PHYSICS

FOR SCIENTISTS AND ENGINEERS **A STRATEGIC APPROACH** 4/E

Chapter 8 Lecture

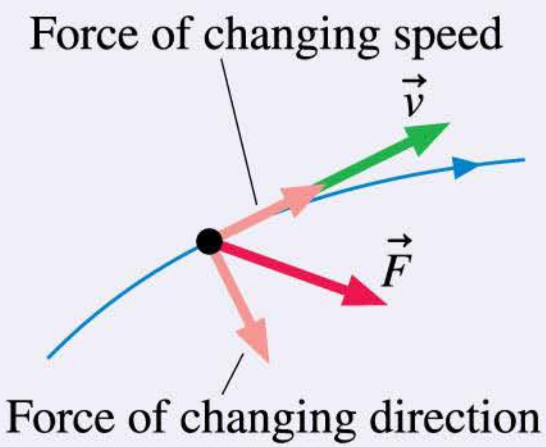
RANDALL D. KNIGHT

Chapter 8. Dynamics II: Motion in a Plane

IN THIS CHAPTER, you will learn to solve problems about motion in two dimensions.

Are Newton's laws different in two dimensions?

No. Newton's laws are vector equations, and they work equally well in two and three dimensions. For motion in a plane, we'll focus on how a force *tangent* to a particle's trajectory changes its speed, while a force *perpendicular* to the trajectory changes the particle's direction.

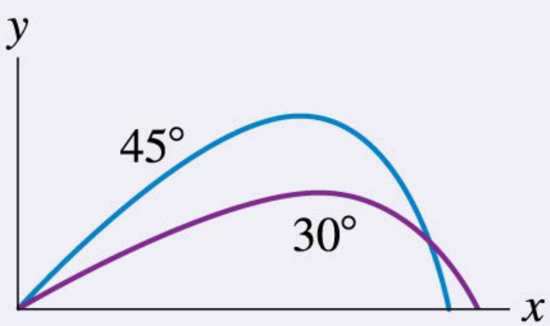


« LOOKING BACK Chapter 4 Kinematics of projectile and circular motion

How do we analyze projectile-like motion?

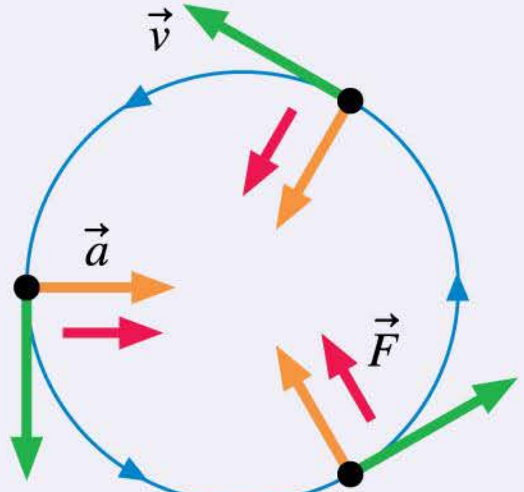
For linear motion, one component of the acceleration was always zero. Motion in a plane generally has acceleration along two axes. If the accelerations are independent, we can use *x*- and *y*-coordinates and we will find motions analogous to the projectile motion we studied in Chapter 4.

Projectile motion with drag



How do we analyze circular motion?

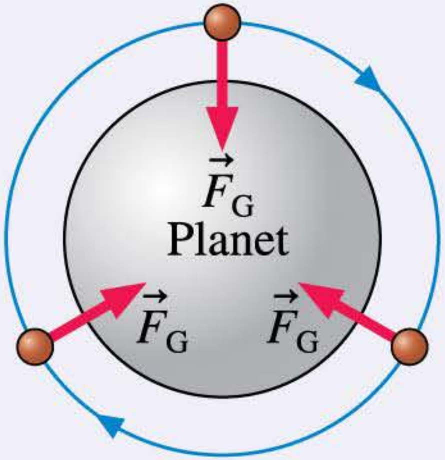
Circular motion must have a force component toward the center of the circle to create the centripetal acceleration. In this case the acceleration components are radial and, perhaps, tangential. We'll use a different coordinate system, *rtz* coordinates, to study the dynamics of circular motion



Does this analysis apply to orbits?

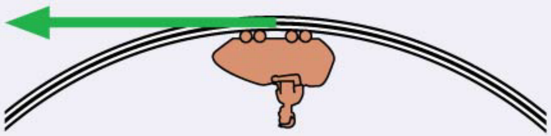
Yes, it does. The circular orbit of a satellite or planet is motion in which the force of gravity is creating the inward centripetal acceleration. You'll see that an orbiting projectile is in free fall.

« LOOKING BACK Section 6.3 Gravity and weight



Why doesn't the water fall out of the bucket?

How can you swing a bucket of water over your head without the water falling out? Why doesn't a car going around a loop-the-loop fall off at the top? Circular motion is not always intuitive, but you'll strengthen your ability to use Newtonian reasoning by thinking about some of these problems.



Chapter 8 Preview

Why is planar motion important?

By starting with linear motion, we were able to develop the ideas and tools of Newtonian mechanics with minimal distractions. But planes and rockets move in a plane. Satellites and electrons orbit in a plane. The points on a rotating hard drive move in a plane. In fact, much of this chapter is a prelude to Chapter 12, where we will study rotational motion. This chapter gives you the tools you need to analyze more complex—and more realistic—forms of motion.

Chapter 8 Reading Questions

Reading Question 8.1

When drag is included, the launch angle of a projectile which maximizes the range is

A. Greater than 45^{o}.
B. Equal to 45^{o}.
C. Less than 45^{o}.

Reading Question 8.1

When drag is included, the launch angle of a projectile which maximizes the range is

A. Greater than 45º.
B. Equal to 45º.
C. **Less than 45º.**

Reading Question 8.2

Circular motion is best analyzed in a coordinate system with

A. x- and y-axes.

B. x-, y-, and z-axes.

C. x- and z-axes.

D. r-, t-, and z-axes.

Reading Question 8.2

Circular motion is best analyzed in a coordinate system with

A. *x*- and *y*-axes.
B. *x*-, *y*-, and *z*-axes.
C. *x*- and *z*-axes.
D. ***r*-, *t*-, and *z*-axes.**

Reading Question 8.3

This chapter studies

A. Uniform circular motion.
B. Nonuniform circular motion.
C. Orbital motion.
D. Both a and b.
E. All of a, b, and c.

Reading Question 8.3

This chapter studies

A. Uniform circular motion.
B. Nonuniform circular motion.
C. Orbital motion.
D. Both a and b.
E. All of a, b, and c.

Reading Question 8.4

For uniform circular motion, the net force

A. Points toward the center of the circle.
B. Points toward the outside of the circle.
C. Is tangent to the circle.
D. Is zero.

Reading Question 8.4

For uniform circular motion, the net force

A. **Points toward the center of the circle.**
B. Points toward the outside of the circle.
C. Is tangent to the circle.
D. Is zero.

Reading Question 8.5

The centrifugal force

A. Is a fictitious force.
B. Points toward the center of the circle.
C. Is provided by static friction.
D. All of the above.
E. B and C, but not A.

Reading Question 8.5

The centrifugal force

A. **Is a fictitious force.**
B. Points toward the center of the circle.
C. Is provided by static friction.
D. All of the above.
E. B and C, but not A.

Chapter 8 Content, Examples, and QuickCheck Questions

Dynamics in Two Dimensions

- Newton's second law determines an object's acceleration; it makes no distinction between linear motion and two-dimensional motion in a plane.
- We began with motion along a line, in order to focus on the essential physics, but now we turn our attention to the motion of projectiles, satellites, and other objects that move in two dimensions.
- We'll continue to follow Problem-Solving Strategy 6.1, which is well worth a review, but we'll find that we need to think carefully about the appropriate coordinate system for each problem.

Dynamics in Two Dimensions

PROBLEM-SOLVING STRATEGY 6.1

Newtonian mechanics

MODEL Model the object as a particle. Make other simplifications depending on what kinds of forces are acting.

VISUALIZE Draw a **pictorial representation**.

- Show important points in the motion with a sketch, establish a coordinate system, define symbols, and identify what the problem is trying to find.
- Use a motion diagram to determine the object's acceleration vector $\vec{a}$. The acceleration is zero for an object in equilibrium.
- Identify all forces acting on the object *at this instant* and show them on a free-body diagram.
- It's OK to go back and forth between these steps as you visualize the situation.

Dynamics in Two Dimensions

PROBLEM-SOLVING STRATEGY 6.1

SOLVE The mathematical representation is based on Newton's second law:

$$\vec{F}_{\text{net}} = \sum_i \vec{F}_i = m\vec{a}$$

The forces are "read" directly from the free-body diagram. Depending on the problem, either

- Solve for the acceleration, then use kinematics to find velocities and positions; or
- Use kinematics to determine the acceleration, then solve for unknown forces.

ASSESS Check that your result has correct units and significant figures, is reasonable, and answers the question.

Exercise 23

Example 8.1 Rocketing in the Wind

EXAMPLE 8.1 | **Rocketing in the wind**

A small rocket for gathering weather data has a mass of 30 kg and generates 1500 N of thrust. On a windy day, the wind exerts a 20 N horizontal force on the rocket. If the rocket is launched straight up, what is the shape of its trajectory, and by how much has it been deflected sideways when it reaches a height of 1.0 km? Because the rocket goes much higher than this, assume there's no significant mass loss during the first 1.0 km of flight.

Example 8.1 Rocketing in the Wind

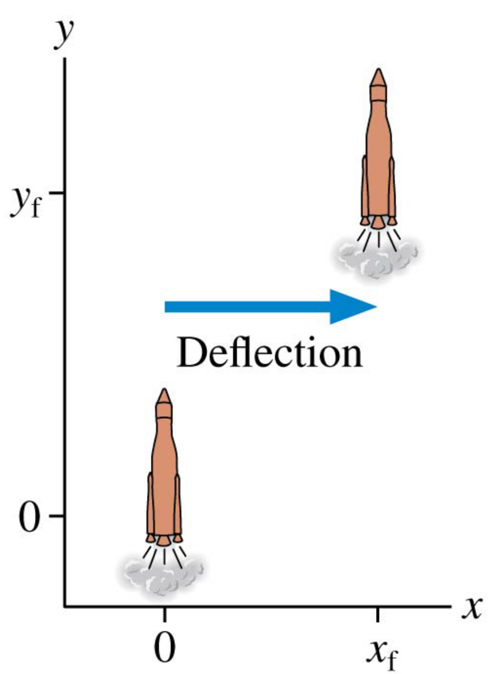
EXAMPLE 8.1 Rocketing in the wind

MODEL Model the rocket as a particle. We need to find the *function* $y(x)$ describing the curve the rocket follows. Because rockets have aerodynamic shapes, we'll assume no vertical air resistance.

Example 8.1 Rocketing in the Wind

EXAMPLE 8.1 | **Rocketing in the wind**

VISUALIZE FIGURE 8.1 shows a pictorial representation. We've chosen a coordinate system with a vertical y-axis. Three forces act on the rocket: two vertical and one horizontal.



Known

$x_i = y_i = 0 \text{ m}$

$v_{ix} = v_{iy} = 0 \text{ m/s}$

$y_f = 1000 \text{ m}$

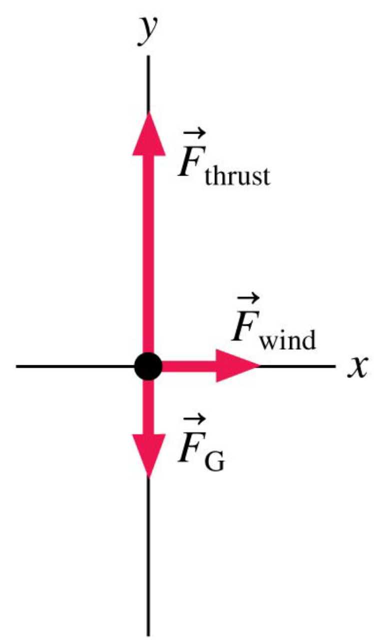
$m = 30 \text{ kg}$

$F_{thrust} = 1500 \text{ N}$

$F_{wind} = 20 \text{ N}$

Find

x_f



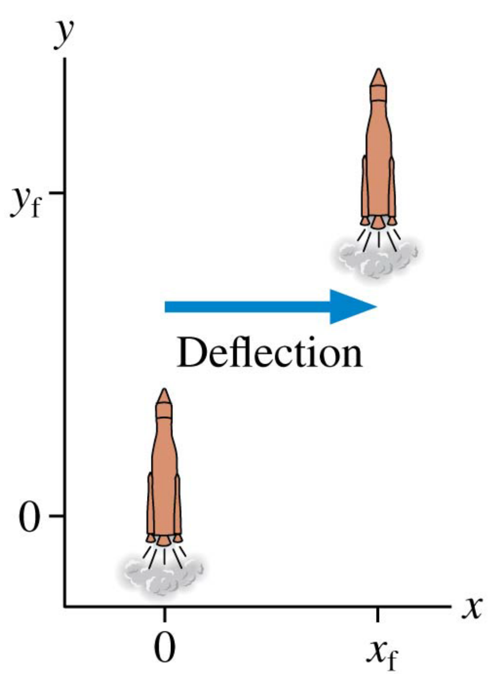
Example 8.1 Rocketing in the Wind

EXAMPLE 8.1 | **Rocketing in the wind**

SOLVE In this problem, the vertical and horizontal forces are independent of each other. Newton's second law is

$$a_x = \frac{(F_{\text{net}})_x}{m} = \frac{F_{\text{wind}}}{m}$$

$$a_y = \frac{(F_{\text{net}})_y}{m} = \frac{F_{\text{thrust}} - mg}{m}$$



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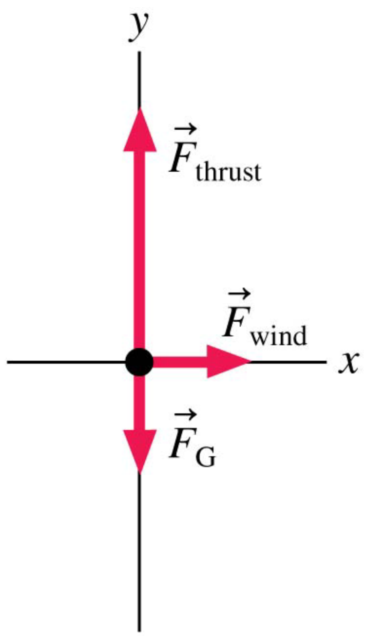


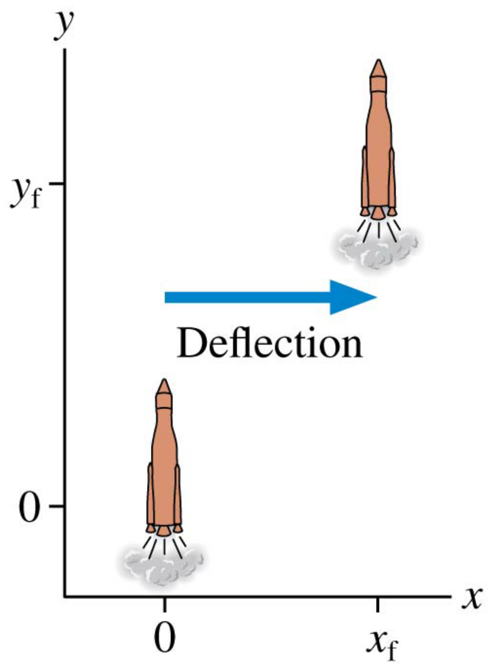
Example 8.1 Rocketing in the Wind

EXAMPLE 8.1 | **Rocketing in the wind**

SOLVE The primary difference from the linear-motion problems you've been solving is that the rocket accelerates along both axes. However, both accelerations are constant, so we can use kinematics to find

$$x = \tfrac{1}{2}a_x(\Delta t)^2 = \frac{F_{\text{wind}}}{2m}(\Delta t)^2$$
$$y = \tfrac{1}{2}a_y(\Delta t)^2 = \frac{F_{\text{thrust}} - mg}{2m}(\Delta t)^2$$

where we used the fact that all initial positions and velocities are zero. From the x-equation, $(\Delta t)^2 = 2mx/F_{\text{wind}}$.



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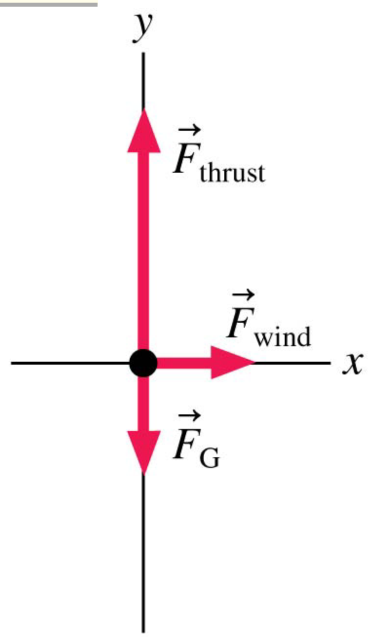
Find

x_f



Example 8.1 Rocketing in the Wind

EXAMPLE 8.1 Rocketing in the wind

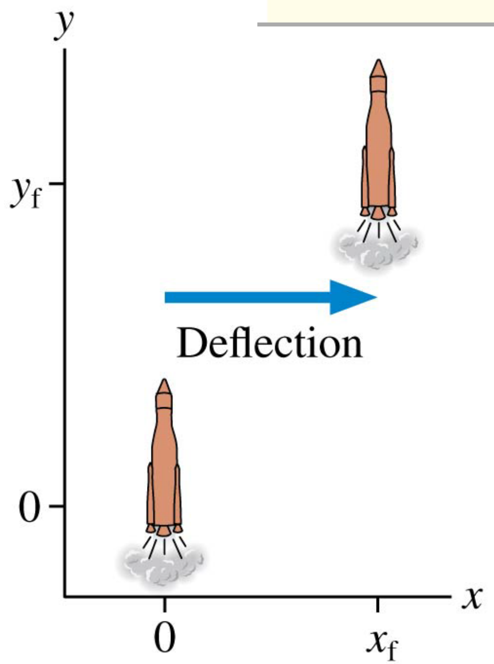
SOLVE Substituting this into the y-equation, we find

$$y(x) = \left(\frac{F_{\text{thrust}} - mg}{F_{\text{wind}}}\right)x$$

This is the equation of the rocket's trajectory. It is a linear equation. Somewhat surprisingly, given that the rocket has both vertical and horizontal accelerations, its trajectory is a *straight line*. We can rearrange this result to find the deflection at height y:

$$x = \left(\frac{F_{\text{wind}}}{F_{\text{thrust}} - mg}\right)y$$

From the data provided, we can calculate a deflection of 17 m at a height of 1000 m.



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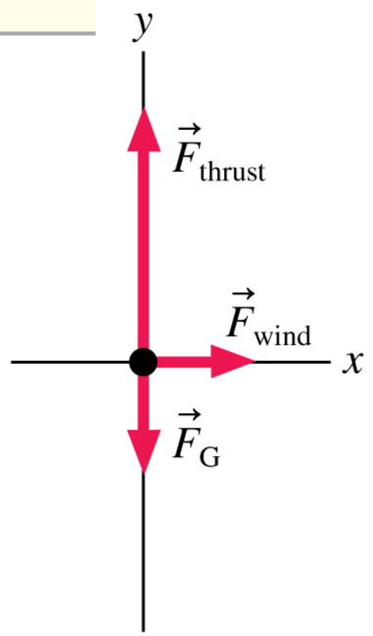
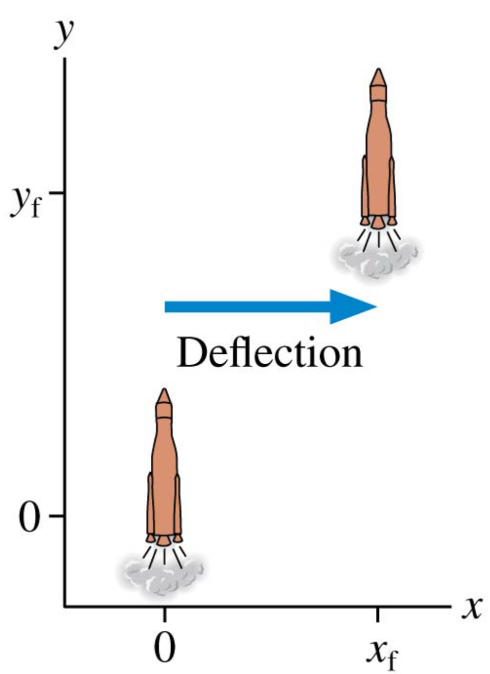


Example 8.1 Rocketing in the Wind

EXAMPLE 8.1 | **Rocketing in the wind**

ASSESS The solution depended on the fact that the time parameter Δt is the *same* for both components of the motion.



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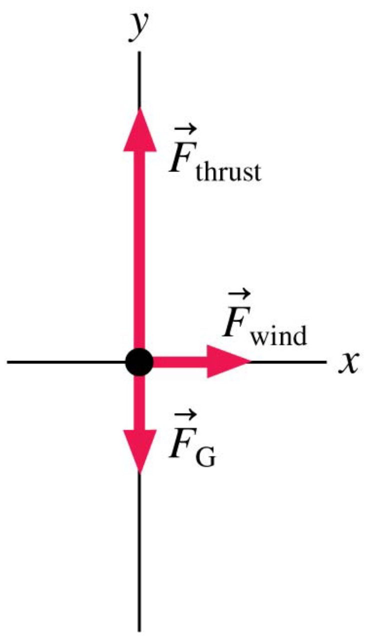
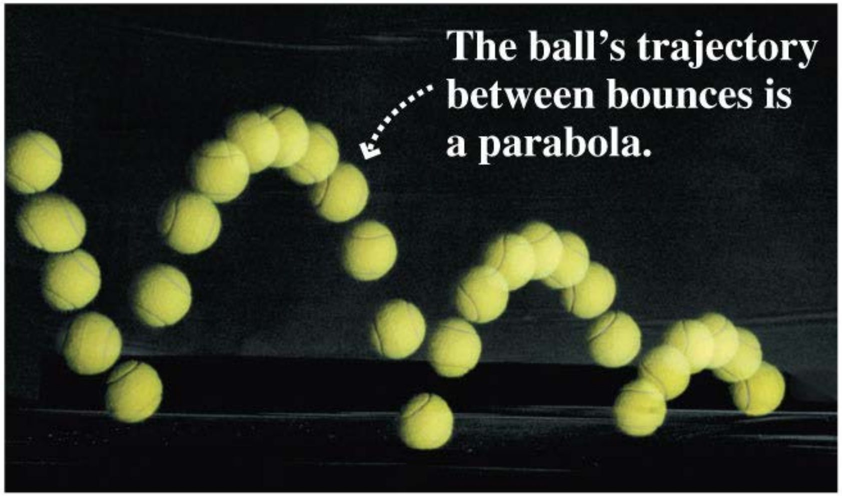


Projectile Motion: Review

- In the absence of air resistance, a projectile moves under the influence of only gravity.
- If we choose a coordinate system with a vertical *y*-axis, then

$$\vec{F}_G = -mg\,\hat{j}$$



- Consequently, from Newton's second law, the acceleration is

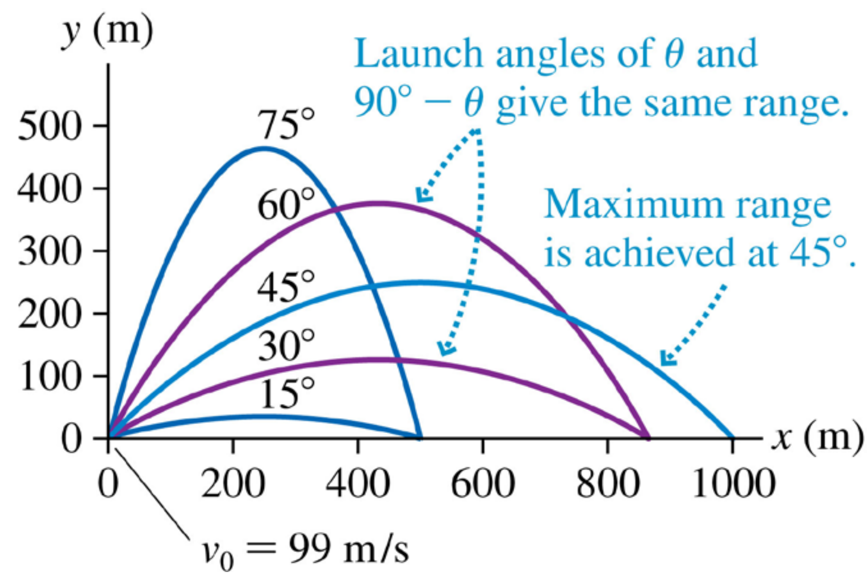
$$a_x = \frac{(F_G)_x}{m} = 0$$

$$a_y = \frac{(F_G)_y}{m} = -g$$

Projectile Motion: Review

- Consider a projectile with initial speed v_0, and a launch angle of θ above the horizontal.
- In Chapter 4 we found that the distance it travels before it returns to the same elevation from which it was launched (the *range*) is

Trajectories of a projectile launched at different angles with a speed of 99 m/s.



$$\text{range} = \frac{v_0^2 \sin(2\theta)}{g}$$

- The maximum range occurs for $\theta = 45°$.
- All of these results *neglect* the effect of air resistance.

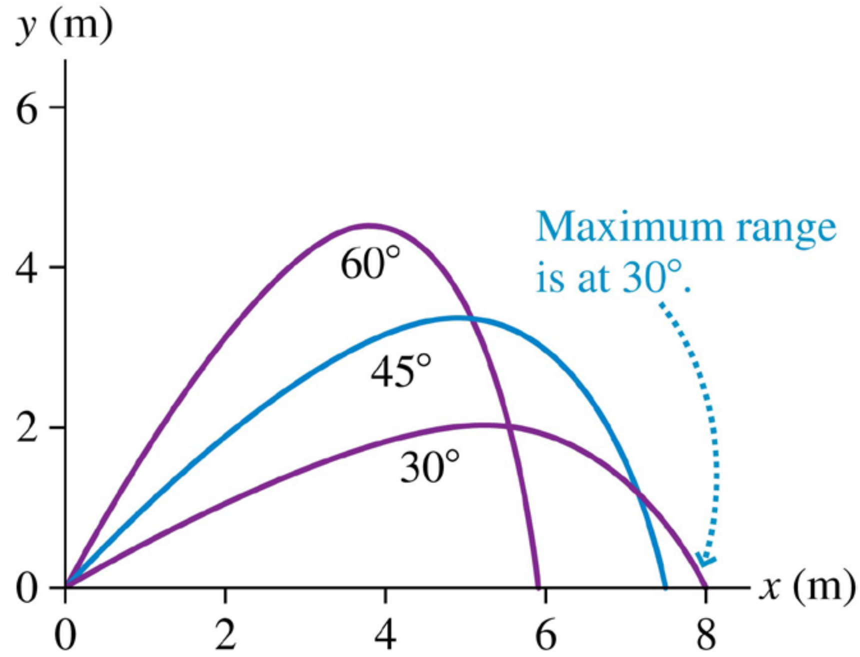
Projectile Motion

- The acceleration of a typical projectile subject to drag force from the air is:

$$a_x = -\frac{\rho C A}{2m} v_x \sqrt{{v_x}^2 + {v_y}^2}$$

$$a_y = -g - \frac{\rho C A}{2m} v_y \sqrt{{v_x}^2 + {v_y}^2}$$

- The components of acceleration are *not* independent of each other.
- These equations can only be solved numerically.
- The figure shows the numerical solution for a 5-g plastic ball.



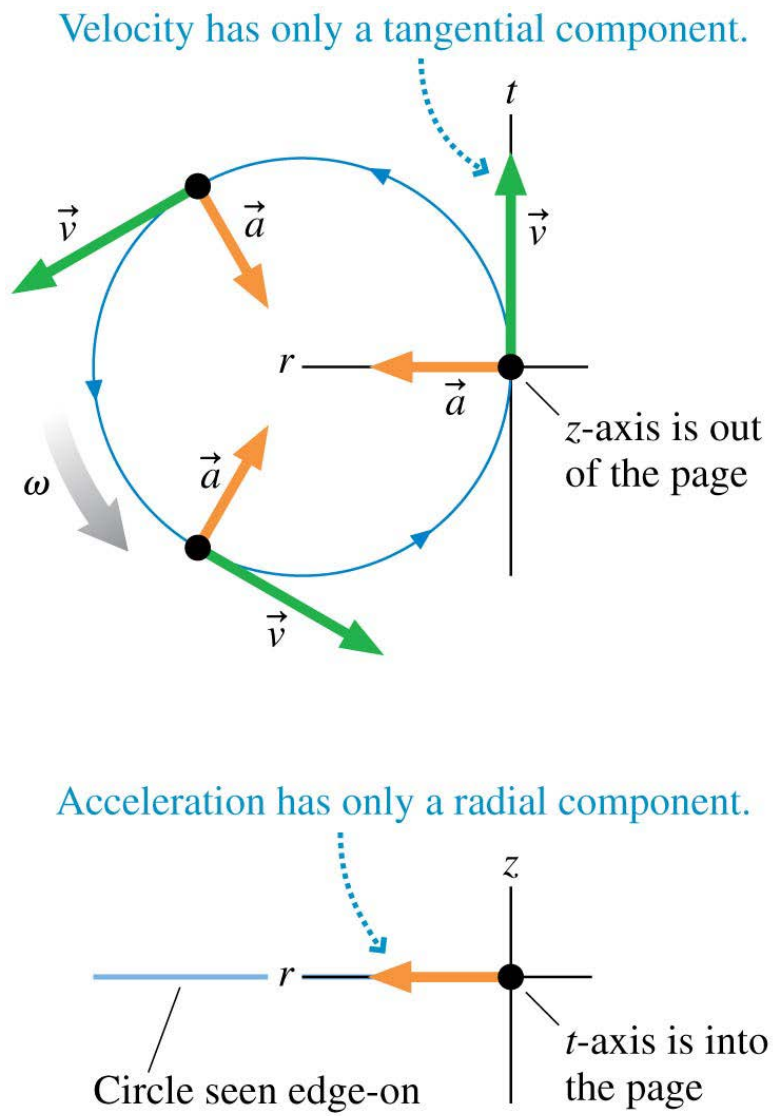
Uniform Circular Motion

- The kinematics of uniform circular motion were introduced in Sections 4.4–4.5, and a review is highly recommended.
- Now we’re ready to study dynamics—how forces cause circular motion.
- The particle’s velocity is tangent to the circle, and its acceleration—a centripetal acceleration—points toward the center.
- If the particle has angular velocity v and speed $v = \omega r$, its centripetal acceleration is

$$\vec{a} = \left(\frac{v^2}{r}, \text{ toward center of circle}\right) = (\omega^2 r, \text{ toward center of circle})$$

Uniform Circular Motion

- When describing circular motion, it is convenient to define a moving *rtz*-coordinate system.
- The origin moves along with a certain particle moving in a circular path.
- The *r*-axis (radial) points *from* the particle *toward* the center of the circle.
- The *t*-axis (tangential) is tangent to the circle, pointing in the ccw direction.
- The *z*-axis is perpendicular to the plane of motion.



