Chapter 11

Oscillations and Waves
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If an object vibrates or oscillates back and forth over the same path, each cycle taking the same amount of time, the motion is called periodic. The mass and spring system is a useful model for a periodic system.
11-1 Simple Harmonic Motion—Spring Oscillations

We assume that the surface is frictionless. There is a point where the spring is neither stretched nor compressed; this is the equilibrium position. We measure displacement from that point ($x = 0$ on the previous figure).

The force exerted by the spring depends on the displacement:

$$F = -kx.$$  

[force exerted by spring] (11-1)
11-1 Simple Harmonic Motion—Spring Oscillations

• The minus sign on the force indicates that it is a restoring force—it is directed to restore the mass to its equilibrium position.

• $k$ is the spring constant

• The force is not constant, so the acceleration is not constant either
11-1 Simple Harmonic Motion—Spring Oscillations

• Displacement is measured from the equilibrium point

• Amplitude is the maximum displacement

• A cycle is a full to-and-fro motion; this figure shows half a cycle

• Period is the time required to complete one cycle

• Frequency is the number of cycles completed per second
If the spring is hung vertically, the only change is in the equilibrium position, which is at the point where the spring force equals the gravitational force.
Any vibrating system where the restoring force is proportional to the negative of the displacement is in simple harmonic motion (SHM), and is often called a simple harmonic oscillator.
11-2 Energy in Simple Harmonic Motion

We already know that the potential energy of a spring is given by:

\[ PE = \frac{1}{2} kx^2 \]

The total mechanical energy is then:

\[ E = \frac{1}{2} mv^2 + \frac{1}{2} kx^2, \quad (11-3) \]

The total mechanical energy will be conserved, as we are assuming the system is frictionless.
11-2 Energy in Simple Harmonic Motion

If the mass is at the limits of its motion, the energy is all potential.

If the mass is at the equilibrium point, the energy is all kinetic.

We know what the potential energy is at the turning points:

\[ E = \frac{1}{2} m (0)^2 + \frac{1}{2} kA^2 = \frac{1}{2} kA^2. \quad (11-4a) \]
The total energy is, therefore $\frac{1}{2} kA^2$

And we can write:

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2.$$  \hspace{3em} (11-4c)

This can be solved for the velocity as a function of position:

$$v = \pm v_{\text{max}} \sqrt{1 - \frac{x^2}{A^2}}.$$  \hspace{3em} (11-5b)

where

$$v_{\text{max}} = \sqrt{\frac{k}{m}} A.$$  \hspace{3em} (11-5a)
If we look at the projection onto the $x$ axis of an object moving in a circle of radius $A$ at a constant speed $v_{\text{max}}$, we find that the $x$ component of its velocity varies as:

$$v = \pm v_{\text{max}} \sqrt{1 - \frac{x^2}{A^2}}. \quad (11-5b)$$

This is identical to SHM.
11-3 The Period and Sinusoidal Nature of SHM

Therefore, we can use the period and frequency of a particle moving in a circle to find the period and frequency:

\[
T = 2\pi \sqrt{\frac{m}{k}}. \tag{11-6a}
\]

\[
f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}. \tag{11-6b}
\]
11-3 The Period and Sinusoidal Nature of SHM

We can similarly find the position as a function of time:

\[ x = A \cos \omega t. \quad (11-8a) \]

\[ x = A \cos(2\pi ft), \quad (11-8b) \]

\[ x = A \cos(2\pi t/T). \quad (11-8c) \]
11-3 The Period and Sinusoidal Nature of SHM

The top curve is a graph of the previous equation. The bottom curve is the same, but shifted $\frac{1}{4}$ period so that it is a sine function rather than a cosine.
The velocity and acceleration can be calculated as functions of time; the results are below, and are plotted at left.

\[ v = -v_{\text{max}} \sin \omega t = -v_{\text{max}} \sin(2\pi ft) = -v_{\text{max}} \sin(2\pi t/T). \quad (11-9) \]

\[ a = \frac{F}{m} = -\frac{kx}{m} = -\left(\frac{kA}{m}\right) \cos \omega t = -a_{\text{max}} \cos(2\pi t/T). \quad (11-10) \]
A simple pendulum consists of a mass at the end of a lightweight cord. We assume that the cord does not stretch, and that its mass is negligible.
In order to be in SHM, the restoring force must be proportional to the negative of the displacement. Here we have \( F = -mg \sin \theta \) which is proportional to \( \sin \theta \) and not to \( \theta \) itself.

However, if the angle is small, \( \sin \theta \approx \theta \).

