CHAPTER 14
Traffic Flow Theory

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1. Basic Equations

Let us derive the basic conservation equation for traffic flow. We consider the flow of vehicles on a long road where the features of the flow we wish to calculate, such as bottlenecks, etc., are long compared with the average distances between vehicles. Let \( n(x, x + \Delta x, t) \) denote the number of vehicles between point \( x \) and point \( x + \Delta x \) on the road at time \( t \) (see Figure 14.1). We shall assume that \( k(x, t) \) exists such that for any \( x, \Delta x, \) and \( t, \)

\[
n(x, x + \Delta x, t) = \int_x^{x + \Delta x} k(\hat{x}, t) \, d\hat{x}.
\]

We note that, by the fundamental theorem of calculus,

\[
k(x, t) = \lim_{\Delta x \to 0} \frac{n(x, x + \Delta x, t)}{\Delta x}
\]

if \( k \) is continuous. We shall assume that we can adequately model the situations of interest with the assumption that \( k \) is continuous.

In terms of infinitesimals, \( k \) is the number of vehicles per unit length in the infinitesimal length between \( x \) and \( x + \Delta x \) at time \( t \). Empirical values of \( k \) can be determined from aerial photographs of roads. We select some "small" (infinitesimal) length \( \Delta x \), count the vehicles between \( x \) and \( x + \Delta x \), and divide by \( \Delta x \).

Now let us define the flow rate \( q(x, t) \). The flow rate \( q \) is simply the rate at which vehicles pass point \( x \) at time \( t \). The total number \( Q \) crossing point \( x \) between time \( t \) and time \( t + \Delta t \) is then given by

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The number of ways to get from the bottom of a hill to the top of a hill is given by the binomial coefficient, which is the number of ways to choose $k$ items from a set of $n$ items. The binomial theorem states that

$$ (a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k $$

where $\binom{n}{k}$ is the binomial coefficient. This theorem is used to expand binomial expressions. For example, if $n = 3$, $a = 2$, and $b = 3$, then

$$ (2 + 3)^3 = \binom{3}{0} 2^3 3^0 + \binom{3}{1} 2^2 3^1 + \binom{3}{2} 2^1 3^2 + \binom{3}{3} 2^0 3^3 $$

The binomial coefficients can be calculated using the formula

$$ \binom{n}{k} = \frac{n!}{k!(n-k)!} $$

where $n!$ denotes the factorial of $n$. The binomial theorem is useful in various fields, including probability, combinatorics, and algebra. It provides a way to express the sum of a power series in terms of its coefficients. The binomial theorem is also used in the study of Taylor series and in the solution of differential equations.
2. Propagation of a Discontinuity

(1)

\[
\frac{\partial}{\partial t} \left( \frac{\rho}{\beta} \right) - \frac{\partial x}{\partial t} = 0
\]

Thus, \( \gamma = 0 \) is the same as \( \gamma + i \frac{\partial \rho}{\partial \beta} = 0 \) if we know the value of \( \gamma \) at each point on the initial curve. Since \( \gamma \) is constant on each of the characteristic curves, we can write:

\[
0 + i \frac{\partial \rho}{\partial \beta} = 0
\]

Let us consider the evolution of the trailing discontinuity on a long road.

(2)

Immediately after (1) and (11), hence we have:

\[
0 = \frac{\partial x}{\partial \beta} + \frac{\partial \rho}{\partial \beta}
\]

where once again we have used the chain rule. Along a characteristic \( \gamma(\xi, t) \) where:

\[
\frac{\partial x}{\partial \beta} = \frac{\partial x}{\partial \beta} + \frac{\partial \rho}{\partial \beta}
\]

Such a curve is called a characteristic of \( \gamma \) and satisfies the initial condition:

\[
0 = \frac{\partial x}{\partial \beta} + \frac{\partial \rho}{\partial \beta}
\]

The function \( \gamma \) also satisfies the differential equation obtained by differentiating (8) and (9) with respect to time. This is thus a function of \( \gamma \) and \( \beta \) and is a function of \( \beta \).

Let us consider the evolution of the trailing discontinuity on a long road.
Two connections on either side of the stock spread on the other hand can be positive or negative, depending on the speed of the stock. The speed of the stock is determined by its impact on the market. The stock spreads are also affected by the speed of their own spread, the speed of the market, and the speed of the stock. The stock spreads are shown in the following equations:

\[ S = \frac{(S_P - S)^2}{(S_P + S)^2} \]

We should also note that the area of the stock spread is given by:

\[ A = \int_{S_P}^{S} S^2 \, dS \]

Next, we can see how the stock spreads correspond to the various sectors of the stock. We can also see how the stock spreads are affected by the speed of the market. The stock spreads are shown in the following equations:

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To compute the area of the stock spread, we must divide by the area of the stock spread, which is given by:

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Exercises

1. Given the following data:

   \[ y \sim \{0.5, 1, 7, 12, 1, 16, 2, 14, 11, 18, 21, 32, 11, 10, 18, 21, 12, 16, 7, 32\} \text{ (in m/s)} \]

   \[ (x, y) \text{ (in m)} \]

   Calculate the following:

   a. Least squares to fit a linear model:

   b. Given the flow data:

   i. Use least squares to fit a model:

   The formation of shocks is similar to a shock wave and how it will propagate.

   The characteristic for small is shown in Figure 1.4 (d). Use your intuition about

   2. Consider the propagation of the initial condition shown in Figure 1.4 (e), where \( \theta = \theta^{*} \).
The module introduces the fundamental balance idea necessary for the Instructor.

In this document, the present state of knowledge in this field is reviewed. New topics and results are presented, which enhance the understanding of the subject—especially in the following sections:


References: