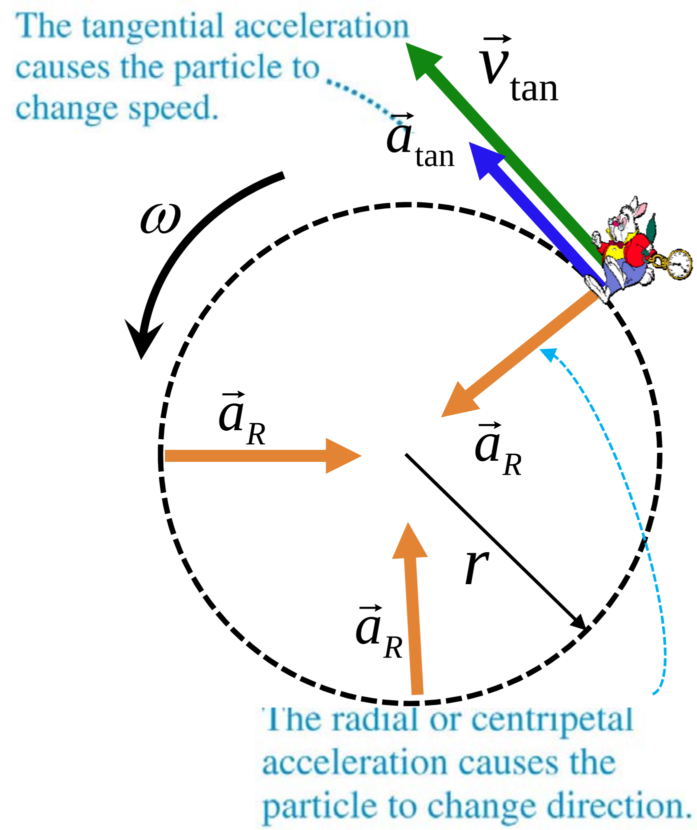
Let’s simplify “our life”

In *uniform circular motion* the speed is constant $v_t = const$

so the tangential acceleration $a_t = 0$

But, the centripetal acceleration is not zero

$$a_r = \frac{v_t^2}{r} = const$$



The best coordinate system

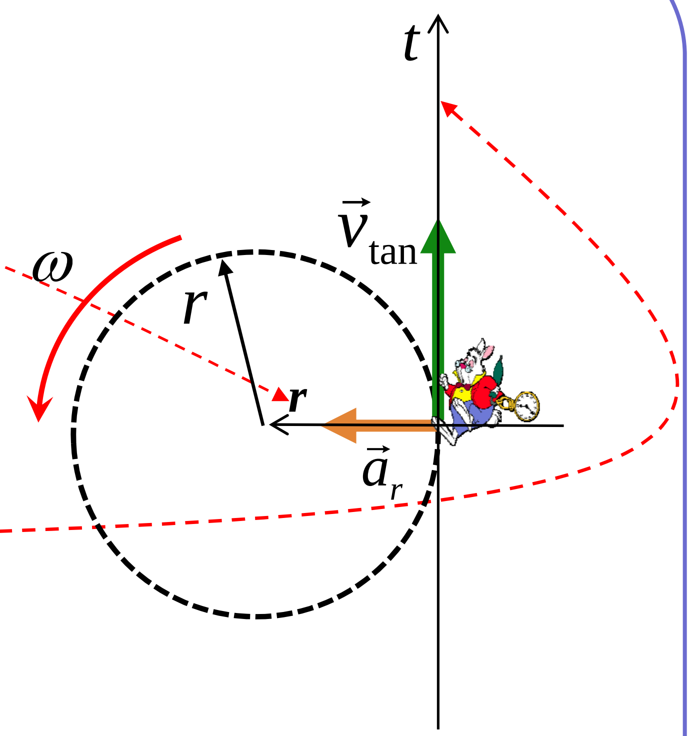
for a Uniform Circular Motion

When describing circular motion, it is convenient to define a moving *rt*-coordinate system.

The *r*-axis (radial) points *from* the particle *toward* the center of the circle.

The *t*-axis (tangential) is tangent to the circle, pointing in the ccw direction.

The origin moves along with a certain particle moving in a circular path.



