Lecture 6

University of Massachusetts
UMASS Lowell

Physics I

Chapter 4

Rotational Motion

Course website:
http://faculty.uml.edu/Andriy_Danylov/Teaching/PhysicsI

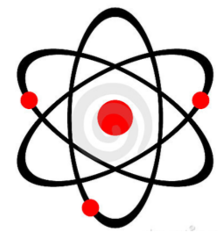
Today we are going to discuss:

Chapter 4:

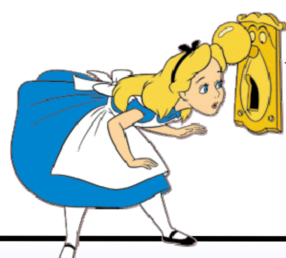
- Uniform Circular Motion**:** *Section 4.4*
- Nonuniform Circular Motion**:** *Section 4.6*

In addition to translation, objects can rotate

There is rotation everywhere you look in the universe, from the nuclei of atoms to spiral galaxies



Rotational Motion



Need to develop a vocabulary for describing rotational motion

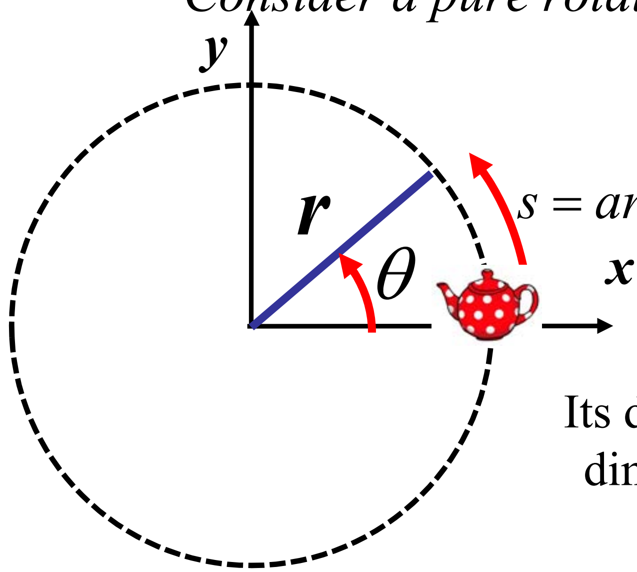
In order to describe rotation, we need to define

How to measure angles?

You know degrees, but in the scientific world *RADIANS* are more popular.
Let’s introduce them.

Angular Position in polar coordinates

Consider a pure rotational motion: an object moves around a fixed axis.



$s = arclength$

Instead of using x and y cartesian coordinates, we will define object's position with: $\boxed{r, \theta}$

$$\boxed{\theta = \frac{s}{r}}$$ <u>*θ in radians!*</u>

Its definition as a ratio of two length makes it a pure number without dimensions. So, the <u>radian is dimensionless</u> and there is no need to mention it in calculations

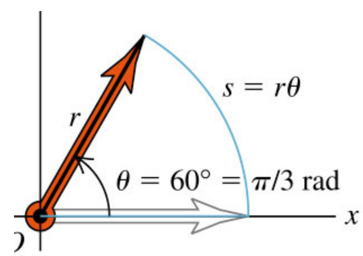
(Thus the unit of angle (radians) is really just a name to remind us that we are dealing with an angle) .