Uniform Circular Motion

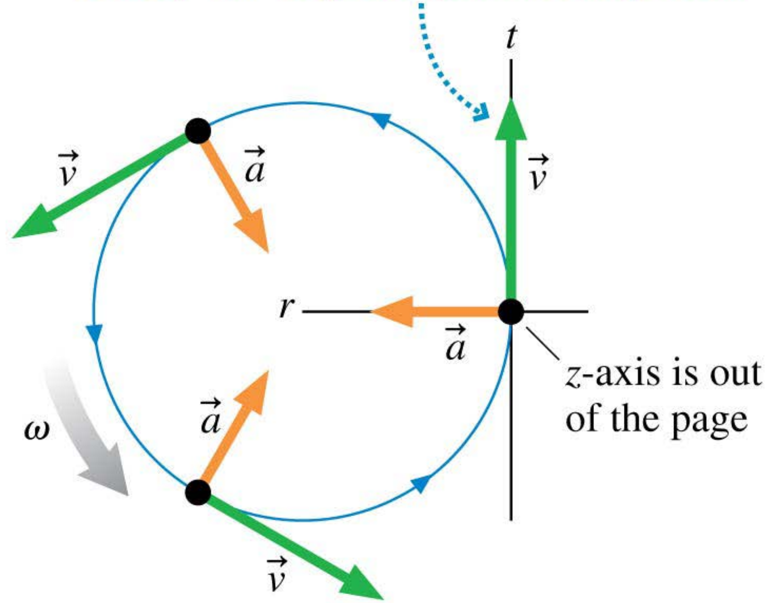
- A particle in uniform circular motion with angular velocity ω has velocity $v = \omega r$, in the tangential direction.
- The acceleration of uniform circular motion points to the center of the circle.
- The rtz-components of $\vec{v}$ and $\vec{a}$ are

$$v_r = 0 \qquad a_r = \frac{v^2}{r} = \omega^2 r$$

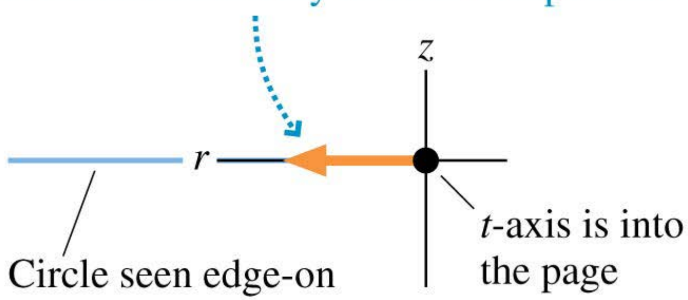
$$v_t = \omega r \qquad a_t = 0$$

$$v_z = 0 \qquad a_z = 0$$

Velocity has only a tangential component.

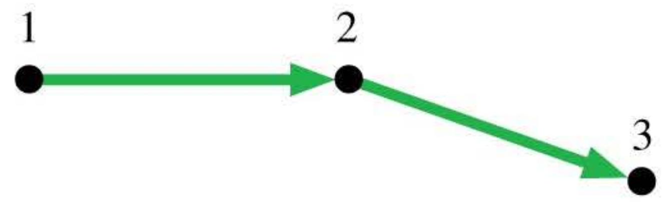


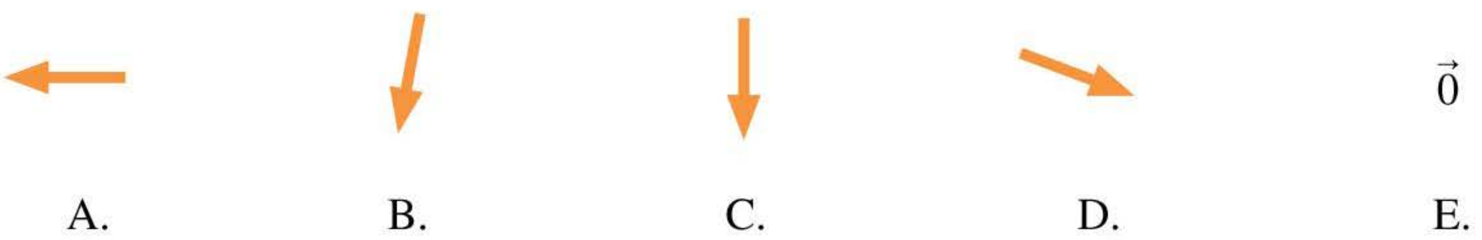
Acceleration has only a radial component.



QuickCheck 8.1

The diagram shows three points of a motion diagram. The particle changes direction with no change of speed. What is the acceleration at point 2?





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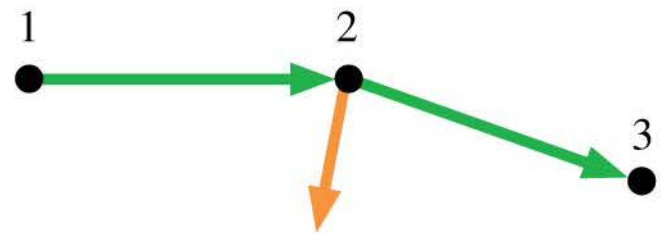

Acceleration of changing direction

A. B. C. D. E. $\vec{0}$

QuickCheck 8.2

A toy car moves around a circular track at constant speed. It suddenly doubles its speed—a change of a factor of 2. As a result, the centripetal acceleration changes by a factor of

A. 1/4.
B. 1/2.
C. No change since the radius doesn't change.
D. 2.
E. 4.

QuickCheck 8.2

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A. 1/4.

B. 1/2.

C. No change since the radius doesn't change.

D. 2.

E. 4.

Dynamics of Uniform Circular Motion

- An object in uniform circular motion is *not* traveling at a constant velocity in a straight line.
- Consequently, the particle must have a net force acting on it

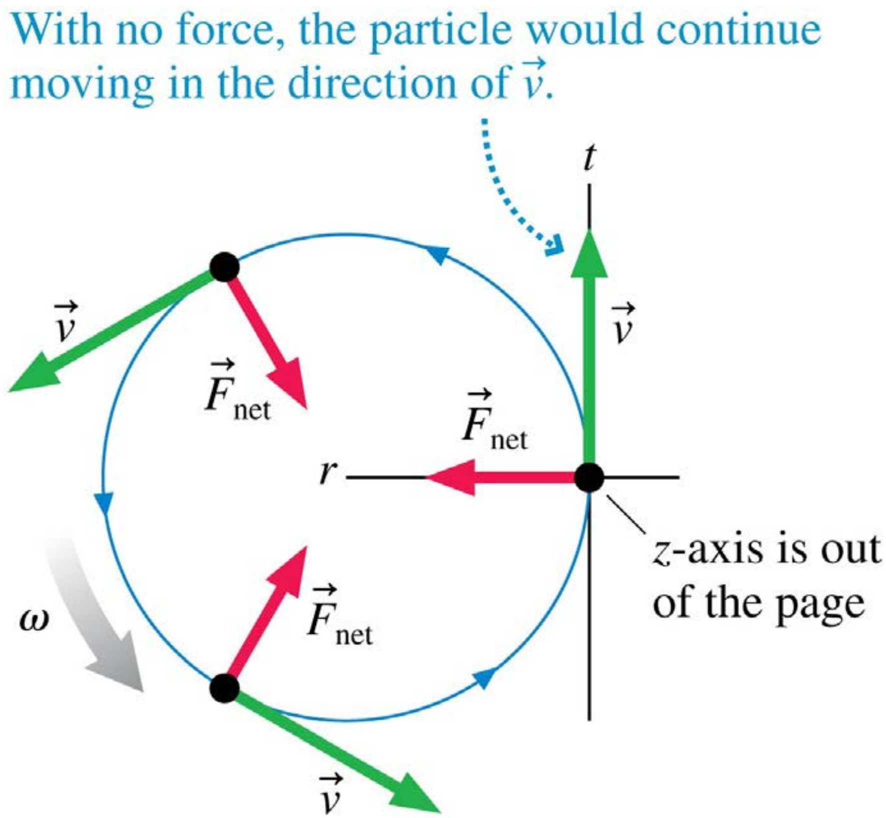
$$\vec{F}_{\text{net}} = m\vec{a} = \left(\frac{mv^2}{r}, \text{ toward center of circle}\right)$$

- Without such a force, the object would move off in a straight line tangent to the circle.
- The car would end up in the ditch!

On banked curves, the normal force of the road assists in providing the centripetal acceleration of the turn.

Dynamics of Uniform Circular Motion

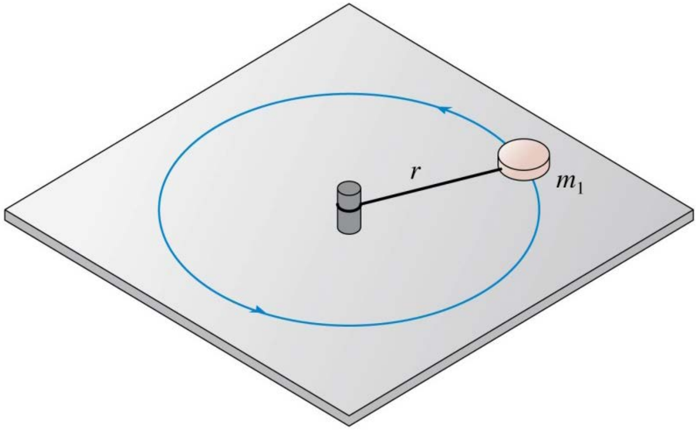
- The figure shows a particle in uniform circular motion.
- The net force must point in the radial direction, toward the center of the circle.
- This centripetal force is not a new force; it must be *provided* by familiar forces.



$$(F_{net})_r = \sum F_r = ma_r = \frac{mv^2}{r} = m\omega^2 r$$
$$(F_{net})_t = \sum F_t = ma_t = 0$$
$$(F_{net})_z = \sum F_z = ma_z = 0$$

QuickCheck 8.3

An ice hockey puck is tied by a string to a stake in the ice. The puck is then swung in a circle. What force or forces does the puck feel?



A. A new force: the centripetal force.
B. A new force: the centrifugal force.
C. One or more of our familiar forces pushing outward.
D. One or more of our familiar forces pulling inward.
E. I have no clue.

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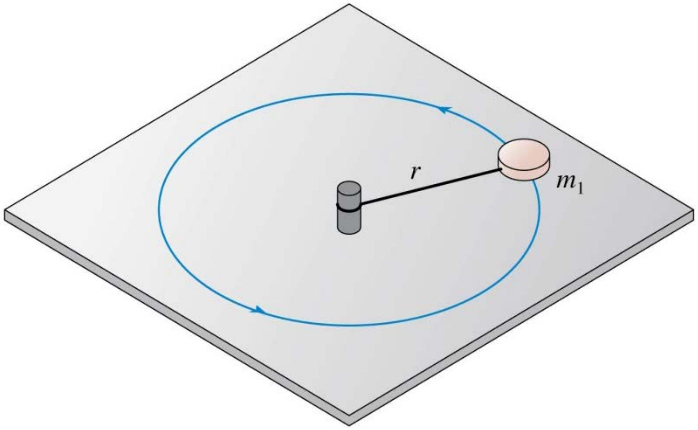

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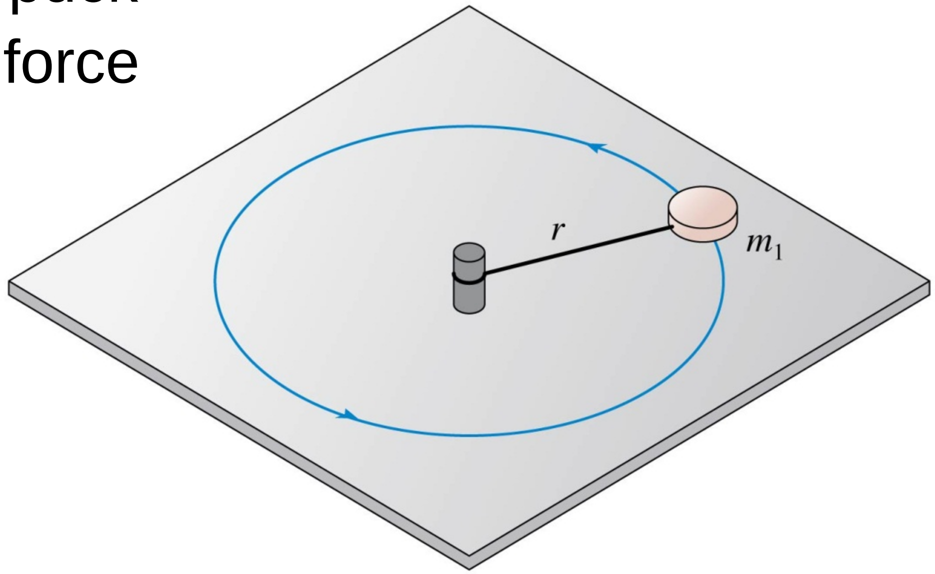
E. I have no clue.

The rules about what is or is not a force haven't changed.
1. Force must be exerted at a point of contact (except for gravity).
2. Force must have an identifiable agent doing the pushing or pulling.
3. The net force must point in the direction of acceleration (Newton's second law).

QuickCheck 8.4

An ice hockey puck is tied by a string to a stake in the ice. The puck is then swung in a circle. What force is producing the centripetal acceleration of the puck?



A. Gravity

B. Air resistance

C. Friction

D. Normal force

E. Tension in the string

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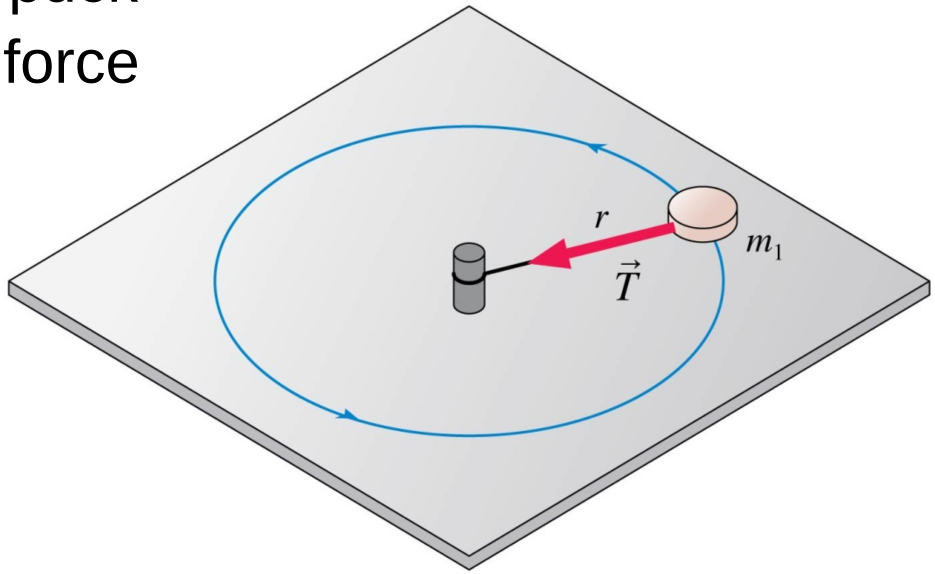

A. Gravity

B. Air resistance

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Example 8.3 Turning the Corner I

EXAMPLE 8.3 **Turning the corner I**

What is the maximum speed with which a 1500 kg car can make a left turn around a curve of radius 50 m on a level (unbanked) road without sliding?

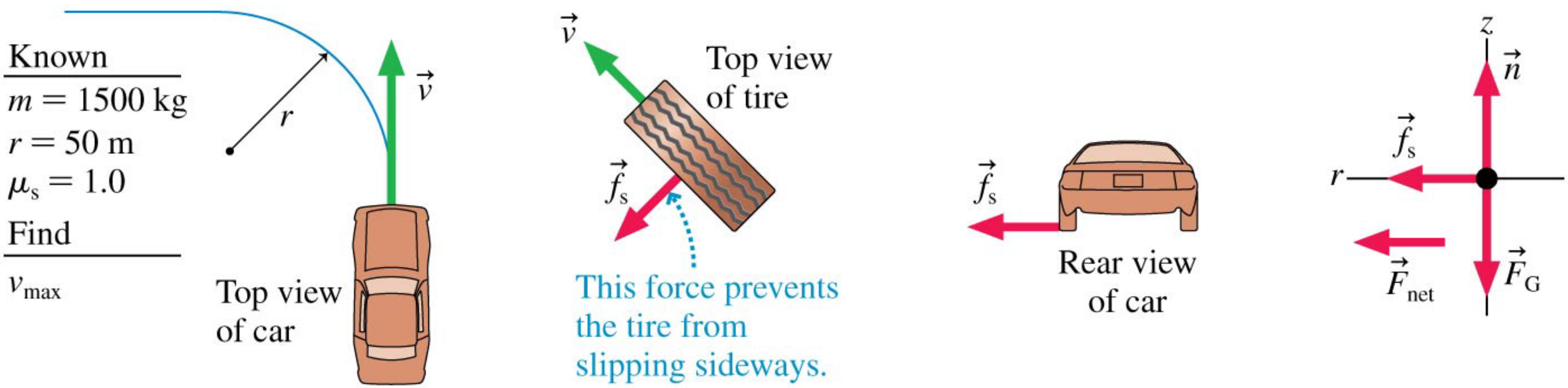
MODEL The car doesn't complete a full circle, but it is in uniform circular motion for a quarter of a circle while turning. We can model the car as a particle subject to a central force. Assume that rolling friction is negligible.