If there is an acceleration, there must be a force

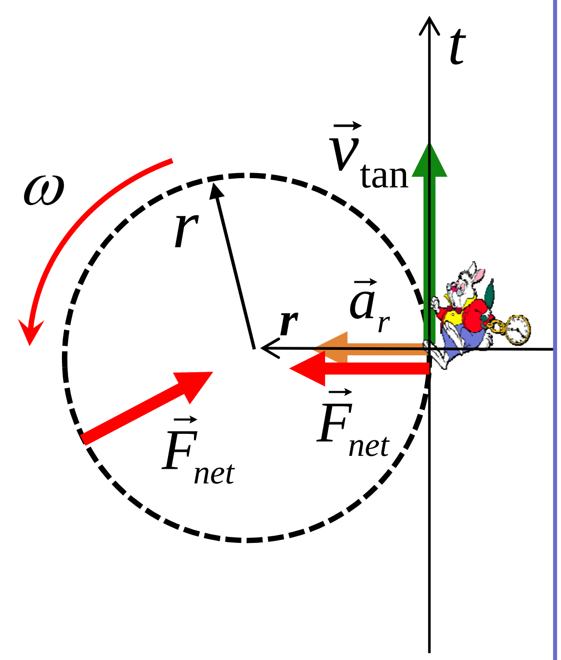
The figure shows a particle in uniform circular motion.

If there is an centripetal (radial) acceleration, there must be a radial force (called *centripetal*) according to N. 2nd law.

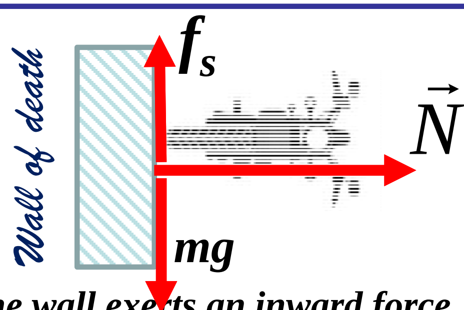
$$\sum F_r = ma_r \quad \Rightarrow \quad \sum F_r = \frac{mv^2}{r}$$

The net force points in the radial direction, toward the center of the circle.

This centripetal force is not a new force. This can be any one of the forces we have already encountered: tension, gravity, normal force, friction, …



Examples



Bike going in a circle: the wall exerts an inward force (normal force) on a bike to make it move in a circle.

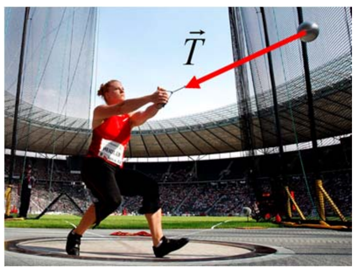
$$\sum F = ma_r \Rightarrow a_r = \frac{v^2}{R} \Rightarrow N = \frac{mv^2}{R}$$

v –velocity of the motorbike

R- radius of the circle

Normal force provides the centripetal acceleration

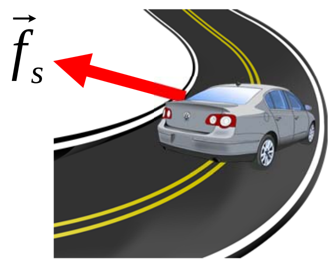
https://www.youtube.com/watch?v=9H4jUptw4Vk



A hammer going in a circle: the cord exerts an inward force (tension) on a hammer to make it move in a circle.

$$\sum F = ma_r \Rightarrow T = \frac{mv^2}{R}$$

Tension provides the centripetal acceleration



Car going in a circle: the road exerts an inward force (friction) on a car to make it move in a circle.

