95.413/513. Classical Mechanics.

First Midterm Exam.

Oct 22, 2013 (Open book exam)

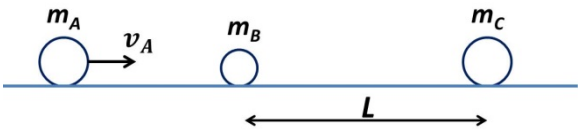
1. (20 points) Three particles *A, B*, and *C* with masses $m_A=2m_B=m_C$ are arranged (in that order) in a straight line. Initially, *B* and *C* are at rest a distance ***L*** apart, and *A* is projected towards *B* with speed v_A. The particles then undergo elastic head-on collisions.

a) *(5 points) Show that for any elastic head-on collision, the relative speed of the two objects after the collision has the same magnitude (but opposite direction) as before the collision*

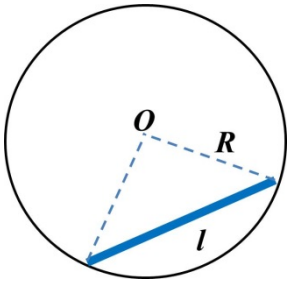
$$v_A - v_B = -(v'_A - v'_B),$$

Where primes define quantities after the collision.
(This equation is simpler to use than the conservation of energy.

b) (15 points) Show that A and B collide twice and that the time interval between these two collisions is $\Delta t = 12L/7v_A$.



2. (10 points) A thin uniform rod of length l and mass ***m*** has its two ends constrained to move on the circumference of a smooth vertical circle of radius ***R***. The circle doesn't move.



a) *Introduce a parameter to describe the system.*
b) *Write down the total PE of the system as a function of the parameter.*
c) *Determine conditions for equilibrium and classify it as stable or unstable.*

3. (10 points) Prove that the force ***F*** = *(yz, zx, xy)* acting on a particle is conservative and find the potential energy *U(x,y,z).*

4. (10 points) Two masses are connected by a cord passing over a pulley of radius R and moment of inertia I. Mass m_1 slides on a frictionless surface, and m_2 hangs freely. Determine

a) *a formula for the angular momentum of the system about the pulley axis, as a function of the speed* ***v*** *of mass* m_1 *or* m_2*;*
b) *if angular momentum is conserved;*
c) *The acceleration of the masses.*