If angle is given in radians, we can get an arclength spanning angle θ

$$\boxed{s = r\theta}$$

Important

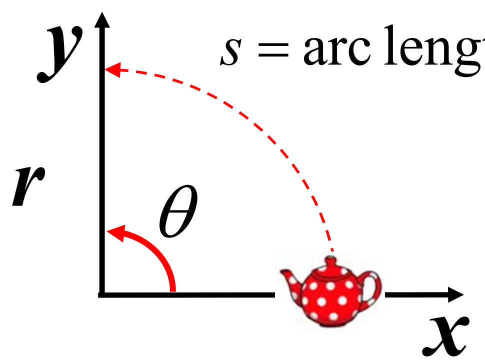
Use *Radians* to get an arc length



INCORRECT $\cancel{s = 60^\circ r}$

CORRECT $s = \frac{\pi}{3} r$

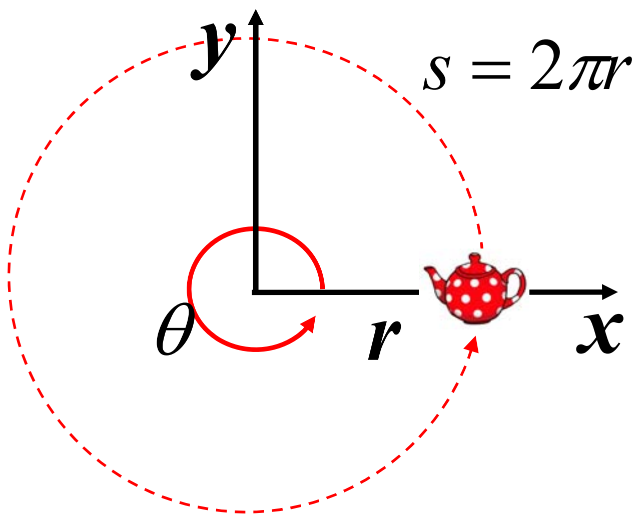
Examples: angles in radians



$$s = \text{arc length} = \frac{2\pi\,\mathrm{r}}{4} = \pi\, r/2$$

Apply $\theta = \dfrac{s}{r} = \dfrac{\pi r/2}{r}$

$$\boxed{\theta = \frac{\pi}{2}\ \textbf{\textit{radians!}}}$$



$$s = 2\pi r$$

$$\theta = \frac{s}{r} = \frac{2\pi r}{r} = \boxed{2\pi\ rad}$$

$$360^\circ = 2\pi\ rad$$

$$\boxed{1\ rad = 360/2\pi \approx 57.3^\circ}$$

Now we need to introduce rotational kinematic quantities like we did for translational motion.

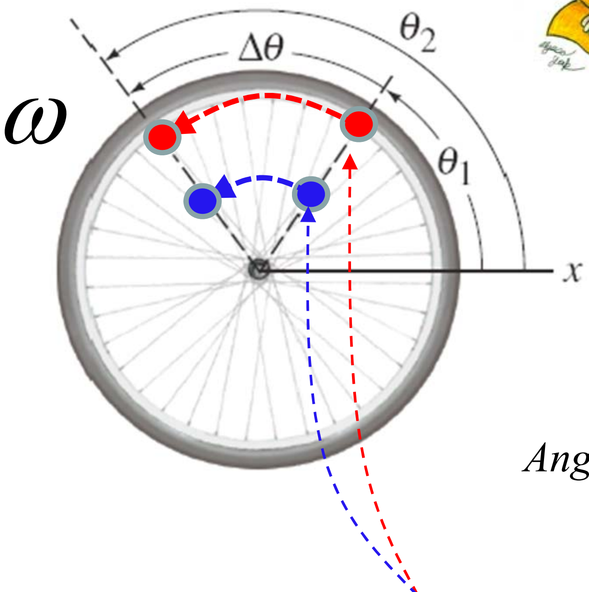
Angular displacement,

Angular velocity

Angular acceleration

for rotational kinematic equations

Angular displacement and velocity



***Angular displacement*:**

$$\boxed{\Delta\theta = \theta_2 - \theta_1}$$

The average angular velocity is defined as the total angular displacement divided by time:

$$\boxed{\overline{\omega} = \frac{\Delta\theta}{\Delta t}}$$

The instantaneous angular velocity:

$$\boxed{\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}}$$

$$\frac{rad}{s};\ \frac{deg}{s};\ \frac{rev}{s};\ \frac{rev}{min} = rpm$$

Angular velocity is the rate at which particle's angular position is changing.

For both points, $\Delta\theta$ and Δt are the same so

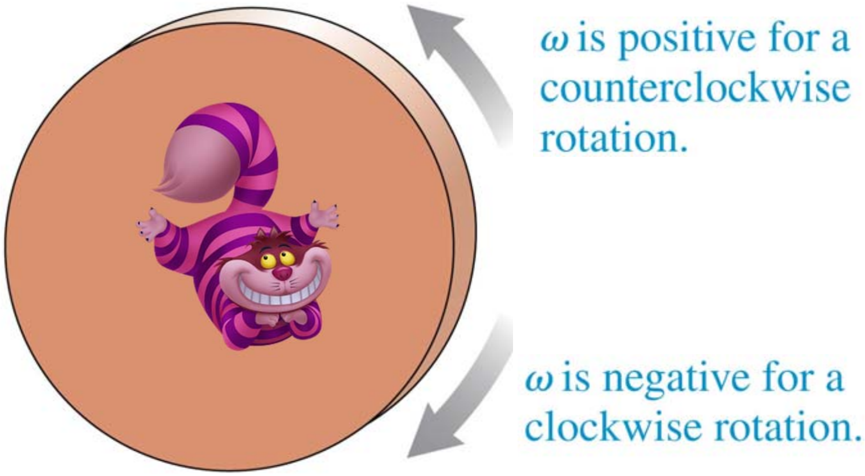
ω is the same for all points of a rotating object.

That is why we can say that Earth's angular velocity is $7.2\text{x}10^{-5}$ rad/sec without connecting to any point on the Earth. All points have the same ω.

So ω is like an intrinsic property of a solid rotating object.

Sign of Angular Velocity

When is the Angular velocity positive/negative?



As shown in the figure, ω can be positive or negative, and this follows from our definition of θ.