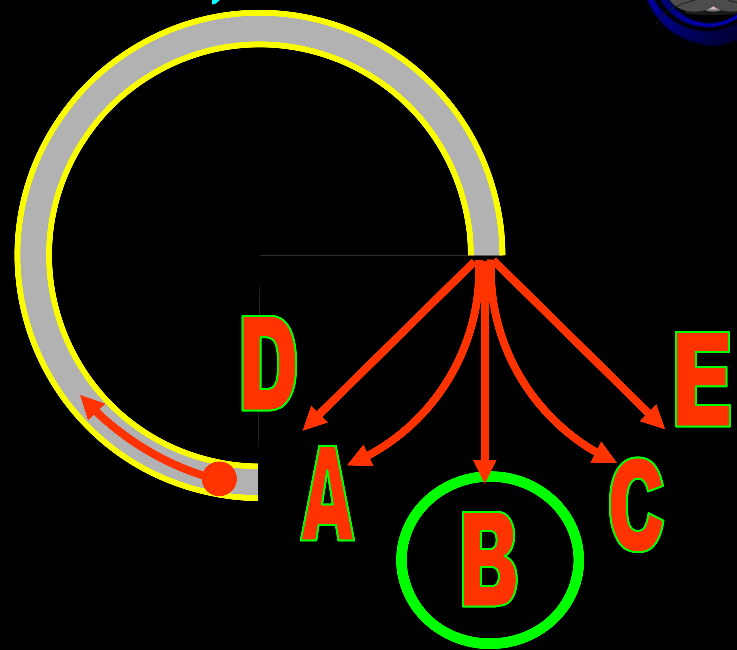
$$\sum F = ma_r \Rightarrow f_s = \frac{mv^2}{R}$$

Friction provides the centripetal acceleration

ConcepTest

Missing Link

A Ping-Pong ball is shot into a circular tube that is lying flat (horizontal) on a tabletop. When the Ping-Pong ball leaves the track, which path will it follow?



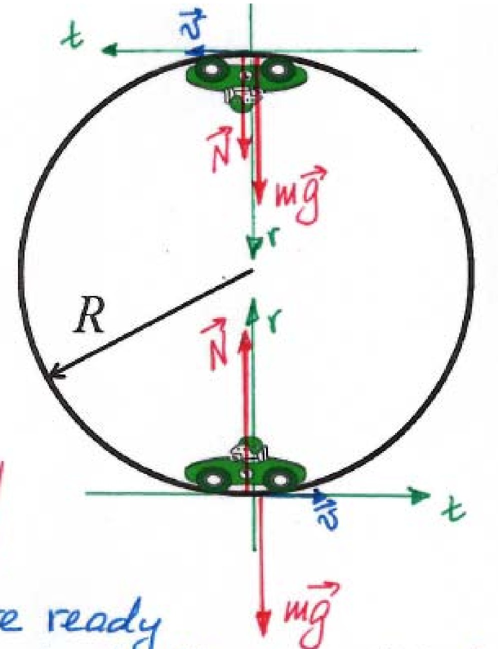
Once the ball leaves the tube, there is no longer a force to keep it going in a circle. Therefore, it simply continues in a straight line, as Newton's First Law requires!

Follow-up: What physical force provides the centripetal acceleration?

Example Loop the Loop

a) To make the loop-the-loop at a constant speed, what minimum speed does the car need?

b) Find an apparent weight at the bottom.



a) Draw a free body diagram for a car at the top

N. 2nd law for a radial direction:

$$\sum_i F_r = m a_r \quad \leftarrow a_r = \frac{v^2}{R}$$

$$N + mg = m\frac{v^2}{R} \Rightarrow \left\| v = \sqrt{\frac{R}{m}(N+mg)} \right\|$$

The critical speed occurs when we are ready to start falling down, i.e. losing contact with the wall ($N=0$).

$$v_{min} = \sqrt{\frac{R}{m}(N + mg)} = \sqrt{g \cdot r}$$

b) apparent weight -? (i.e N-?) at the bottom.

$$\sum_i F_r = m a_r \Rightarrow N - mg = m\frac{v^2}{R} \Rightarrow \left\| N = mg + m\frac{v^2}{R} \right\|$$

Thus, $N > mg$. You would feel heavier (similar to a case when a person is in an elevator)

http://phys23p.sl.psu.edu/phys_anim/mech/

End of the class

ConcepTest Going in Circles

You're on a Ferris wheel moving in a vertical circle. When the Ferris wheel is at rest, the normal force N exerted by your seat is equal to your weight *mg*. How does N change at the top of the Ferris wheel when you are in motion?