### TABLE 11-1

<table>
<thead>
<tr>
<th>( \theta ) (degrees)</th>
<th>( \theta ) (radians)</th>
<th>( \sin \theta )</th>
<th>% Difference</th>
</tr>
</thead>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1°</td>
<td>0.01745</td>
<td>0.01745</td>
<td>0.005%</td>
</tr>
<tr>
<td>5°</td>
<td>0.08727</td>
<td>0.08716</td>
<td>0.1%</td>
</tr>
<tr>
<td>10°</td>
<td>0.17453</td>
<td>0.17365</td>
<td>0.5%</td>
</tr>
<tr>
<td>15°</td>
<td>0.26180</td>
<td>0.25882</td>
<td>1.1%</td>
</tr>
<tr>
<td>20°</td>
<td>0.34907</td>
<td>0.34202</td>
<td>2.0%</td>
</tr>
<tr>
<td>30°</td>
<td>0.52360</td>
<td>0.50000</td>
<td>4.5%</td>
</tr>
</tbody>
</table>

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11-4 The Simple Pendulum

Therefore, for small angles, the force is approximately proportional to the angular displacement.

The period and frequency are:

\[ T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{m}{mg/\ell}} \]

or

\[ T = 2\pi \sqrt{\frac{\ell}{g}}. \quad [\theta \text{ small}] \quad (11-11a) \]

\[ f = \frac{1}{2\pi} \sqrt{\frac{g}{\ell}}. \quad [\theta \text{ small}] \quad (11-11b) \]
So, as long as the cord can be considered massless and the amplitude is small, the period does not depend on the mass.
Damped harmonic motion is harmonic motion with a frictional or drag force. If the damping is small, we can treat it as an “envelope” that modifies the undamped oscillation.
However, if the damping is large, it no longer resembles SHM at all.

A: underdamping: there are a few small oscillations before the oscillator comes to rest.

B: critical damping: this is the fastest way to get to equilibrium.

C: overdamping: the system is slowed so much that it takes a long time to get to equilibrium.
There are systems where damping is unwanted, such as clocks and watches.

Then there are systems in which it is wanted, and often needs to be as close to critical damping as possible, such as automobile shock absorbers and earthquake protection for buildings.
Forced vibrations occur when there is a periodic driving force. This force may or may not have the same period as the natural frequency of the system.

If the frequency is the same as the natural frequency, the amplitude becomes quite large. This is called resonance.
The sharpness of the resonant peak depends on the damping. If the damping is small (A), it can be quite sharp; if the damping is larger (B), it is less sharp.

Like damping, resonance can be wanted or unwanted. Musical instruments and TV/radio receivers depend on it.
A wave travels along its medium, but the individual particles just move up and down.
All types of traveling waves transport energy.

Study of a single wave pulse shows that it is begun with a vibration and transmitted through internal forces in the medium.

Continuous waves start with vibrations too. If the vibration is SHM, then the wave will be sinusoidal.
Wave characteristics:

- Amplitude, $A$
- Wavelength, $\lambda$
- Frequency $f$ and period $T$
- Wave velocity $v = \lambda f$. (11-12)
The motion of particles in a wave can either be perpendicular to the wave direction (transverse) or parallel to it (longitudinal).
11-8 Types of Waves and Their Speeds: Transverse and Longitudinal

Sound waves are longitudinal waves:

Drum membrane
Compression
Expansion
Earthquakes produce both longitudinal and transverse waves. Both types can travel through solid material, but only longitudinal waves can propagate through a fluid—in the transverse direction, a fluid has no restoring force.

Surface waves are waves that travel along the boundary between two media.
11-9 Energy Transported by Waves

Just as with the oscillation that starts it, the energy transported by a wave is proportional to the square of the amplitude.

Definition of intensity:

\[ I = \frac{\text{energy/time}}{\text{area}} = \frac{\text{power}}{\text{area}}. \]

The intensity is also proportional to the square of the amplitude:

\[ I \propto A^2. \quad (11-15) \]
If a wave is able to spread out three-dimensionally from its source, and the medium is uniform, the wave is spherical.

Just from geometrical considerations, as long as power output is constant, we see:

\[ I \propto \frac{1}{r^2}. \]  

[11-16b] [spherical wave]
By looking at the energy of a particle of matter in the medium of the wave, we find:

$$E = 2\pi^2 \rho Svtf^2 A^2. \quad (11-17a)$$

Then, assuming the entire medium has the same density, we find:

$$I = \frac{\overline{P}}{S} = 2\pi^2 \rho vf^2 A^2. \quad (11-18)$$

Therefore, the intensity is proportional to the square of the frequency and to the square of the amplitude.
A wave reaching the end of its medium, but where the medium is still free to move, will be reflected (b), and its reflection will be upright.