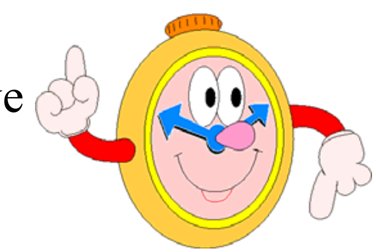
(Definition of θ: An angle θ is measured (convention) from the positive x-axis in a counterclockwise direction.)

Example:

ω is negative for a clockwise rotation.

For a clock hand, the angular velocity is negative

$$\omega < 0$$



ConcepTest *Bonnie and Klyde*

Bonnie **sits on the outer rim of a merry-go-round, and** **Klyde** **sits midway between the center and the rim. The merry-go-round makes one complete revolution every 2 seconds.**

Klyde's angular velocity is:

A) same as Bonnie's

B) twice Bonnie's

C) half of Bonnie's

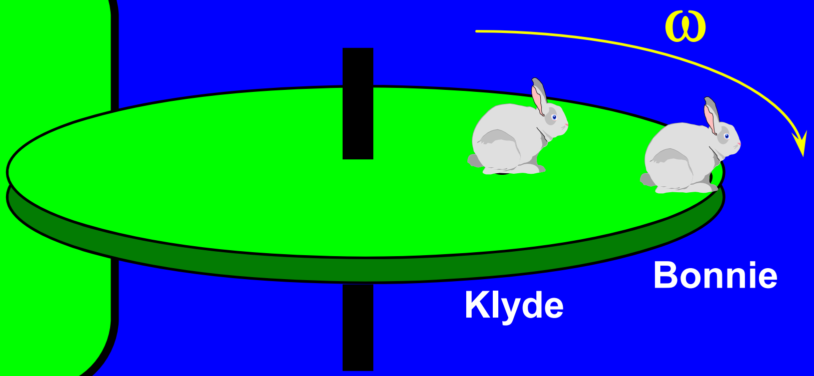
D) one-quarter of Bonnie's

E) four times Bonnie's

$$\overline{\omega} = \frac{\Delta\theta}{\Delta t} = \frac{1\,rev}{2\sec} = \frac{2\pi\, rad}{2\sec} = \pi\, {}^{rad}\!/_{\sec}$$

ω is the same for both rabbits

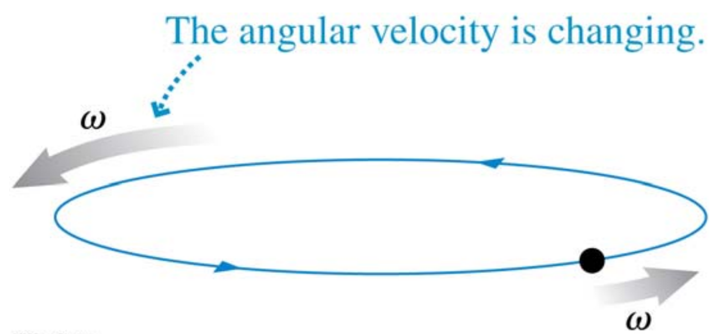
The **angular velocity ω** of any point on a solid object rotating about a fixed axis ***is the same***. Both Bonnie and Klyde go around one revolution (2π radians) every 2 sec.



A particle moves with ***uniform circular motion*** *if its* ***angular velocity is constant.***

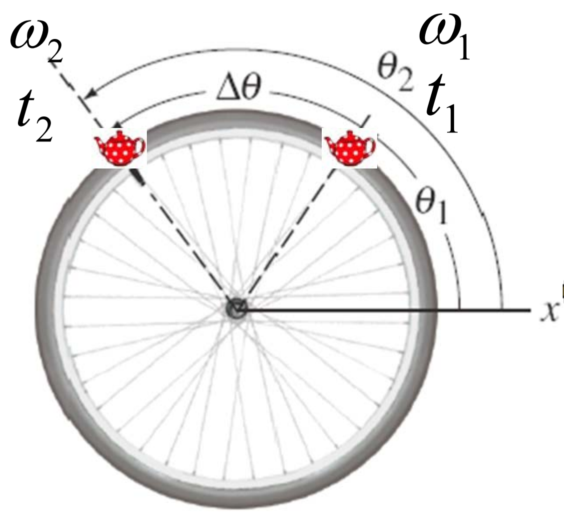
Now, if a *rotation is not uniform* (angular velocity is not constant), we can introduce angular acceleration

Angular acceleration



Angular Acceleration

The angular acceleration is the rate at which the angular velocity changes with time:



Average angular acceleration:

$$\boxed{\overline{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_2 - \omega_1}{t_2 - t_1}}$$

Units rad/s^2

Instantaneous angular acceleration:

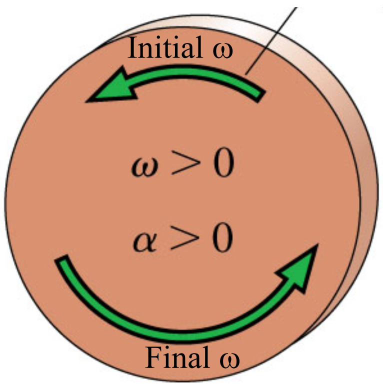
$$\boxed{\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}}$$

The units of angular acceleration are rad/s^2

Since ω is the same for all points of a rotating object, angular acceleration also will be the same for all points.