A) N remains equal to *mg*

B) N is smaller than *mg*

C) N is larger than *mg*

D) none of the above

$$mg - N = m\frac{v^2}{R}$$

$$mg - m\frac{v^2}{R} = N$$

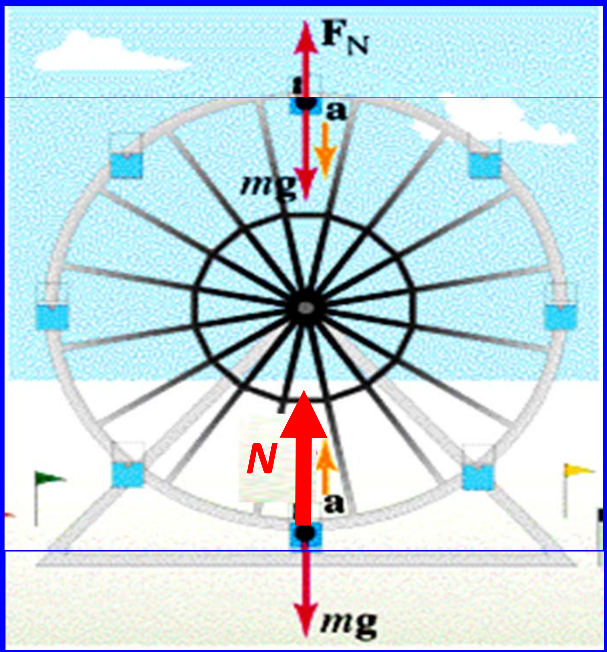
You are in circular motion, so there has to be a centripetal force pointing *inward*. At the top, the only two forces are *mg* (down) and *N* (up), so *N* must be smaller than *mg*.

Follow-up: Where is N larger than *mg*?

Bottom

$$N - mg = m\frac{v^2}{R}$$

$$N = mg + m\frac{v^2}{R}$$



ConcepTest Going in Circles

A skier goes over a small round hill with radius *R*. Because she is in circular motion, there has to be a *centripetal force.* At the top of the hill, what is F_c of the skier equal to?

A) $F_c = N + mg$

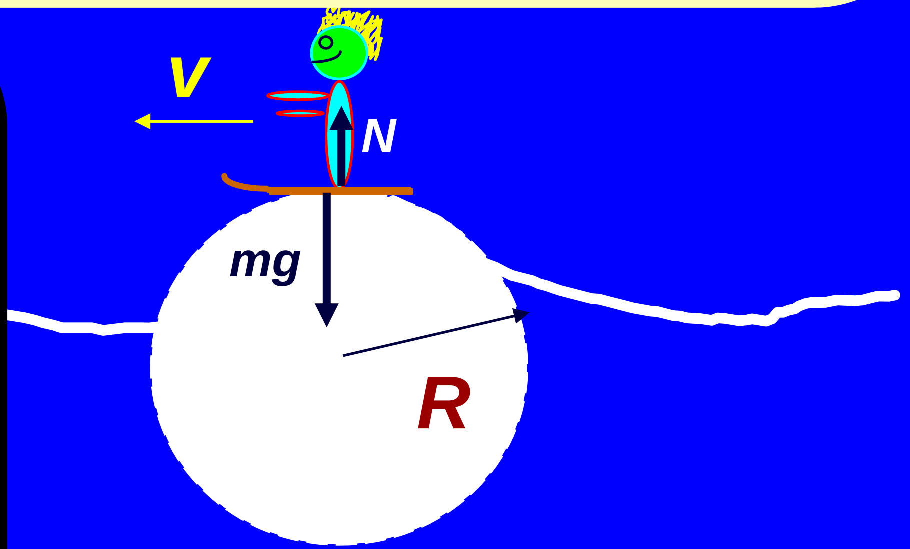
B) $F_c = mg - N$

C) $F_c = T + N - mg$

D) $F_c = N$

E) $F_c = mg$

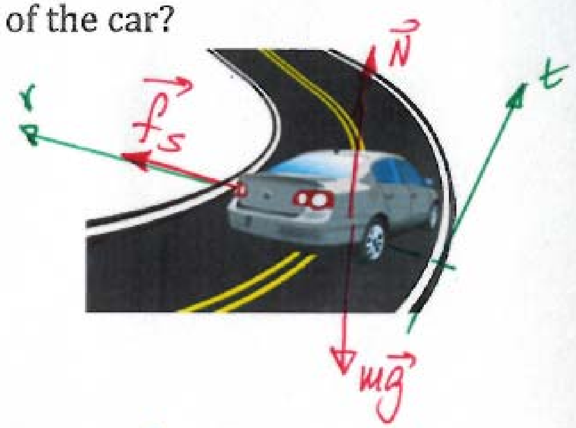
F_c points toward the center of the circle (*i.e.*, downward in this case). The weight vector points down and the normal force (exerted by the hill) points up. The magnitude of the net force, therefore, is $F_c = mg - N$.