Example 8.3 Turning the Corner I

EXAMPLE 8.3 **Turning the corner I**

VISUALIZE

- The second figure below shows the top view of a tire as it turns a corner.
- The force that prevents the tire from sliding across a surface is *static friction*.
- Static friction pushes sideways on the tire, perpendicular to the velocity, since the car is not speeding up or slowing down.
- The free-body diagram, drawn from behind the car, shows the static friction pointing toward the center of the circle.



Example 8.3 Turning the Corner I

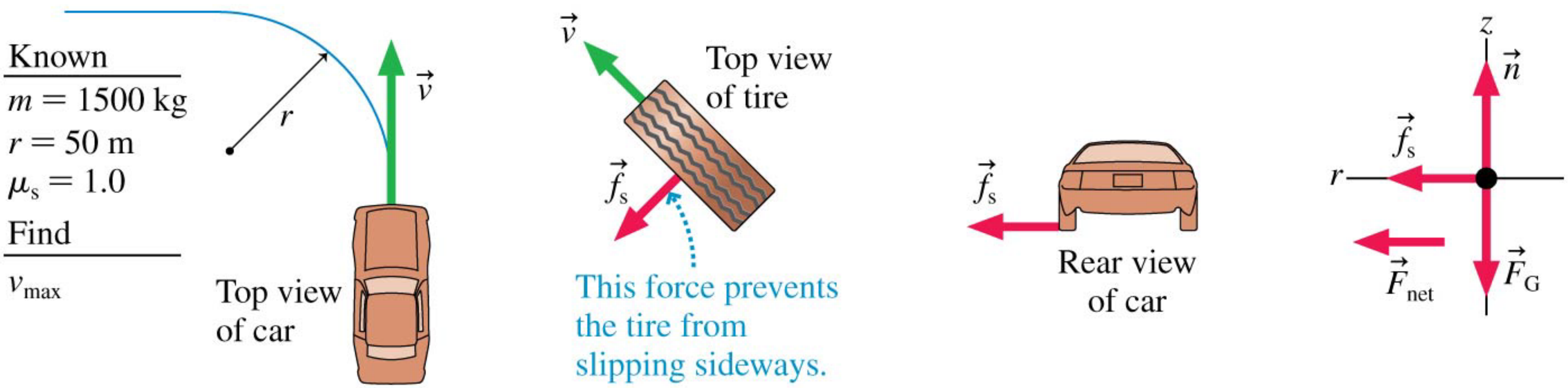
EXAMPLE 8.3 Turning the corner I

SOLVE The maximum turning speed is reached when the static friction force reaches its maximum $f_{s\,max} = \mu_s n$. If the car enters the curve at a speed higher than the maximum, static friction will not be large enough to provide the necessary centripetal acceleration and the car will slide.

The static friction force points in the positive r-direction, so its radial component is simply the magnitude of the vector: $(f_s)_r = f_s$. Newton's second law in the rtz-coordinate system is

$$\sum F_r = f_s = \frac{mv^2}{r}$$

$$\sum F_z = n - mg = 0$$



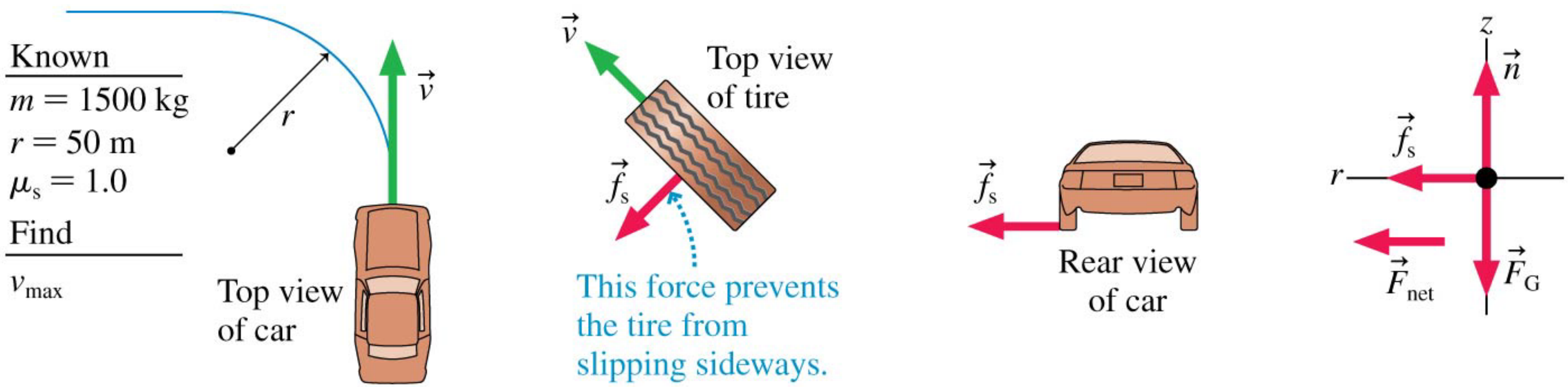
Example 8.3 Turning the Corner I

EXAMPLE 8.3 | **Turning the corner I**

[redacted]

The speed will be a maximum when f_s reach[redacted]ue:

$$v = \sqrt{\frac{rf_s}{m}}$$



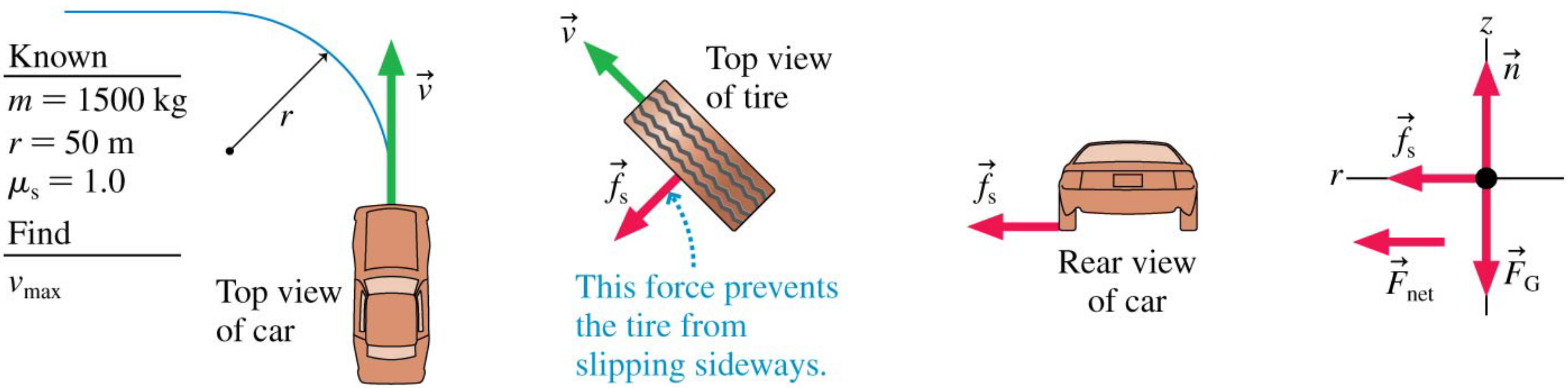
Example 8.3 Turning the Corner I

EXAMPLE 8.3 Turning the corner I

SOLVE At that point,

$$v_{max} = \sqrt{\frac{rf_{s\,max}}{m}} = \sqrt{\mu_s rg}$$
$$= \sqrt{(1.0)(50\text{ m})(9.80\text{ m/s}^2)} = 22\text{ m/s}$$

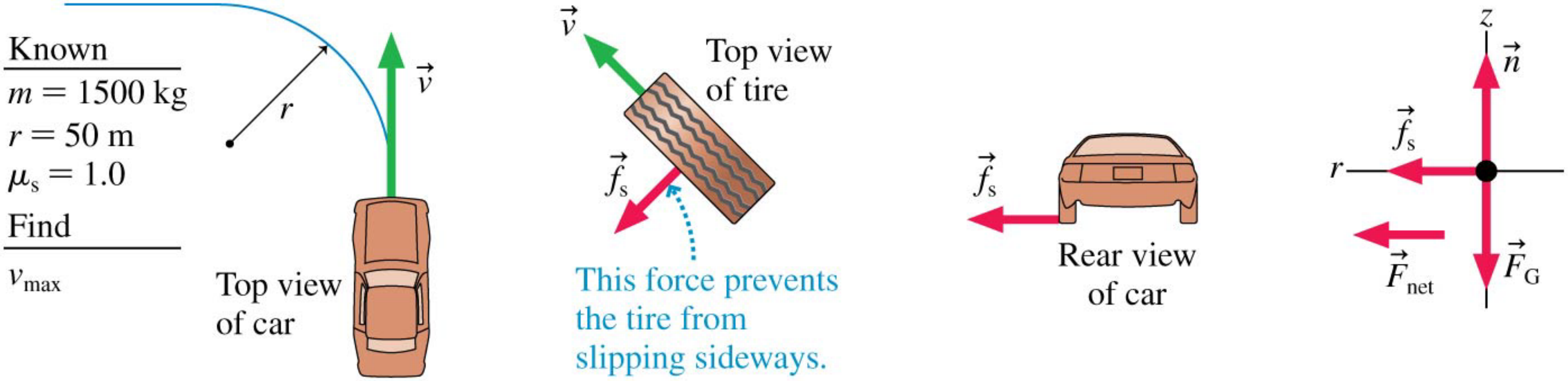
where the coefficient of static friction was taken from Table 6.1.



Example 8.3 Turning the Corner I

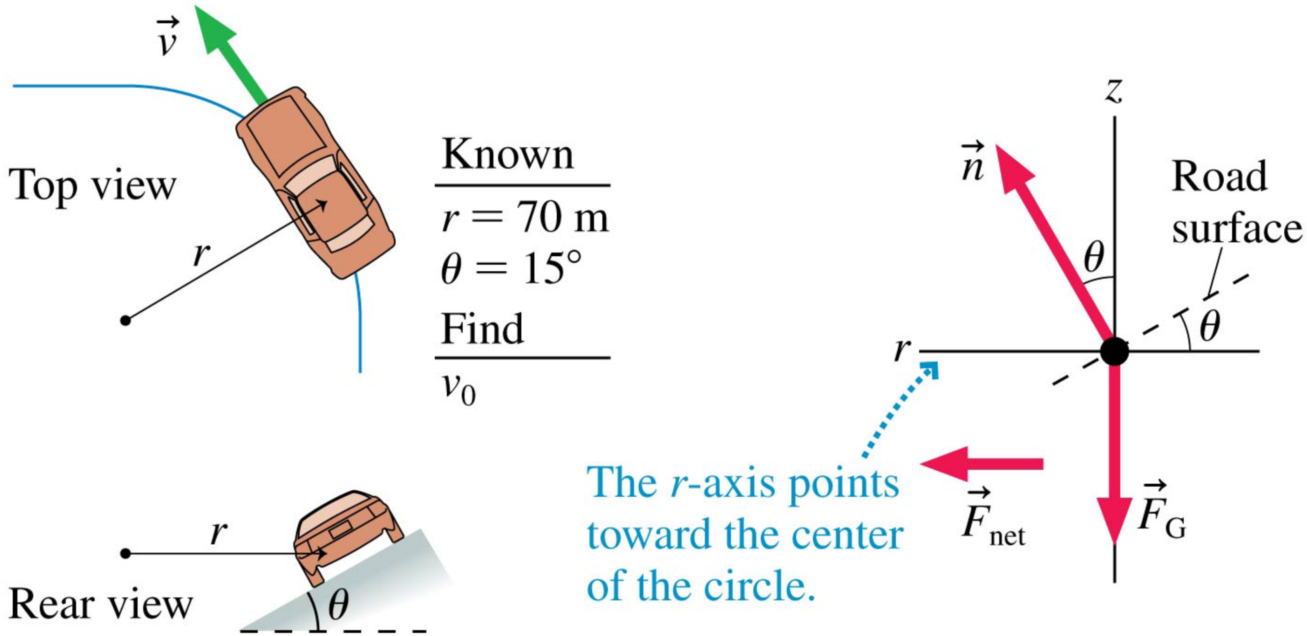
EXAMPLE 8.3 Turning the corner I

ASSESS 22 m/s ≈ 45 mph, a reasonable answer for how fast a car can take an unbanked curve. Notice that the car's mass canceled out and that the final equation for v_{max} is quite simple. This is another example of why it pays to work algebraically until the very end.



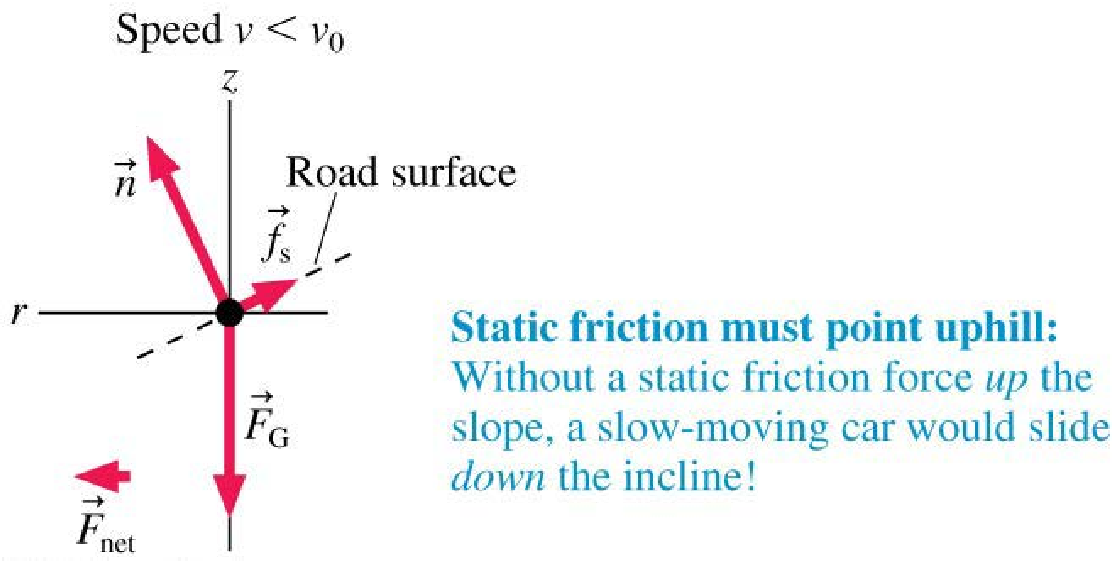
Banked Curves

- Real highway curves are *banked* by being tilted up at the outside edge of the curve.
- The radial component of the normal force can provide centripetal acceleration needed to turn the car.
- For a curve of radius r banked at an angle θ, the exact speed at which a car must take the curve without assistance from friction is $v_0 = \sqrt{rg\tan\theta}$.



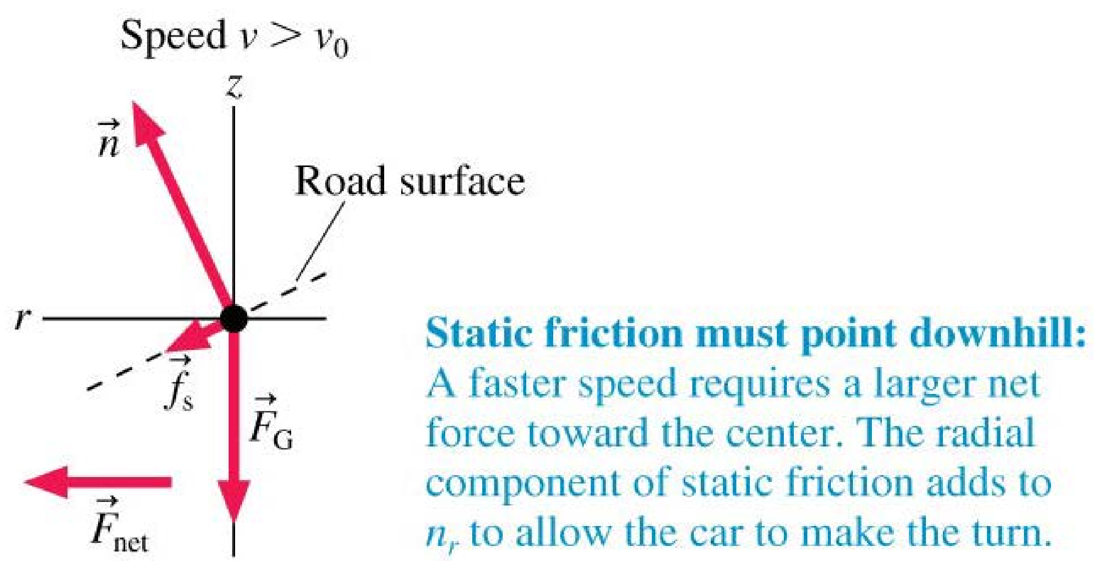
Banked Curves

- Consider a car going around a banked curve at a speed *slower* than $v_0 = \sqrt{rg \tan\theta}$.
- In this case, static friction must prevent the car from slipping *down* the hill.