Thus, ω and α are properties of a rotating object

The Sign of Angular Acceleration, α

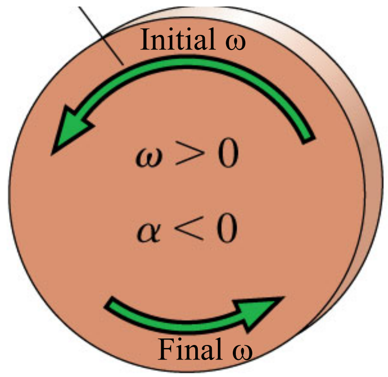


Speeding up ccw

Positive and larger — Positive and smaller

$$\alpha = \frac{\omega_f - \omega_i}{t_f - t_i} > 0$$

*So, **α is positive** if |ω| is increasing and is counter-clockwise.*

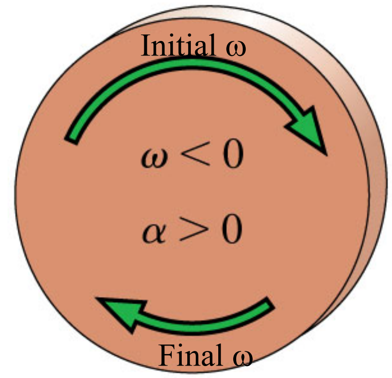


Slowing down ccw

Positive and smaller — Positive and larger

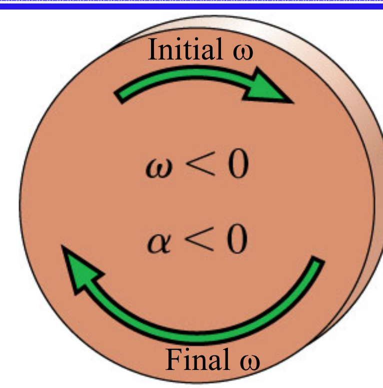
$$\alpha = \frac{\omega_f - \omega_i}{t_f - t_i} < 0$$

***α is negative** if |ω| is decreasing and ω is counterclockwise.*



Slowing down cw

***α is positive** if |ω| is decreasing and ω is clockwise.*



Speeding up cw

***α is negative** if |ω| is increasing and ω is clockwise.*

ConcepTest *The signs of ω and α*

The fan blade is slowing down. What are the signs of ω and α?

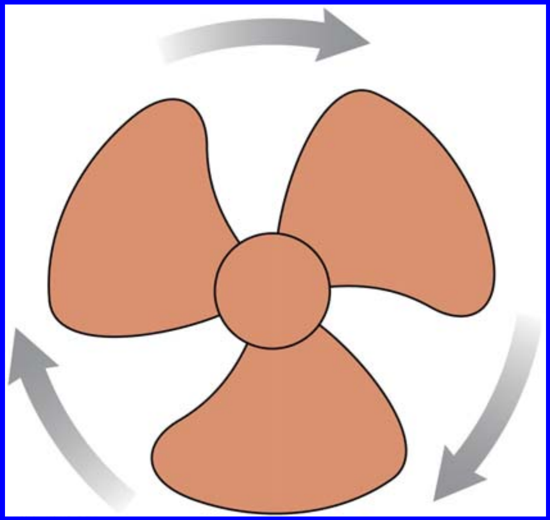
A) ω is positive and α is positive.

B) ω is positive and α is negative.

C) ω is negative and α is positive

D) ω is negative and α is negative

E) ω is positive and α is zero



1) ω is negative (rotation CW)

2) ω is slowing down ($|\omega_f| < |\omega_i|$)

Less Negative More Negative

$$\alpha = \frac{\omega_f - \omega_i}{t_f - t_i} > 0$$

For example

$$\alpha = \frac{-2-(-5)}{3-2} = +3\, rad/s^2$$

Case 3 (the previous slide)

Now, since we have introduced all angular quantities, we can write down

Rotational Kinematic Equations

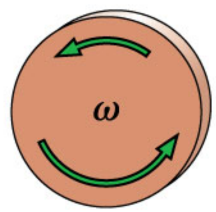
For motion with constant angular acceleration