Follow-up: What happens when the skier goes into a small dip? $F_c = N - mg$

Example Car on a circular flat road

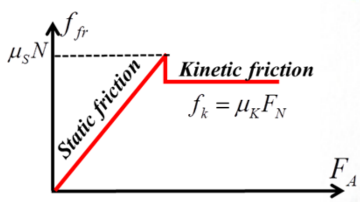
What is the maximum speed with which a 1200-kg car can round a turn of radius 80 m ona flat road if the coefficient of static friction between tires and road is 0.65? Is the result independent of the mass of the car?



● The radial force required to keep the car in the curved path is supplied by the force of static friction between the tires and the road.

The max static friction force is

$$f_s^{max} = \mu_s \cdot N = \mu_s \cdot mg$$



In this case the car would be on a verge of skidding. Let's find the speed corresponding to this centripetal force (f_s^{max}) and that would be the max speed.

● N. 2nd law in the r direction

$$\sum F_r = ma_r \Rightarrow f_s^{max} = m\frac{v_{max}^2}{R}$$

$$\mu_s \cdot m \cdot g = m \cdot \frac{v_{max}^2}{R} \Rightarrow \left\| v_{max} = \sqrt{\mu_s \cdot gR} \right\| = \sqrt{0.65 \cdot 9.8\,m/s^2 \cdot 80\,m} = 22.6\ m/s$$

It's independent of the car's mass.

ConcepTest Around the Curve

You drive your car too fast around a curve and the car starts to skid. What is the correct description of this situation?

A) car's engine is not strong enough to keep the car from being pushed out

B) friction between tires and road is not strong enough to keep car in a circle

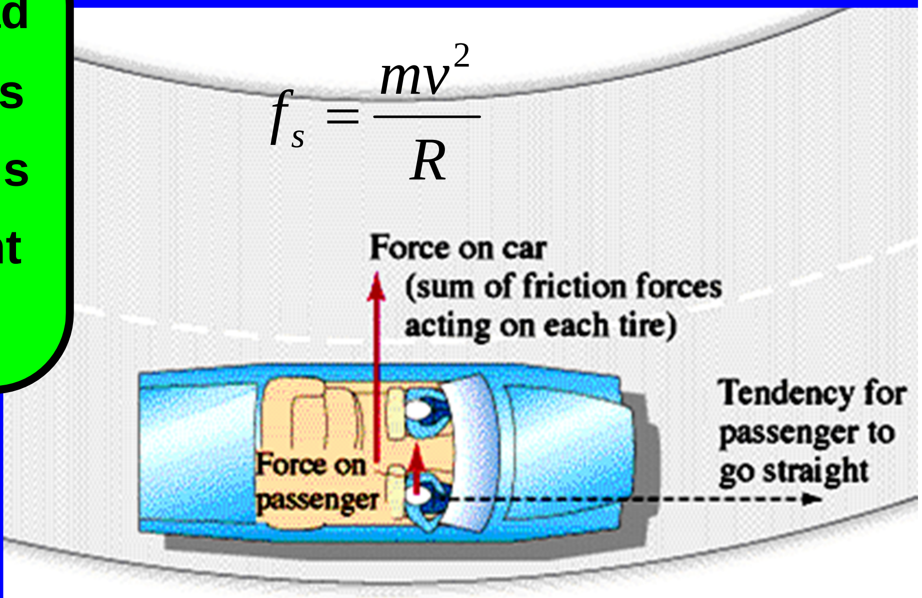
C) car is too heavy to make the turn

D) a deer caused you to skid

E) none of the above

The friction force between tires and road provides the centripetal force that keeps the car moving in a circle. If this force is too small, the car continues in a straight line!