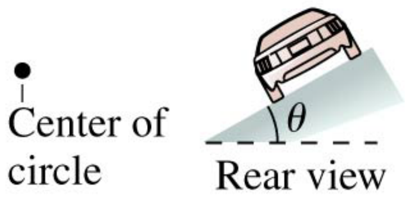
Banked Curves

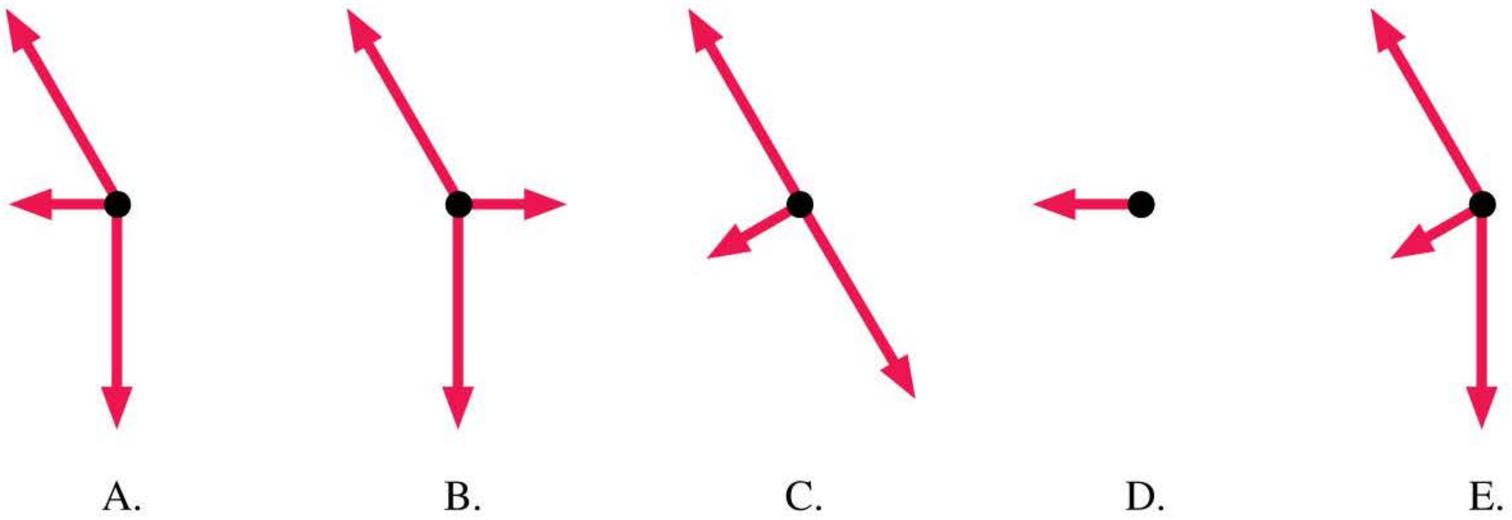
- Consider a car going around a banked curve at a speed *faster* than $v_0 = \sqrt{rg \tan\theta}$.
- In this case, static friction must prevent the car from slipping *up* the hill.



QuickCheck 8.5

A car turns a corner on a banked road. Which of the diagrams <u>could</u> be the car's free-body diagram?





QuickCheck 8.5

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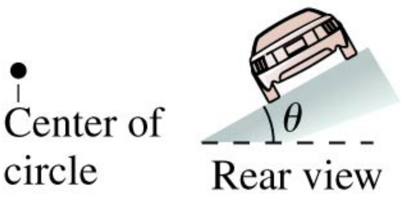
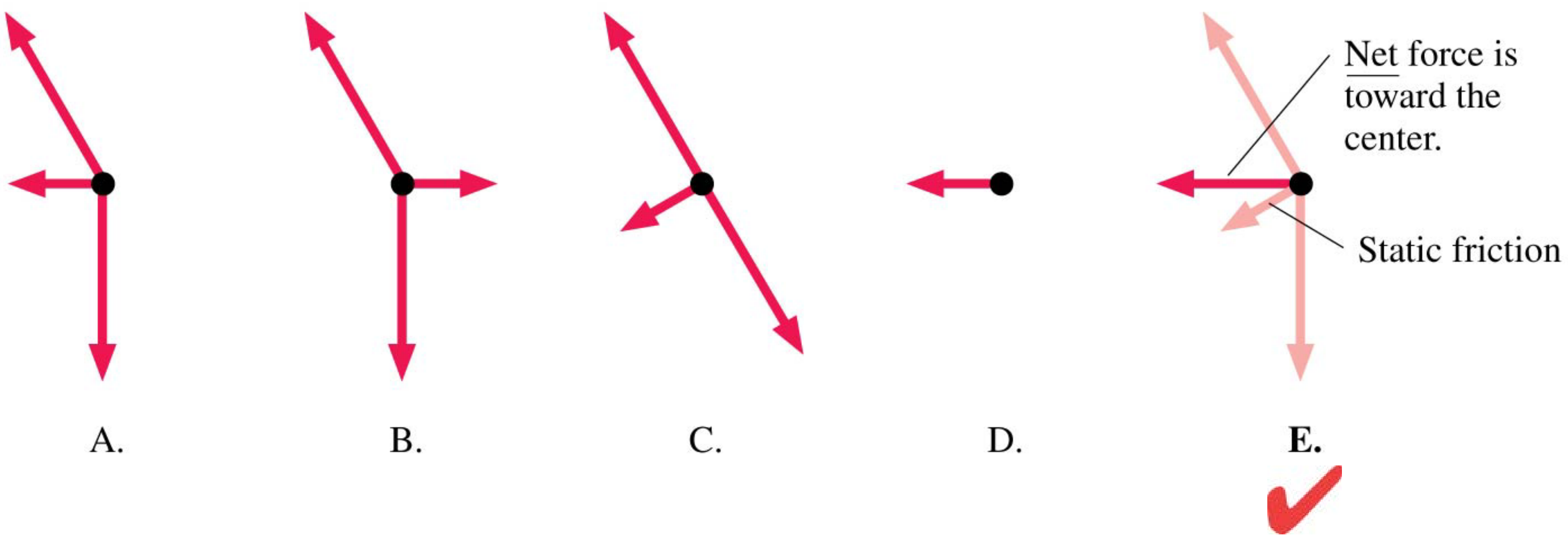
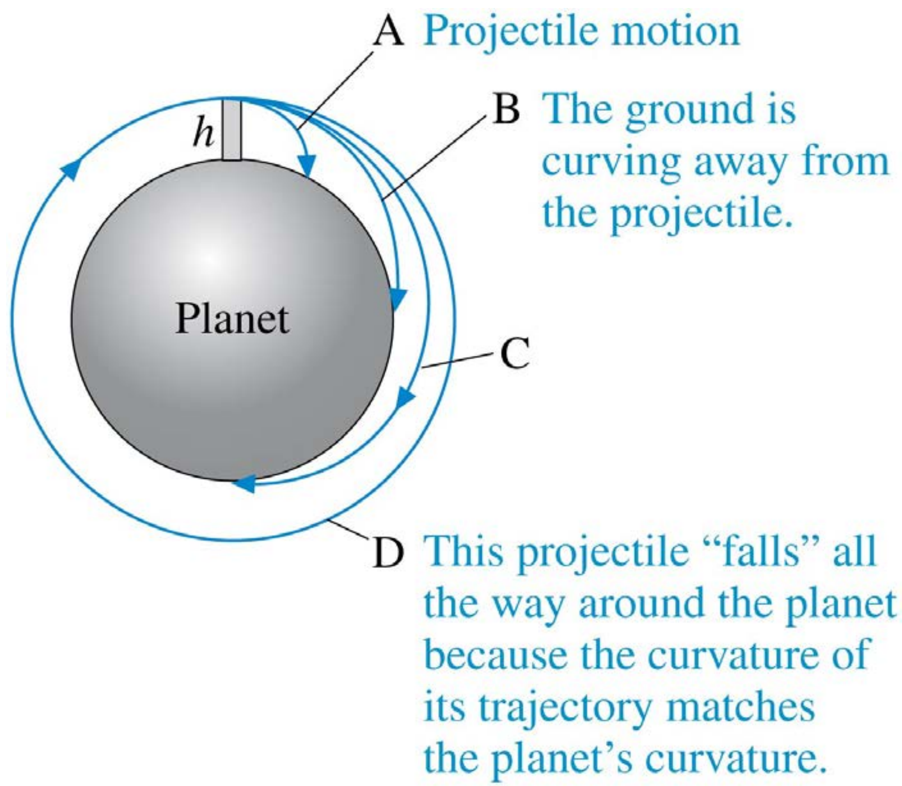




Circular Orbits

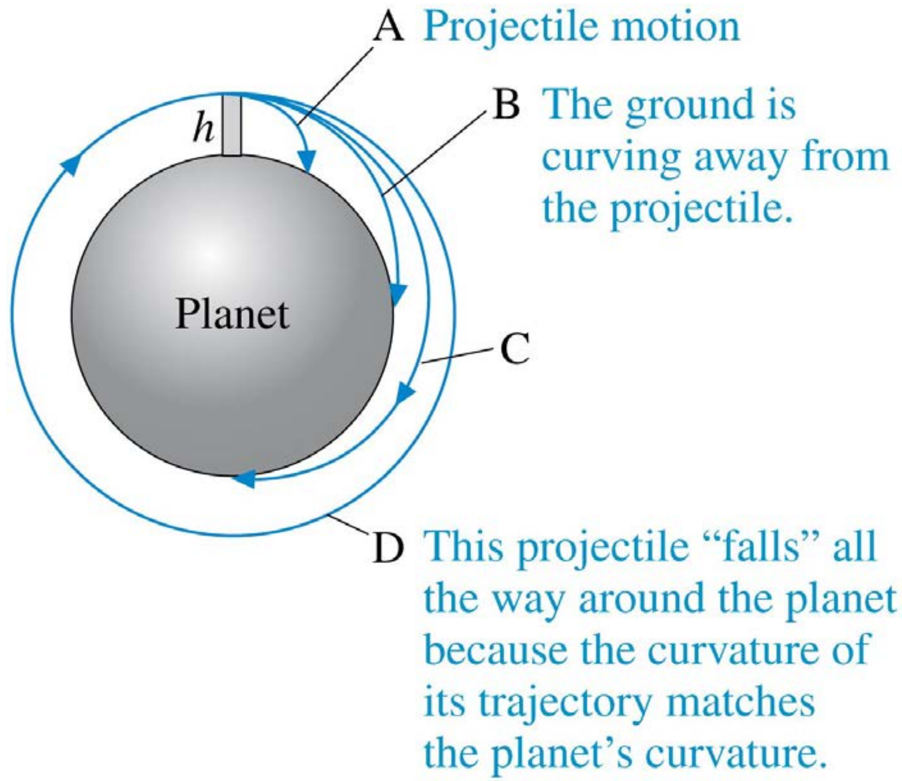
- The figure shows a perfectly smooth, spherical, airless planet with one tower of height h.
- A projectile is launched parallel to the ground with speed v_0.
- If v_0 is very small, as in trajectory A, it simply falls to the ground along a parabolic trajectory.



- This is the "flat-earth approximation."

Circular Orbits

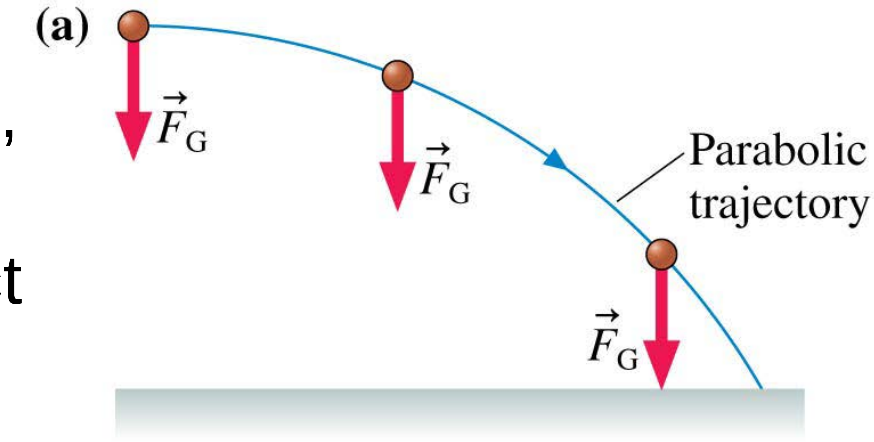
- As the initial speed v_0 is increased, the range of the projectile increases as the ground curves away from it.
- Trajectories B and C are of this type.
- If v_0 is sufficiently large, there comes a point where the trajectory and the curve of the earth are parallel.
- In this case, the projectile "falls" but it never gets any closer to the ground!
- This is trajectory D, called an **orbit**.



Circular Orbits

- In the flat-earth approximation, shown in figure (a), the gravitational force on an object of mass m is

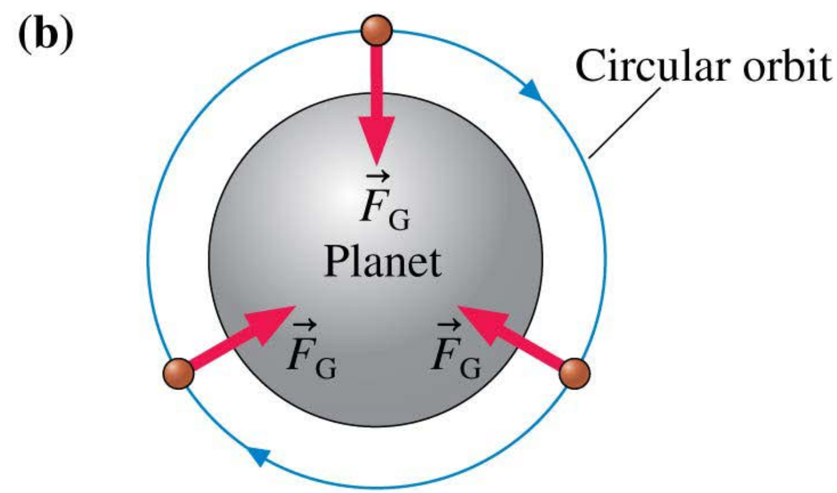
$$\vec{F}_G = (mg, \text{vertically downward})$$



Flat-earth approximation

- Since actual planets are spherical, the real force of gravity is toward the center of the planet, as shown in figure (b):

$$\vec{F}_G = (mg, \text{toward center})$$



Spherical planet

Circular Orbits

- An object in a low circular orbit has acceleration:

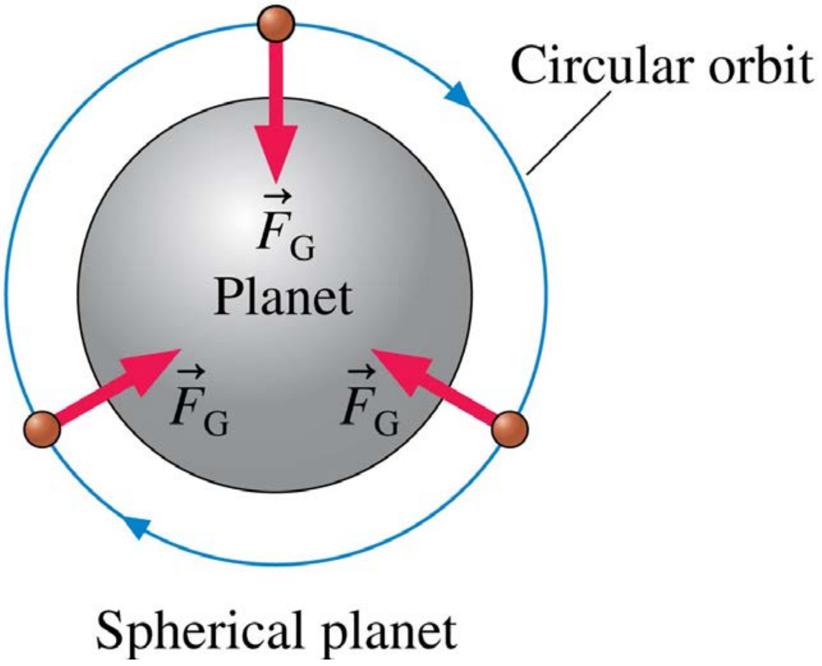
$$\vec{a} = \frac{\vec{F}_{net}}{m} = (g, \text{ toward center})$$

- If the object moves in a circle of radius r at speed v_{orbit} the centripetal acceleration is

$$a_r = \frac{(v_{orbit})^2}{r} = g$$

- The required speed for a circular orbit near a planet's surface, neglecting air resistance, is

$$v_{orbit} = \sqrt{rg}$$

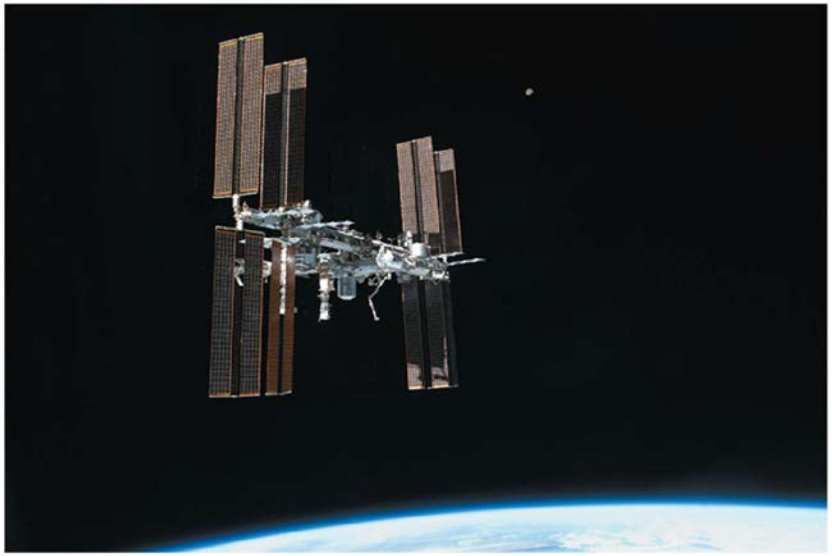


Circular Orbits

- The period of a low-earth-orbit satellite is

$$T = \frac{2\pi r}{v_{\text{orbit}}} = 2\pi\sqrt{\frac{r}{g}}$$

- If r is approximately the radius of the earth $R_e = 6400\ \text{km}$, then T is about 90 minutes.

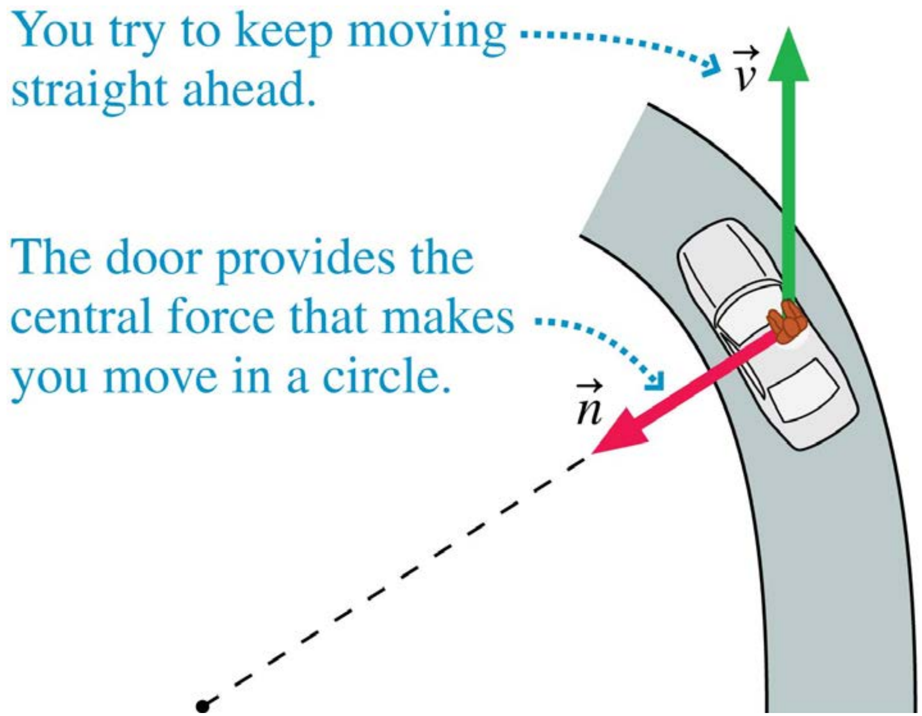


The International Space Station is in free fall.

- An orbiting spacecraft is constantly in free fall, falling under the influence only of the gravitational force.
- This is why astronauts feel *weightless* in space.

Centrifugal Force?

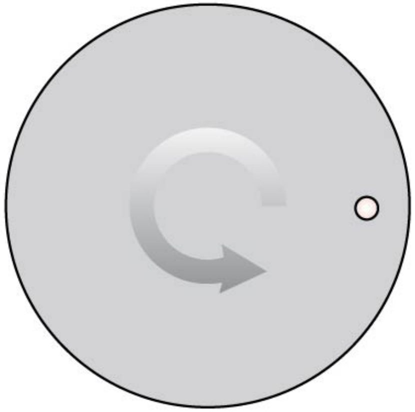
- The figure shows a bird's-eye view of you riding in a car as it makes a left turn.
- From the perspective of an inertial reference frame, the normal force from the door points *inward*, keeping you on the road with the car.

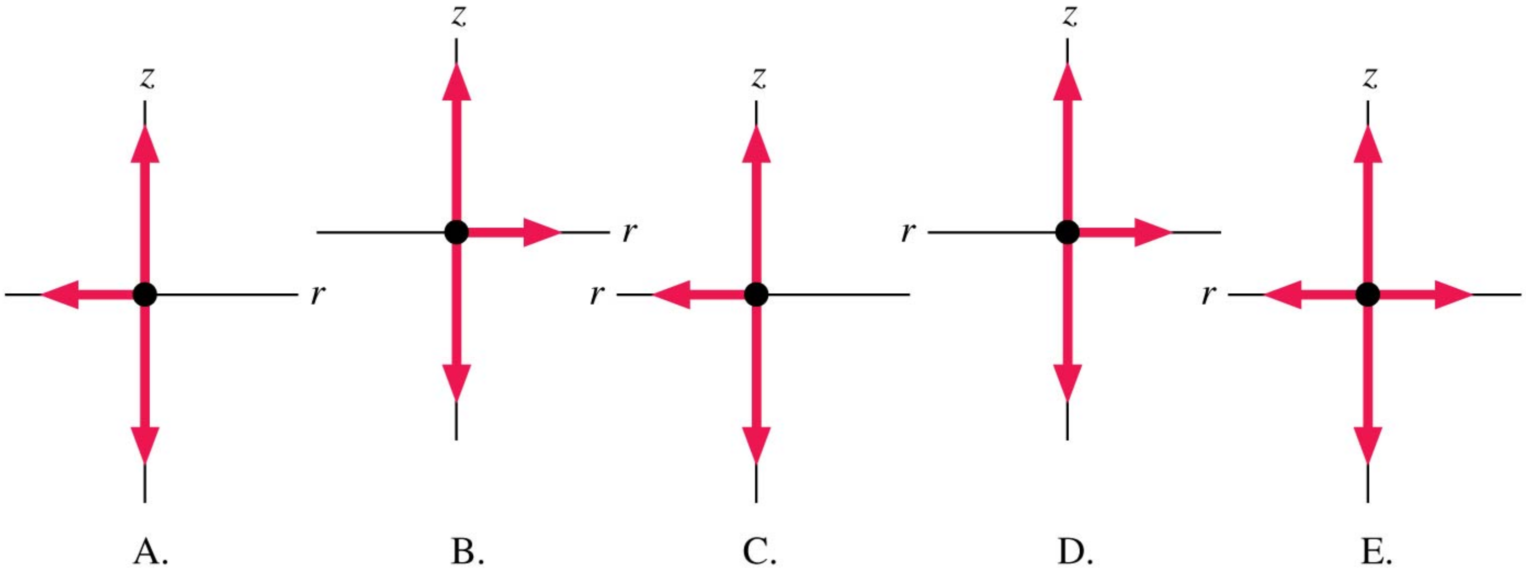


- *Relative to the noninertial reference frame* of the car, you feel pushed toward the *outside* of the curve.
- The fictitious force that seems to push an object to the outside of a circle is called the **centrifugal force**.
- There really is no such force in an inertial reference frame.

QuickCheck 8.6

A coin sits on a turntable as the table steadily rotates ccw. The free-body diagrams below show the coin from behind, moving away from you. Which is the correct diagram?





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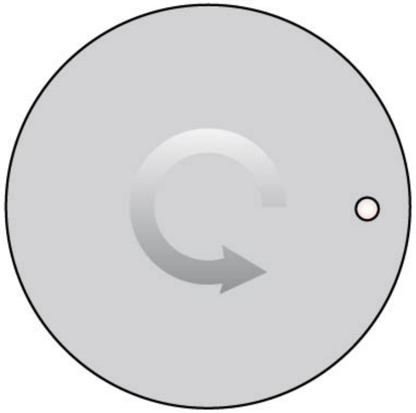
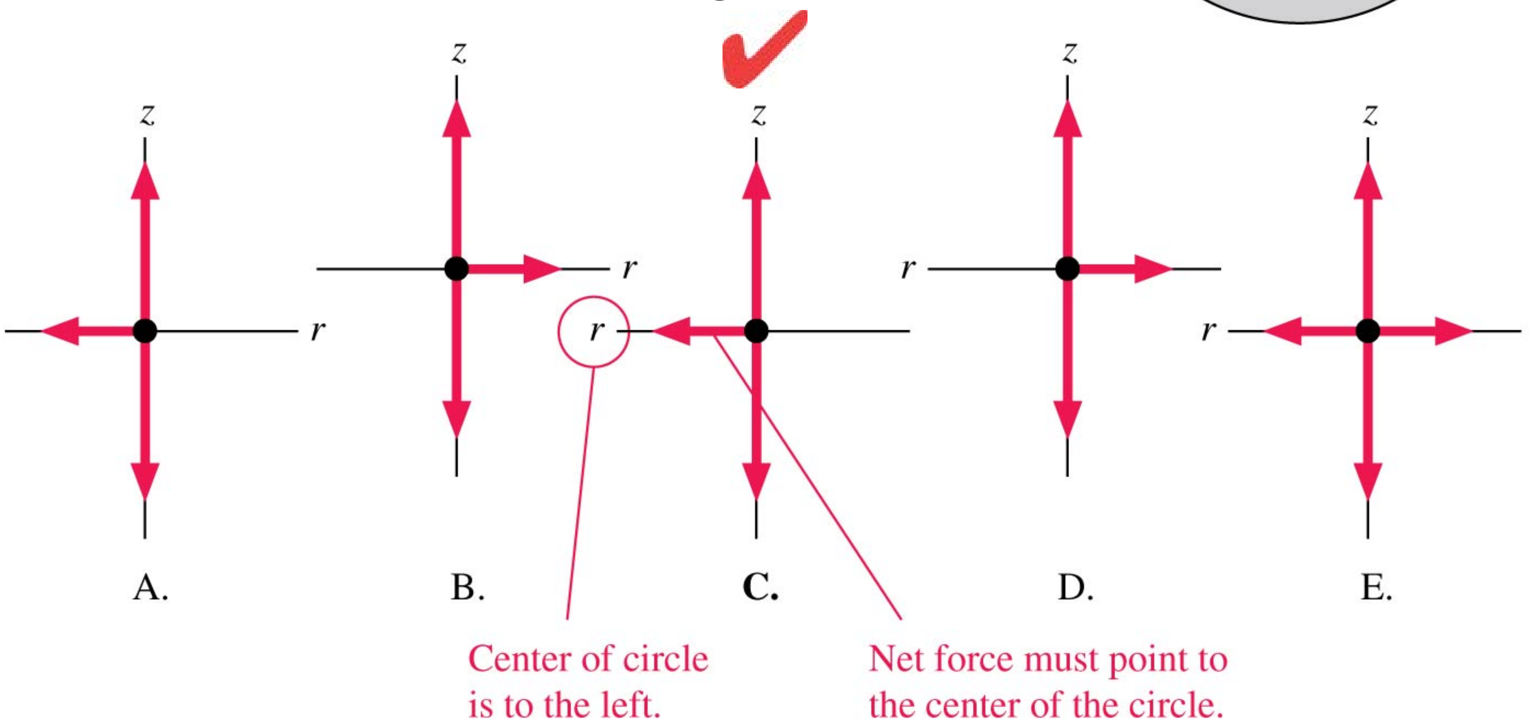
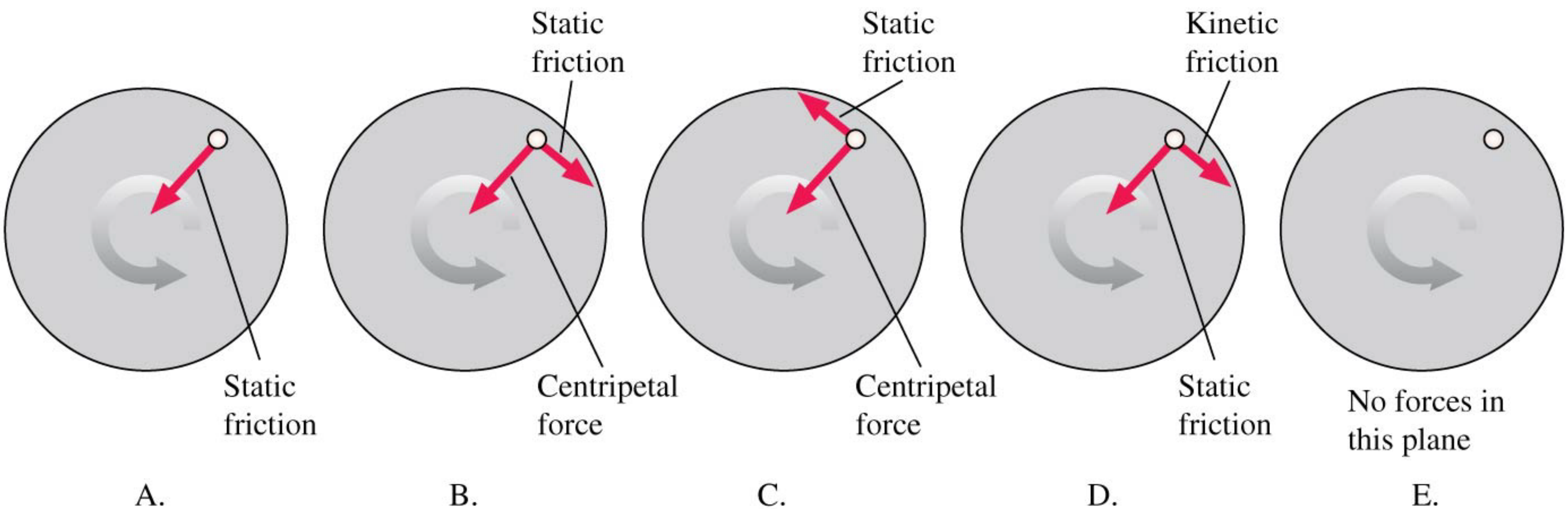