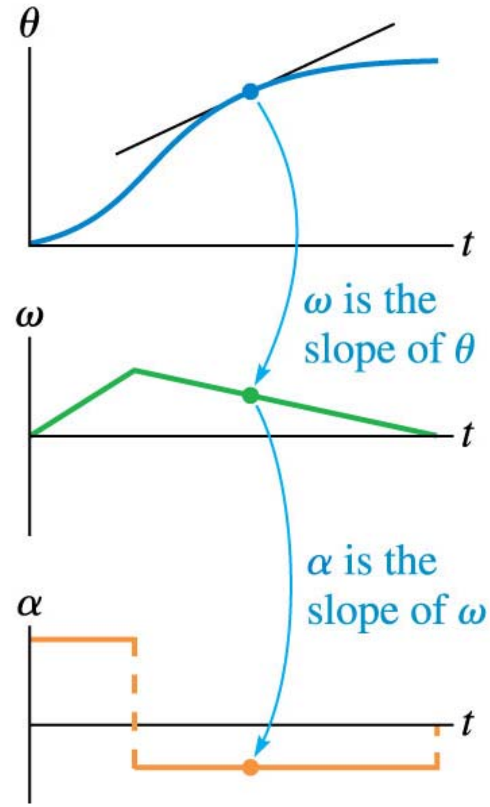
$$\alpha = const$$

Rotational kinematic equations

The equations of motion for translational and rotational motion (for constant acceleration) are identical

Applies to particles with circular trajectories and to rotating solid objects.

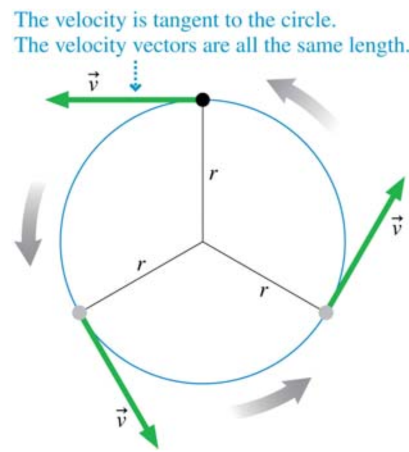




Translational kinematic equations $a = const$	*Analogs*	***Rotational kinematic equations*** $\alpha = const$
$v = v_o + at$	$v \to \omega$	$\omega = \omega_o + \alpha t$
$x = x_o + v_o t + \frac{1}{2} a t^2$	$x \to \theta$	$\theta = \theta_o + \omega_o t + \frac{1}{2} \alpha t^2$
$v^2 = v_o^2 + 2a(x - x_o)$	$a \to \alpha$	$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$

For a rotating object we can also introduce a linear velocity which is called the

Tangential velocity

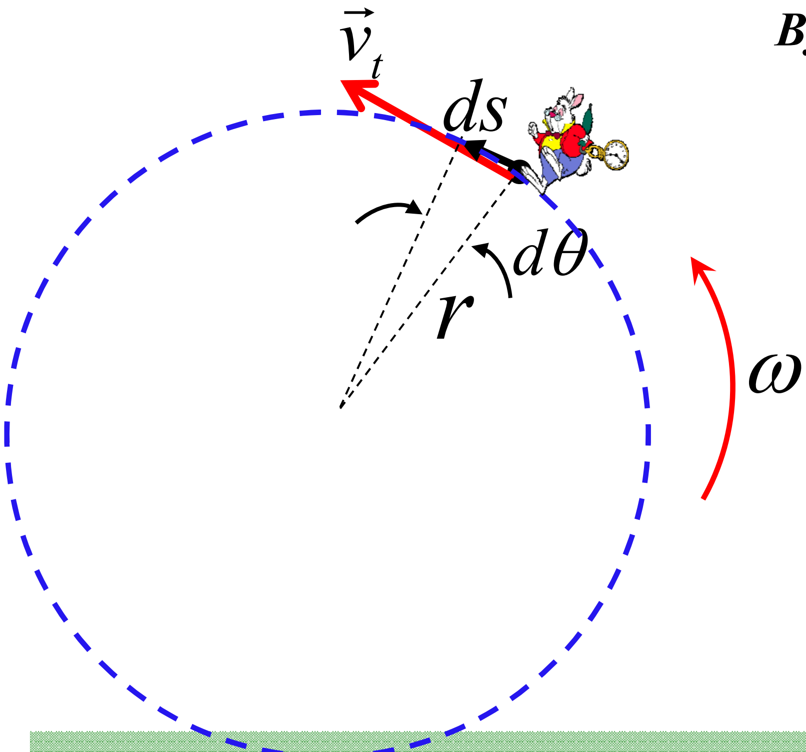


Now we need to introduce a useful expression relating

linear velocity and angular velocity

Relation between tangential and angular velocities

The tangential velocity component v_t is the rate ds/dt at which the particle moves around the circle, where s is the arc length.



By definition, linear velocity:

In the 1st slide, we defined:

$$v_t = \frac{ds}{dt} = \left\| \begin{matrix} s = r\theta \\ ds = r d\theta \end{matrix} \right\| = r\frac{d\theta}{dt} = \left\| \omega = \frac{d\theta}{dt} \right\| = r\omega$$

$$\boxed{v_t = r\omega}$$

Remember it!!!!
You will use it often!!!