$$f_s = \frac{mv^2}{R}$$

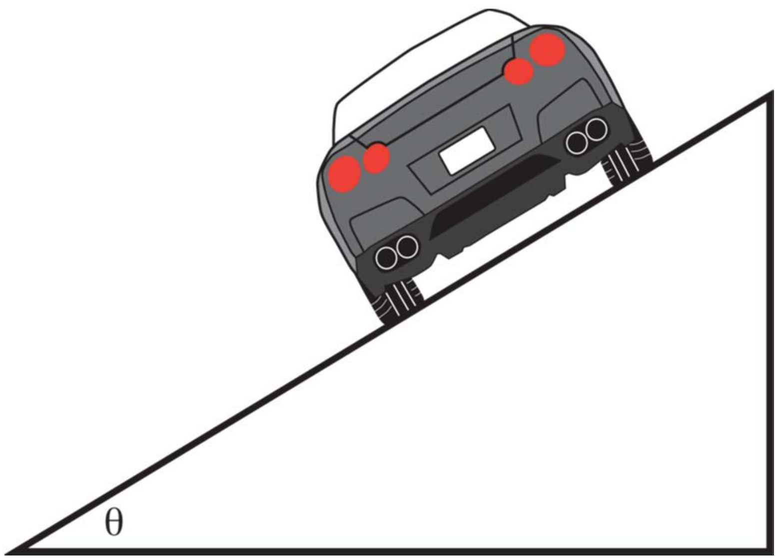


Follow-up: What could be done to the road or car to prevent skidding?

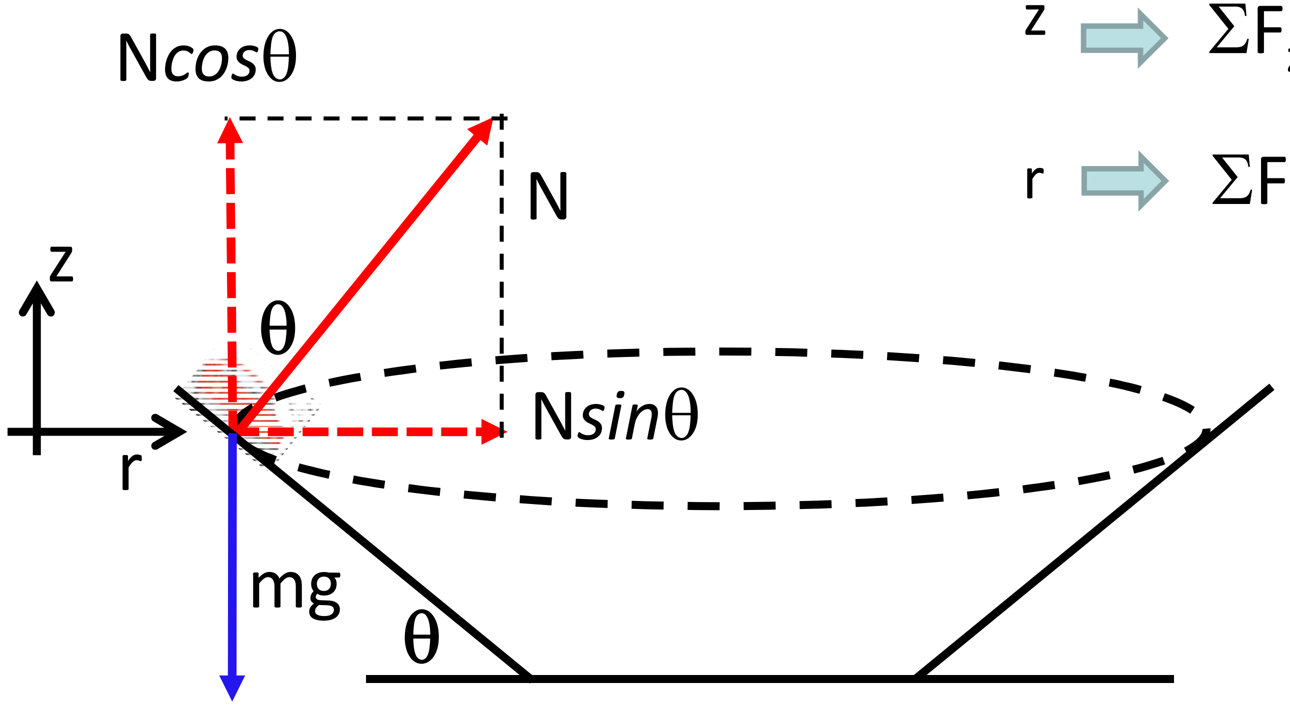

Thank you
Bye Bye For Now

Examples. Banked curve

- ***But sometimes, friction force is not enough to keep a car on a circular road.***
- ***Banking the curve can help to keep cars from skidding.***



