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IEEE TRANSACTIONS ON

MEDICAL IMAGING

A PUBLICATION OF
THE IEEE ENGINEERING IN MEDICINE AND BIOLOGY SOCIETY
THE IEEE NUCLEAR AND PLASMA SCIENCES SOCIETY
THE IEEE SIGNAL PROCESSING SOCIETY
THE IEEE ULTRASONICS, FERROELECTRICS, AND FREQUENCY CONTROL SOCIETY

APRIL 2000 VOLUME 19 NUMBER 4 ITMID4 (ISSN 0278–0062)

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Editorial
Recent Developments in Iterative Image Reconstruction for PET and SPECT

THREE articles that begin this issue of TMI describe distinct regularized approaches to iterative image reconstruction from emission tomography data [24], [27], [39]. Their publication in this issue provides us with the opportunity to explain the background to this work and speculate on the future of such methods.

Model-based iterative approaches to image reconstruction in PET and SPECT allow optimal noise handling [37] and accurate system response modeling [38], [34]. Research in model-based image reconstruction methods addresses two key issues: how to select a cost function that produces images with the desired properties and how to find these images quickly. In the first category we include work addressing statistical and physical models for the data, selection of image smoothing terms or priors that regularize the solution and the choice of cost function to be optimized over the image space [30]. The second area addresses the issue of rapidly finding a solution once a cost function has been selected. In principle, the solutions to the concave maximization problems typically encountered in image reconstruction are independent of the numerical algorithm selected to find them. In practice however, fast algorithms are often terminated before convergence so that the solution becomes a function of the algorithm. Nevertheless, it is useful to maintain the distinction between classes of algorithms that compute, ostensibly, the same solution and those that optimize different cost criteria and, hence, result in different solutions. Here we are primarily concerned with the choice of iterative algorithm rather than issues relating to cost function selection.

The early iterative algorithms for image reconstruction, which form the broad class of algebraic reconstruction techniques (ART's), solve sets of simultaneous, possibly under-determined, linear equations [4], [17], [21]. While the ART methods have much in common with more recently developed statistically-based iterative methods, they do not themselves directly model noise in the data. Shepp and Vardi's maximum likelihood (ML) algorithm, based on the EM (expectation maximization) methods of Dempster, Laird, and Rubin, was among the first to explicitly model the Poisson distribution of noise in photon limited imaging systems such as PET and SPECT [37]. The EM formalism for this problem gives rise to an elegant update equation reminiscent of the earlier multiplicative ART algorithms.

The improvements in image quality the EMML produced inspired a tremendous amount of subsequent research. Much of this work has addressed the problem of speeding up EMML's slow convergence, e.g., [25]. A more fundamental problem with the EMML algorithm is that it exhibits instabilities, manifested in high variance in the estimated voxel intensities. These can be controlled in practice by early termination of the algorithm or by smoothing of the images after reconstruction. A more formal approach to controlling instabilities is to regularize the solution using maximum *a posteriori* (MAP) Bayesian (or equivalently penalized-ML) methods in which some measure of smoothness is appended to the log-likelihood to penalize excessively noisy images. The MAP solution can be computed using generalizations of the EMML algorithm [10], [18], [20].

While the EMML algorithm and its MAP generalizations can be used to compute, respectively, ML and MAP solutions, convergence of these methods is very slow. The first three papers in this issue all describe MAP algorithms that exhibit much faster convergence. The key differences among them lie in the path taken to the solution. We will attempt to place these methods in the broader context of fast iterative methods for iterative image reconstruction.

The classical approach to the numerical solution of nonlinear optimization problems is to update all pixels simultaneously using some function of the gradient [31]. The EMML algorithm itself can be viewed as a preconditioned form of steepest ascent [25]. While both steepest ascent and EMML are rather slow to converge, modifications using conjugate gradient or quasi-Newton methods, in combination with preconditioners, can produce rapidly convergent algorithms. A complication in these gradient-based methods is that of ensuring that the solution is nonnegative. This can be achieved using active set methods and bent line searches [26], [33]. An alternative is to convert the constrained problem into an unconstrained one using penalty or barrier function methods [31]. The paper in the current issue by Johnson *et al.* [24] describes a pair of gradient-based algorithms for MAP estimation, one of which uses a barrier function, the other an alternative interior point methodology. Fast convergence is achieved using, respectively, a truncated Newton method and a preconditioned conjugate gradient method.

An alternative approach to speeding up convergence is to use algorithms that involve only a subset of either the data or the solution space at each iteration. We consider first the class of block-iterative (or ordered-subset) methods that are based on partitioning of the data into disjoint subsets, or blocks. At each iteration only the data in one of the blocks are used. The use of blocks arises naturally in tomography, where a single family of parallel or fan rays contributes an obvious block of data. Algorithms that Censor called row-action methods in [4], such as ART and multiplicative ART [17], employ blocks that contain only a single data value. These methods can be faster to converge than simultaneous methods that use all the data at each step,

Publisher Item Identifier S 0278-0062(00)05725-6.

but if the time required to access the current estimated image is significant, forward and backward projection for a single ray can be wasteful. A compromise, involving the use of blocks and the calculation of the forward and backward projection for several rays at a time, can be a reasonable course of action [12], [7]. It should be noted that, apart from the efficiencies just mentioned, block-iterative methods are not automatically faster than their simultaneous counterparts. However, the use of blocks provides an opportunity to access the data in an advantageous ordering and to over relax in a way that is impossible if all the data were used at each step, both leading to accelerated convergence. Block-iterative versions of ART and MART were discussed by several authors, e.g., [8]. A good source for details concerning these and other related algorithms is the book by Censor and Zenios [9].