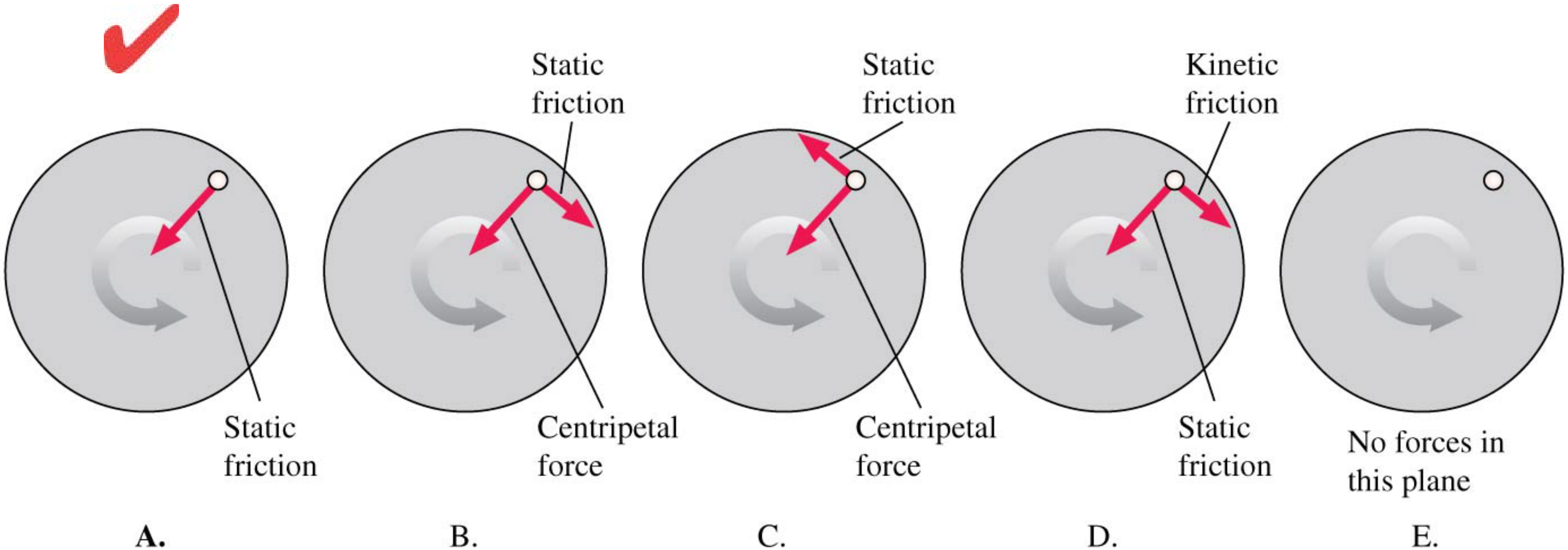
QuickCheck 8.7

A coin sits on a turntable as the table steadily rotates ccw. What force or forces act in the plane of the turntable?



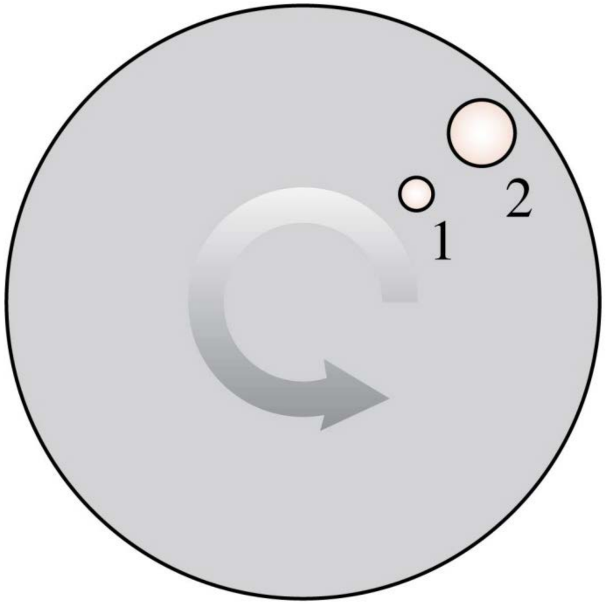
QuickCheck 8.7

A coin sits on a turntable as the table steadily rotates ccw. What force or forces act in the plane of the turntable?



QuickCheck 8.8

Two coins are on a turntable that steadily speeds up, starting from rest, with a ccw rotation. Which coin flies off the turntable first?



A. Coin 1 flies off first.

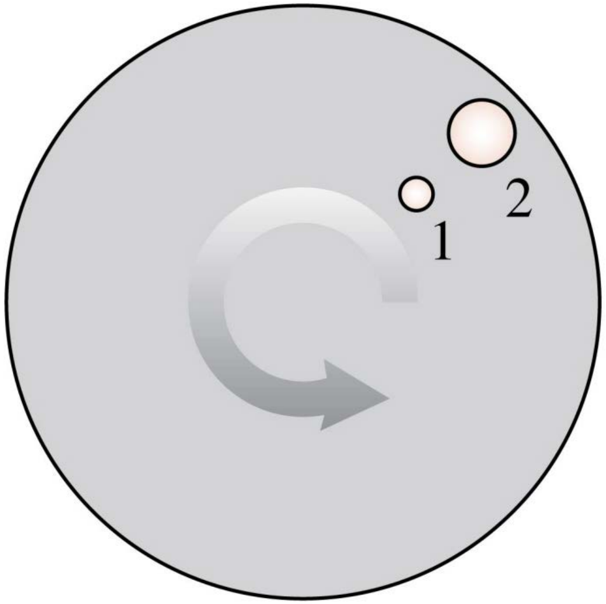
B. Coin 2 flies off first.

C. Both coins fly off at the same time.

D. We can't say without knowing their masses.

QuickCheck 8.8

Two coins are on a turntable that steadily speeds up, starting from rest, with a ccw rotation. Which coin flies off the turntable first?



A. Coin 1 flies off first.

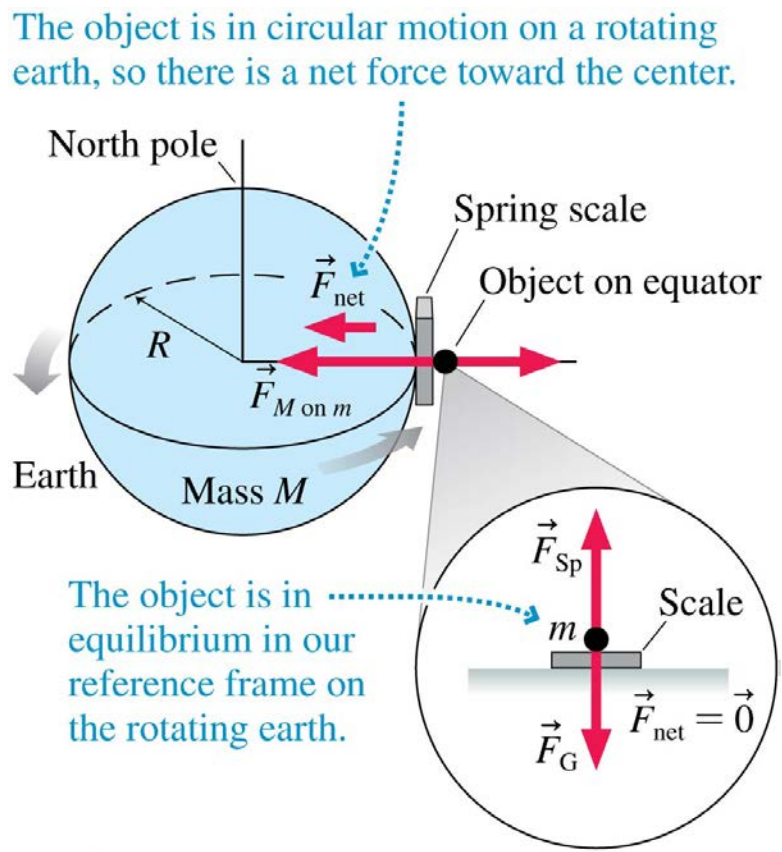
B. Coin 2 flies off first.

C. Both coins fly off at the same time.

D. We can't say without knowing their masses.

Gravity on a Rotating Earth

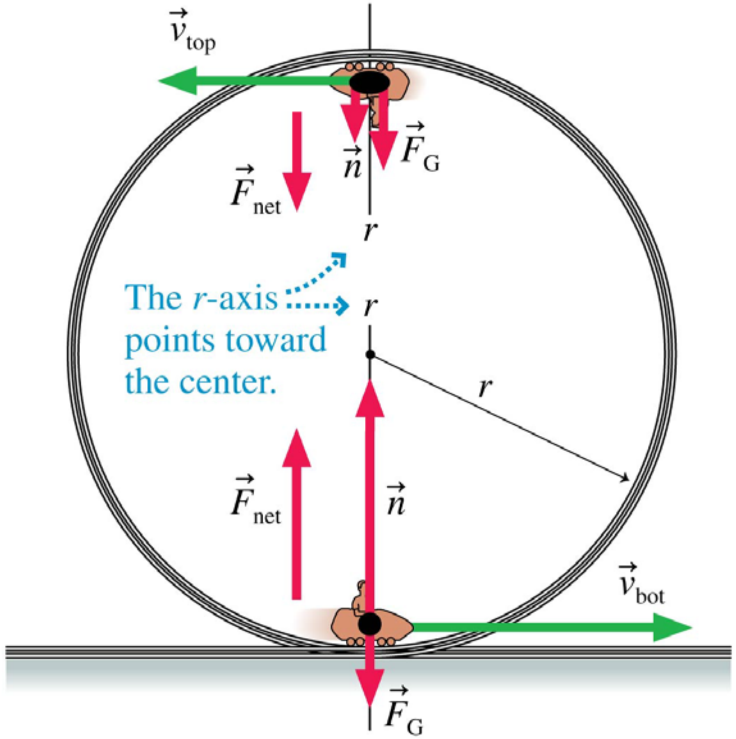
- The figure shows an object being weighed by a spring scale on the earth's equator.
- The observer is hovering in an inertial reference frame above the north pole.
- If we pretend the spring-scale reading is $F_{\mathrm{Sp}} = F_G = mg$, this has the effect of "weakening" gravity.
- The free-fall acceleration we measure in our rotating reference frame is



$$g = \frac{F_{\mathrm{G}}}{m} = \frac{F_{M\text{ on }m} - m\omega^2 R}{m} = \frac{GM}{R^2} - \omega^2 R = g_{\mathrm{earth}} - \omega^2 R$$

Loop-the-Loop

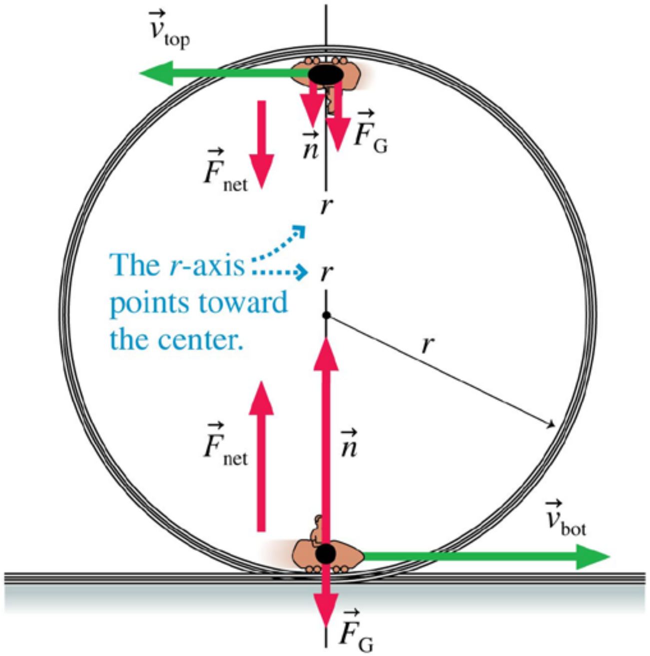
- The figure shows a roller-coaster going around a vertical loop-the-loop of radius *r*.
- Note this is *not* uniform circular motion: The car slows down going up one side, and speeds up going down the other.



- At the very top and very bottom points, only the car's direction is changing, so the acceleration is purely centripetal.
- **Because the car is moving in a circle, there must be a net force toward the center of the circle**.

Loop-the-Loop

- Consider the roller-coaster free-body diagram at the *bottom* of the loop.
- Since the net force is toward the center (upward at this point), $n > F_G$.
- This is why you "feel heavy" at the bottom of the valley on a roller coaster.



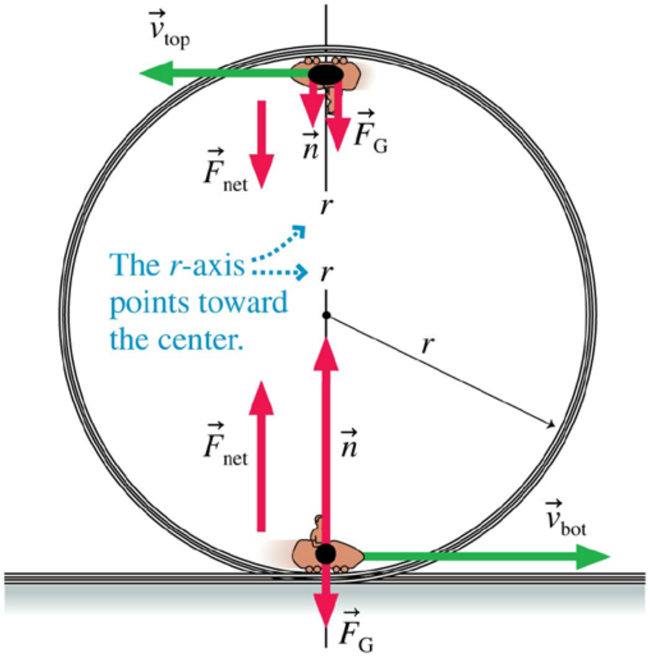
$$\sum F_r = n_r + (F_G)_r = n - mg = ma_r = \frac{m(v_{bot})^2}{r}$$

$$n = mg + \frac{m(v_{bot})^2}{r}$$

- The normal force at the bottom is *larger* than mg.

Loop-the-Loop

- The figure shows the roller-coaster free-body diagram at the *top* of the loop.
- The track can only push on the wheels of the car, it cannot pull, therefore $\vec{n}$ presses *downward.*
- The car is still moving in a circle, so the net force is also downward:



$$\sum F_r = n_r + (F_G)_r = n + mg = \frac{m(v_{top})^2}{r}$$

$$n = \frac{m(v_{top})^2}{r} - mg$$

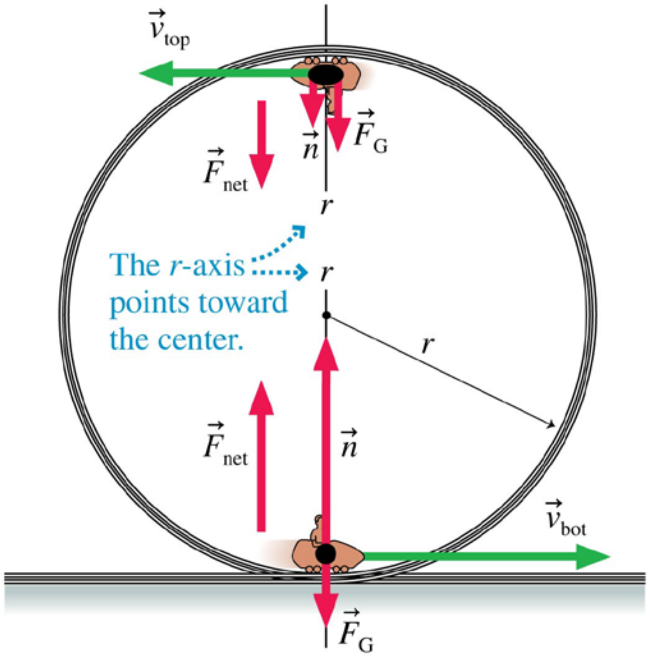
- The normal force at the at the top can exceed mg if v_{top} is large enough.

Loop-the-Loop

- At the top of the roller coaster, the normal force of the track on the car is

$$n = \frac{m(v_{top})^2}{r} - mg$$

- As v_{top} decreases, there comes a point when n reaches zero.