Relation between linear and angular velocities (**<u>ω in rad/sec</u>**)

Each point on a rotating rigid body has ***the same angular*** displacement, velocity, and acceleration!

The corresponding ***linear (or tangential) variables depend on the radius*** and ***the linear velocity is greater for points farther from the axis.***

The end of the class

ConcepTest Bonnie and Klyde II

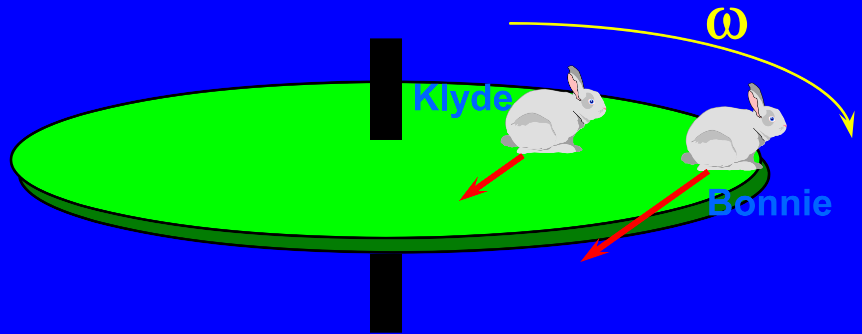
Bonnie sits on the outer rim of a merry-go-round, and **Klyde** sits midway between the center and the rim. The merry-go-round makes one revolution every 2 seconds. **Who has the larger linear (tangential) velocity?**

A) Klyde
B) Bonnie
C) both the same
D) linear velocity is zero for both of them

We already know that all points of a rotating body have the same angular velocity ω.

But their linear speeds v will be different because $v_t = r\omega$ and Bonnie is located farther out (larger radius R) than Klyde.

$$V_{Klyde} = \frac{1}{2} V_{Bonnie}$$

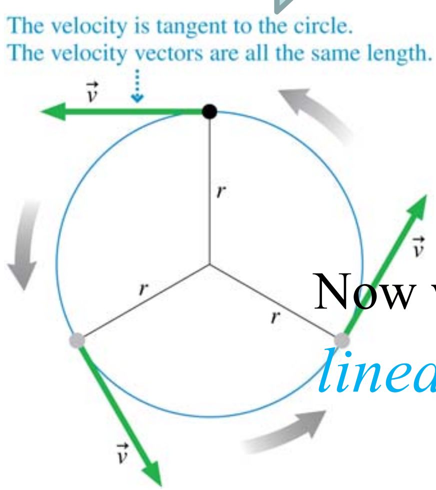


For a rotating object we can also introduce the

Acceleration

Tangential acceleration

Centripetal acceleration



Now we need to introduce a useful expression relating

linear acceleration and angular acceleration

a  α

Tangential acceleration

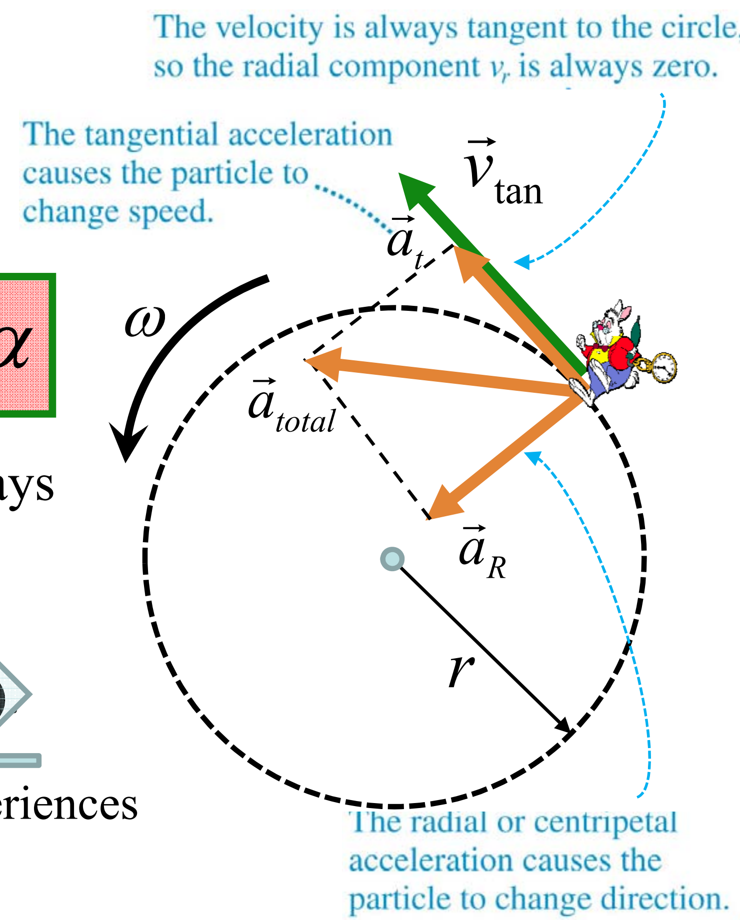
The particle in the figure is moving along a circle and is speeding up.