Banked Curves (solution)



z ⇨ $\Sigma F_z = N\cos\theta - mg = 0$

r ⇨ $\Sigma F_r = N\sin\theta = ma_r$

$a_r = v^2/R$

$$\begin{cases} N\sin\theta = mv^2/R \\ N\cos\theta = mg \end{cases}$$

Take a ratio

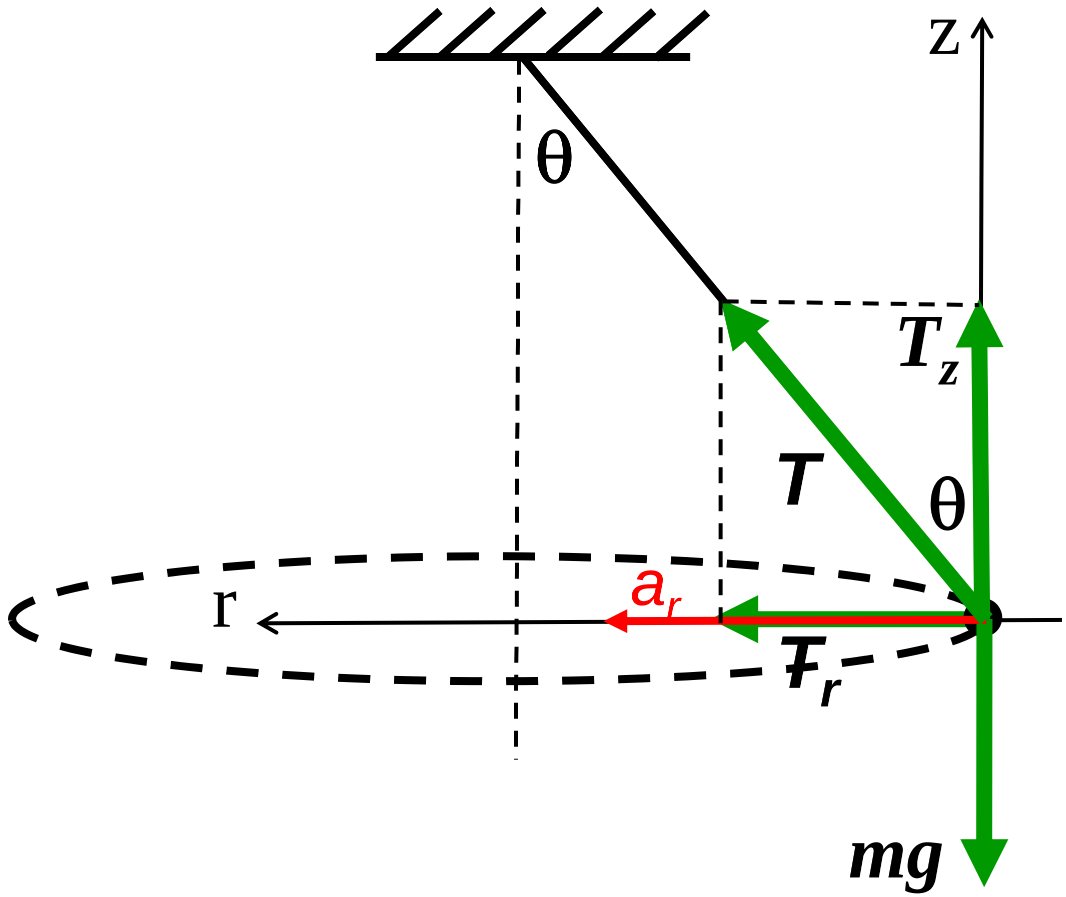
Independent of object mass !!!

$$\tan\theta = v^2/gR$$

r component of normal force provides the centripetal acceleration

http://phys23p.sl.psu.edu/phys_anim/mech/car_banked_new.avi

Example: Conical pendulum



$$\sum F_z = T_z - mg = 0 \quad (1)$$

$$\sum F_r = T_r = ma_r$$

$$T_r = T \cdot Sin\theta = \frac{mv^2}{R} \quad (2)$$

r component of tension provides the centripetal acceleration