The slow convergence of EMML and its resemblance to the multiplicative ART algorithm leads naturally to the following questions. Does EMML have block-iterative versions? If so, are they faster than EMML? Hudson, Hutton and Larkin provided the first answer to these questions with their ordered-subset EM (OSEM) algorithm [22], [23]. The OSEM algorithm does not apply to the case of singleton blocks, nor does it converge for arbitrarily chosen blocks (and noise-free data) but, in many cases of practical interest, OSEM has been shown to perform as well as EMML, with an order-of-magnitude fewer iterations. Manglos *et al.* noted, however, that noisy reconstructions can result if too many subsets are used in OSEM [32]. With OSEM, as with ART, the careful selection of subset ordering can greatly accelerate convergence (see [21] and [19]). However, as with the earlier block-iterative methods, limit cycles appear in the noisy case. The lack of convergence of OSEM algorithms is to some degree of theoretical importance since, in practice, the algorithm is terminated after only a few iterations.

To remove limit cycles from OSEM, Browne and De Pierro introduced their row-action maximum likelihood algorithm (RAMLA), which is a modification of OSEM that uses strong under relaxation [2]. The rescaled block-iterative version of EMML, called RBI-EMML [3], differs from RAMLA only in the nature of the rescaling involved. In RBI-EMML the rescaling is larger, for acceleration, while in RAMLA it is going to zero, to remove the limit cycles. The RBI-EMML is OSEM for certain special cases but converges, in the noise-free case, for any choice of blocks. When there is only one data value per block the RBI-EMML becomes a row-action version of EMML. The use of variable-sized blocks has also been suggested by Censor [5]. Guan and Gordon [19] consider block-ART with the number of blocks, decreasing as the iteration proceeds, as a way to avoid the limit cycle.

As noted above, unregularized reconstruction methods, such as ART, EMML and OSEM, can have high variance when applied to noisy data. Just as the EM algorithm can be extended to solve MAP estimation problems, so the block-iterative methods can be extended to the MAP problem. The regularization of block-iterative methods is the theme of [27]. Lalush *et al.* extend the RBI-EMML approach to compute a MAP solution using their RBI-MAP algorithm. Also of interest in this paper is the integration of an automated procedure for selecting the smoothing parameter. De Pierro and Yamagishi [11] describe a MAP extension of the RAMLA approach and investigate its convergence properties. This work may appear in a future issue of IEEE TRANSACTIONS ON MEDICAL IMAGING.

A second type of block-iterative method, which groups the pixels rather than data, has been used by a number of authors ([36], [13]). When blocks consist of a single pixel this method reduces to coordinate ascent [36], [14] which has been shown to exhibit very rapid convergence. As with the other block-iterative approaches, the ordering can have a major impact on convergence rates. As well as exhibiting rapid convergence, handling the nonnegativity constraint in coordinate ascent is trivial. The primary potential drawback of coordinate ascent is that pixelwise updates may be computationally inefficient; this can be remedied using block-based methods.

The space-alternating generalized EM (SAGE) algorithm, introduced by Fessler and Hero [16], is a general formalism for constructing pixel-based block-iterative methods for ML and MAP estimation. As with the EM algorithm, the methods use unobserved data spaces to effectively modify the function to be optimized at each iteration so as to simplify the update equations. The hidden data spaces are chosen to ensure that the sequence generated is monotonically increasing in the original cost function. Convergence of SAGE algorithms under certain restrictions is shown in [15].

The EM and SAGE algorithms are, in turn, special cases of a very general class of algorithms, which have been termed functional substitution methods in which, at each iteration, a modified or surrogate cost function is introduced that is easier to maximize than the original function and whose maximization guarantees monotonic increase in the original function. Other examples of functional substitution methods include De Pierro's MAP-EM algorithm [10], the coordinate ascent method of Saquib *et al.* [35], and the grouped coordinate ascent method of Fessler *et al.* [13].

Preconditioned gradient-based optimization and block-iterative methods can produce high-quality reconstructions in only a few iterations. Improvements in the choice of preconditioners or block iterative methods that involve both sets of indexes may lead to further improvements in convergence rates. Other advances in iterative methods will undoubtedly include improvements in the modeling of the systems, development of priors optimized for clinical and research applications (including integration of anatomical information) and the further development of analytical tools for investigating the resolution and noise properties of these nonlinear algorithms.

We close by noting that, recently, the OSEM approach has been adopted by the manufacturers of a number of nuclear medicine imaging systems. Now that the door has been opened to the use of iterative approaches in a clinical setting, it will be interesting to see how the relative merits of OSEM and other block-iterative approaches, as well as coordinatewise and gradient-based schemes, play out in terms of both computational cost and clinically relevant measures of image quality.

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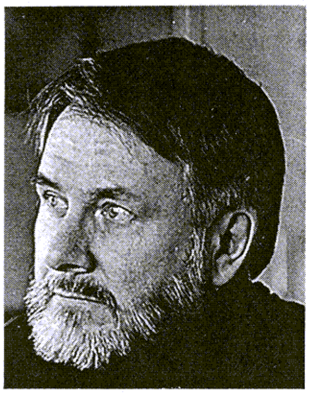

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