(Definition) ***Tangential acceleration*** is the rate at which the tangential velocity changes, $a_t = dv_t/dt$.

$$a_t = \frac{dv_t}{dt} = \| v_t = r\omega \| = r\frac{d\omega}{dt} = r\alpha \qquad \boxed{a_t = r\alpha}$$

There is a tangential acceleration a_t, which is always tangent to the circle.

There is also the ***centripetal acceleration*** is $a_r = v_t^2/r$, where v_t is the tangential speed (next slide).



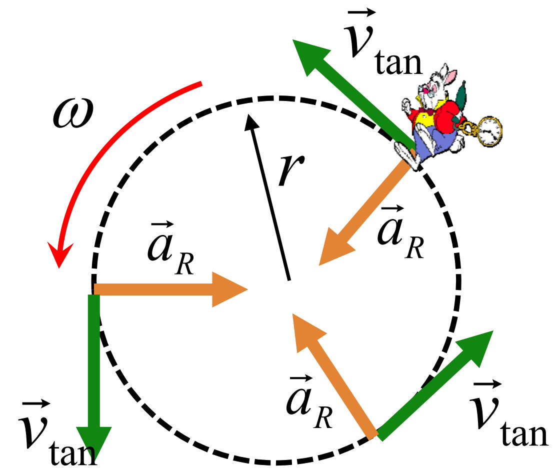
Finally, any object that is undergoing circular motion experiences two accelerations: centripetal and tangential.

Let's get a total acceleration:

$$\vec{a}_{total} = \vec{a}_t + \vec{a}_r \qquad a_{total} = \sqrt{a_t^2 + a_r^2}$$

Centripetal acceleration

- In uniform circular motion *(ω=const)*, although the speed is constant, there is an acceleration because the *direction* of the velocity vector is always changing.
- The acceleration of uniform circular motion is called **centripetal acceleration**.
- The direction of the centripetal acceleration is toward the center of the circle.
- The magnitude of the centripetal acceleration is constant for uniform circular motion:



The velocity is tangent to the circle.

$$\boxed{a_r = \frac{v_t^2}{r} \qquad \textit{(toward center of circle)}}$$

(centripetal acceleration)

(without derivation)

Centripetal acceleration can be rewritten in term of angular velocity, ω

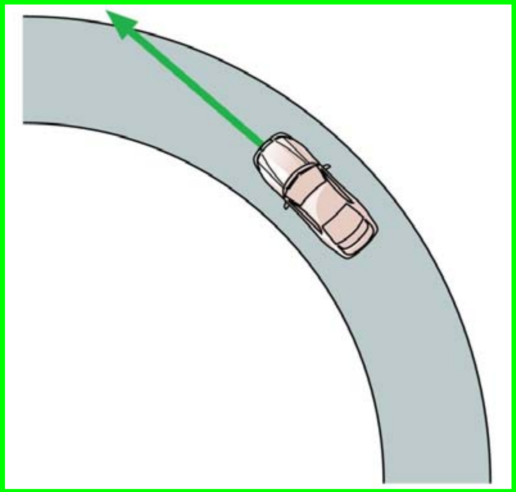
$$a_r = \frac{v_t^2}{r} = \omega^2 r \qquad v_{\tan} = r\omega$$

ConcepTest *Car on a curve*

A car is traveling around a curve at a steady 45 mph. Is the car accelerating?

A) Yes

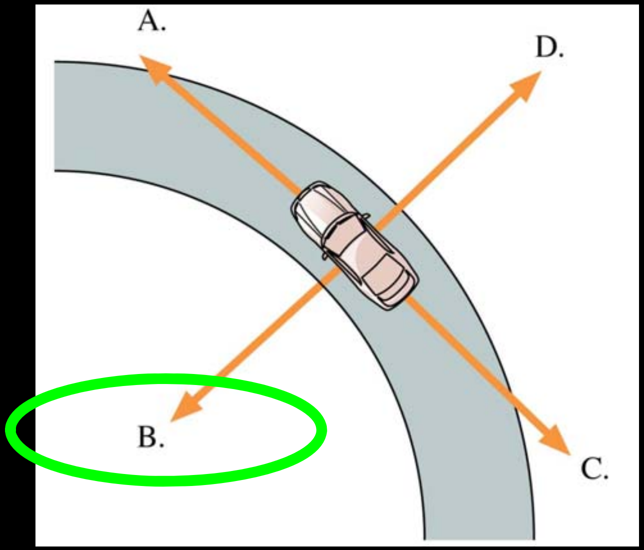
B) No



There is a Centripetal acceleration

ConcepTest Car on a curve

A car is traveling around a curve at a steady 45 mph. Which vector shows the direction of the car's acceleration?