A wave hitting an obstacle will be reflected (a), and its reflection will be inverted.
A wave encountering a denser medium will be partly reflected and partly transmitted; if the wave speed is less in the denser medium, the wavelength will be shorter.
Two- or three-dimensional waves can be represented by wave fronts, which are curves of surfaces where all the waves have the same phase.

Lines perpendicular to the wave fronts are called rays; they point in the direction of propagation of the wave.
The law of reflection: the angle of incidence equals the angle of reflection.
The superposition principle says that when two waves pass through the same point, the displacement is the arithmetic sum of the individual displacements.

In the figure below, (a) exhibits destructive interference and (b) exhibits constructive interference.
These figures show the sum of two waves. In (a) they add constructively; in (b) they add destructively; and in (c) they add partially destructively.
Standing waves occur when both ends of a string are fixed. In that case, only waves which are motionless at the ends of the string can persist. There are nodes, where the amplitude is always zero, and antinodes, where the amplitude varies from zero to the maximum value.
The frequencies of the standing waves on a particular string are called resonant frequencies.

They are also referred to as the fundamental and harmonics.
The wavelengths and frequencies of standing waves are:

\[ \lambda_n = \frac{2L}{n}, \quad n = 1, 2, 3, \ldots \]  \hspace{1cm} \text{[string fixed at both ends]} \quad \text{(11-19a)}

\[ f_n = \frac{v}{\lambda_n} = n \frac{v}{2L} = nf_1, \quad n = 1, 2, 3, \ldots \] \quad \text{(11-19b)}
If the wave enters a medium where the wave speed is different, it will be refracted—its wave fronts and rays will change direction.

We can calculate the angle of refraction, which depends on both wave speeds:

\[
\frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1}. \quad (11-20)
\]
The law of refraction works both ways—a wave going from a slower medium to a faster one would follow the red line in the other direction.
When waves encounter an obstacle, they bend around it, leaving a “shadow region.” This is called diffraction.
The amount of diffraction depends on the size of the obstacle compared to the wavelength. If the obstacle is much smaller than the wavelength, the wave is barely affected (a). If the object is comparable to, or larger than, the wavelength, diffraction is much more significant (b, c, d).
To the left, we have a snapshot of a traveling wave at a single point in time. Below left, the same wave is shown traveling.
11-15 Mathematical Representation of a Traveling Wave

A full mathematical description of the wave describes the displacement of any point as a function of both distance and time:

\[ y = A \sin \left( \frac{2\pi}{\lambda} (x - vt) \right) \]. (11-22)
Summary of Chapter 11

- For SHM, the restoring force is proportional to the displacement.
- The period is the time required for one cycle, and the frequency is the number of cycles per second.
- Period for a mass on a spring: \( T = 2\pi \sqrt{\frac{m}{k}} \) \hspace{1cm} (11-6a)
- SHM is sinusoidal.
- During SHM, the total energy is continually changing from kinetic to potential and back.
A simple pendulum approximates SHM if its amplitude is not large. Its period in that case is:

\[ T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{m}{mg/\ell}} \]

or

\[ T = 2\pi \sqrt{\frac{\ell}{g}} \quad [\theta \text{ small}] \quad (11-11a) \]

When friction is present, the motion is damped.

If an oscillating force is applied to a SHO, its amplitude depends on how close to the natural frequency the driving frequency is. If it is close, the amplitude becomes quite large. This is called resonance.
Vibrating objects are sources of waves, which may be either a pulse or continuous.

- Wavelength: distance between successive crests.
- Frequency: number of crests that pass a given point per unit time.
- Amplitude: maximum height of crest.
- Wave velocity: $v = \lambda f$
Summary of Chapter 11

- Transverse wave: oscillations perpendicular to direction of wave motion.
- Longitudinal wave: oscillations parallel to direction of wave motion.
- Intensity: energy per unit time crossing unit area (W/m²):
  \[ I \propto \frac{1}{r^2}. \]  
  [spherical wave] \hspace{1cm} (11-16b)
- Angle of reflection is equal to angle of incidence.
Summary of Chapter 11

• When two waves pass through the same region of space, they interfere. Interference may be either constructive or destructive.

• Standing waves can be produced on a string with both ends fixed. The waves that persist are at the resonant frequencies.

• Nodes occur where there is no motion; antinodes where the amplitude is maximum.

• Waves refract when entering a medium of different wave speed, and diffract around obstacles.