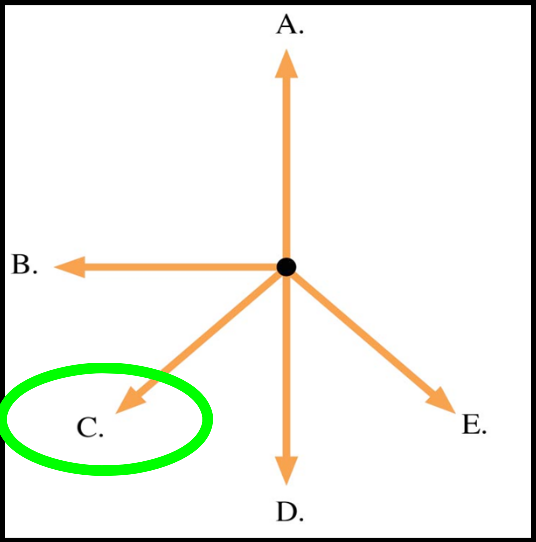
*There is a **Centripetal acceleration pointing toward the center***

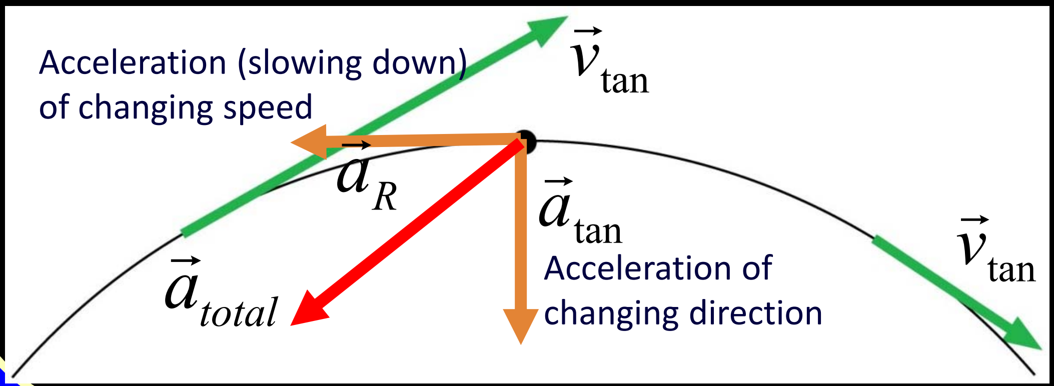
ConcepTest

Car on a curve

A car is slowing down as it drives over a circular hill.

Which of these is the acceleration vector at the highest point?





Uniform circular motion

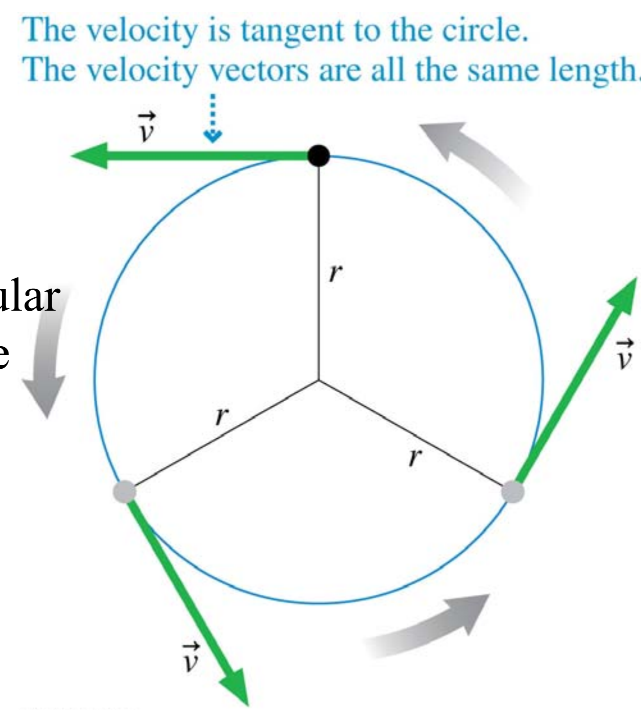
ω =const

A particle moves with ***uniform circular motion*** *if its* ***angular velocity is constant.***

The time interval to complete one revolution is called the period, T.

The period T is related to the speed v:

$$v = \frac{1 \text{ circumference}}{1 \text{ period}} = \frac{2\pi r}{T}$$



In this case, as the particle goes around a circle one time, its angular displacement is $\Delta\theta = 2\pi$ during one period $\Delta t = T$. Then , the angular velocity is related to the period of the motion:

$$\omega = \frac{d\theta}{dt} = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T}$$

$$|\omega| = \frac{2\pi \text{ rad}}{T} \qquad \text{or} \qquad T = \frac{2\pi \text{ rad}}{|\omega|}$$

Thank you
See you on Wednesday