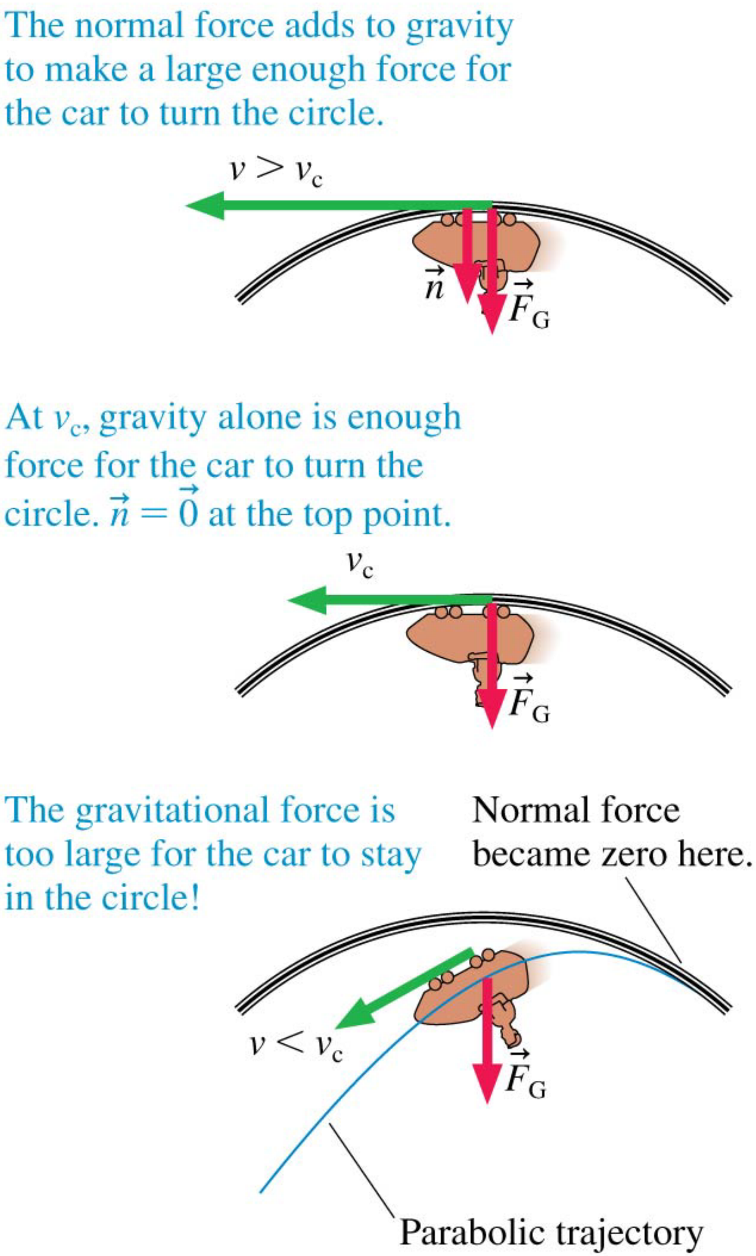
- The speed at which $n = 0$ is called the **critical speed**:

$$v_c = \sqrt{\frac{rmg}{m}} = \sqrt{rg}$$

- This is the slowest speed at which the car can complete the circle without falling off the track near the top.

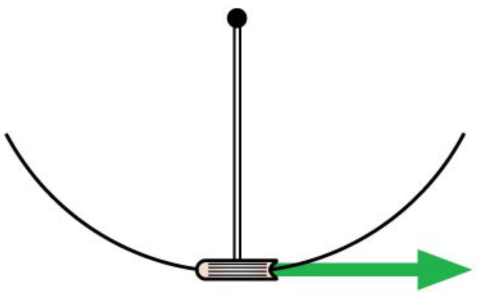
Loop-the-Loop

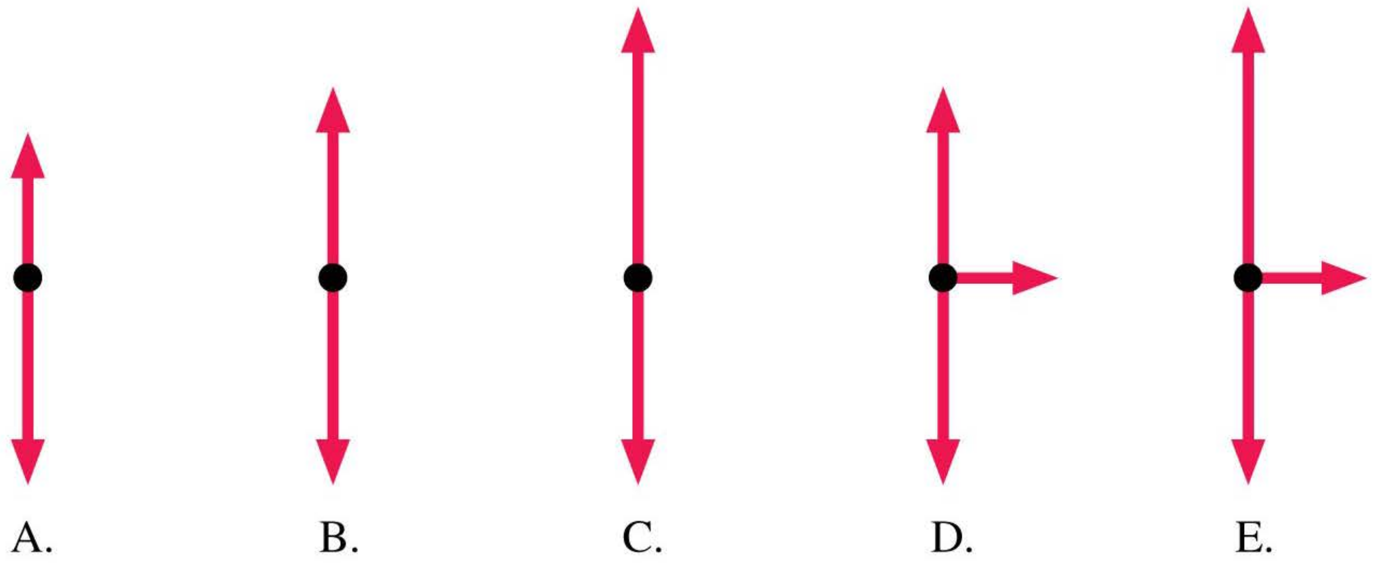
A roller-coaster car at the top of the loop.



QuickCheck 8.9

A physics textbook swings back and forth as a pendulum. Which is the correct free-body diagram when the book is at the bottom and moving to the right?





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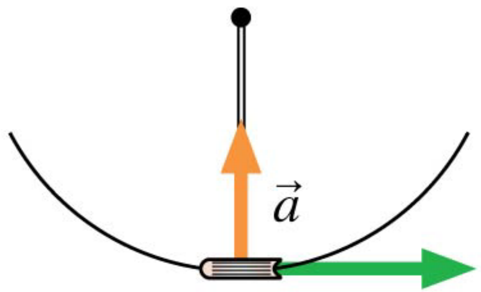
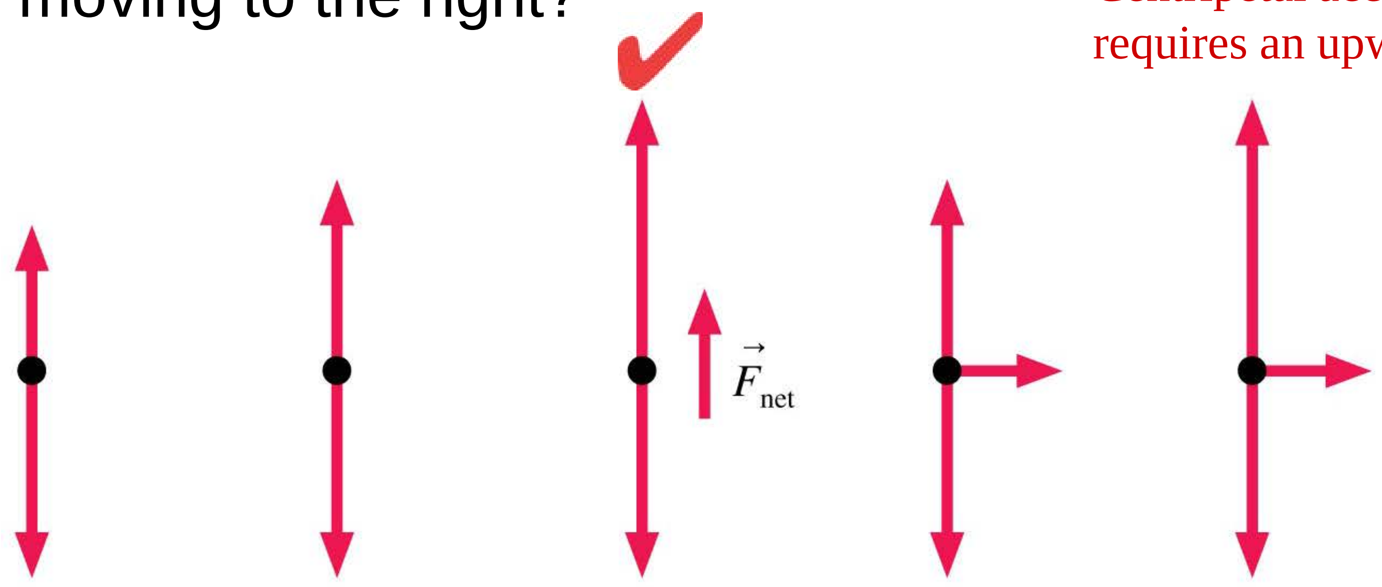


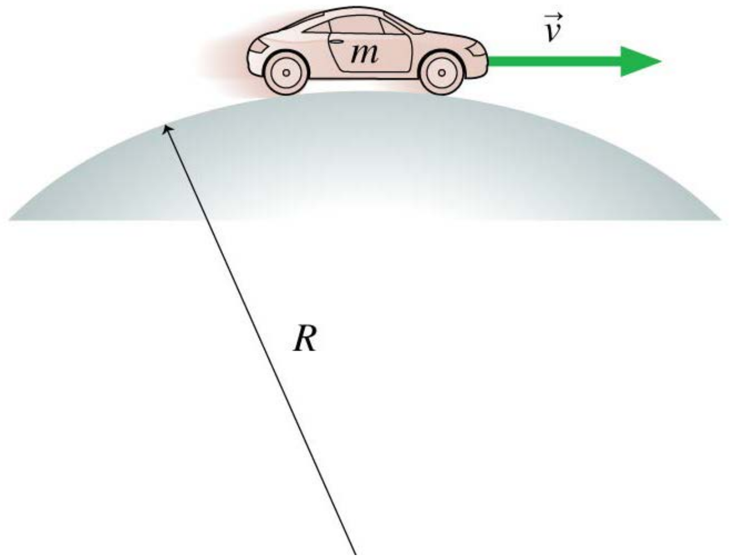
Centripetal acceleration requires an upward force.

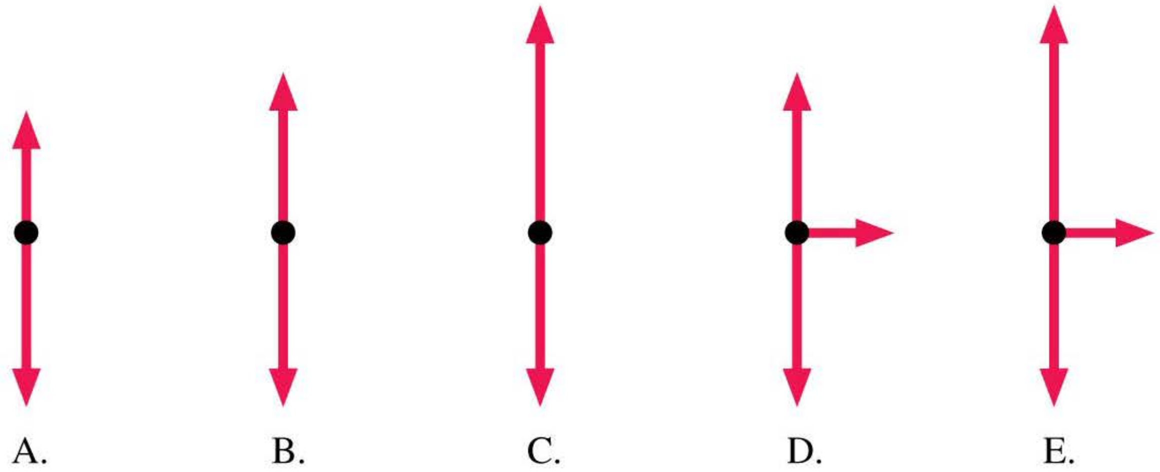


A. B. C. D. E.

QuickCheck 8.10

A car that's out of gas coasts over the top of a hill at a steady 20 m/s. Assume air resistance is negligible. Which free-body diagram describes the car at this instant?





QuickCheck 8.10

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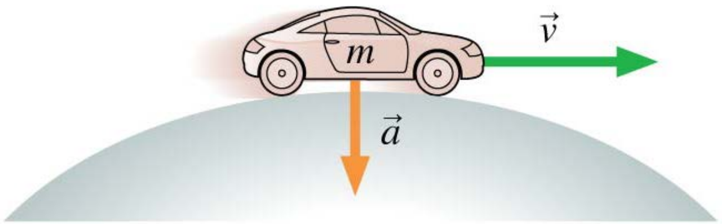
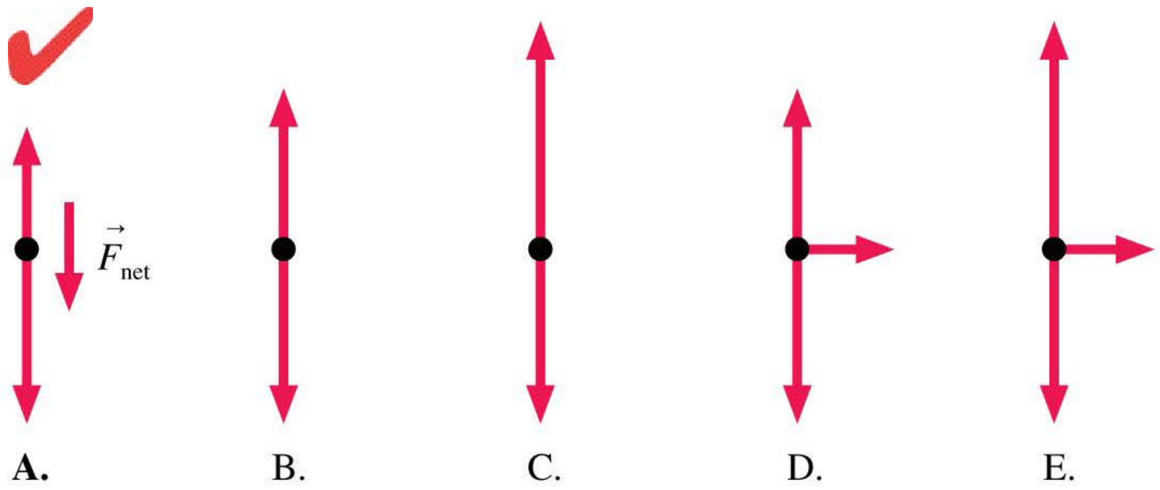
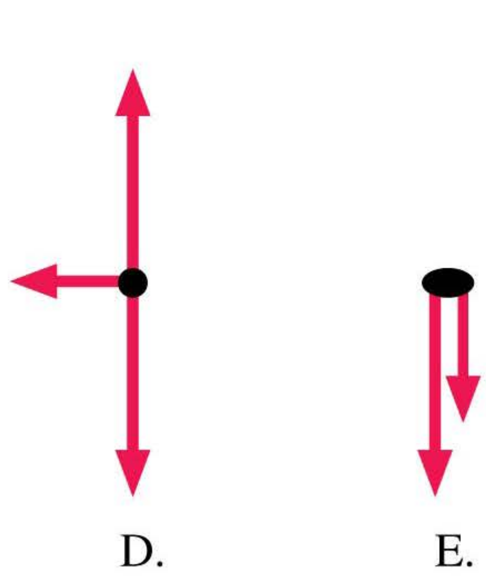


Now the centripetal acceleration points down.



QuickCheck 8.11

A roller coaster car does a loop-the-loop. Which of the free-body diagrams shows the forces on the car at the top of the loop? Rolling friction can be neglected.



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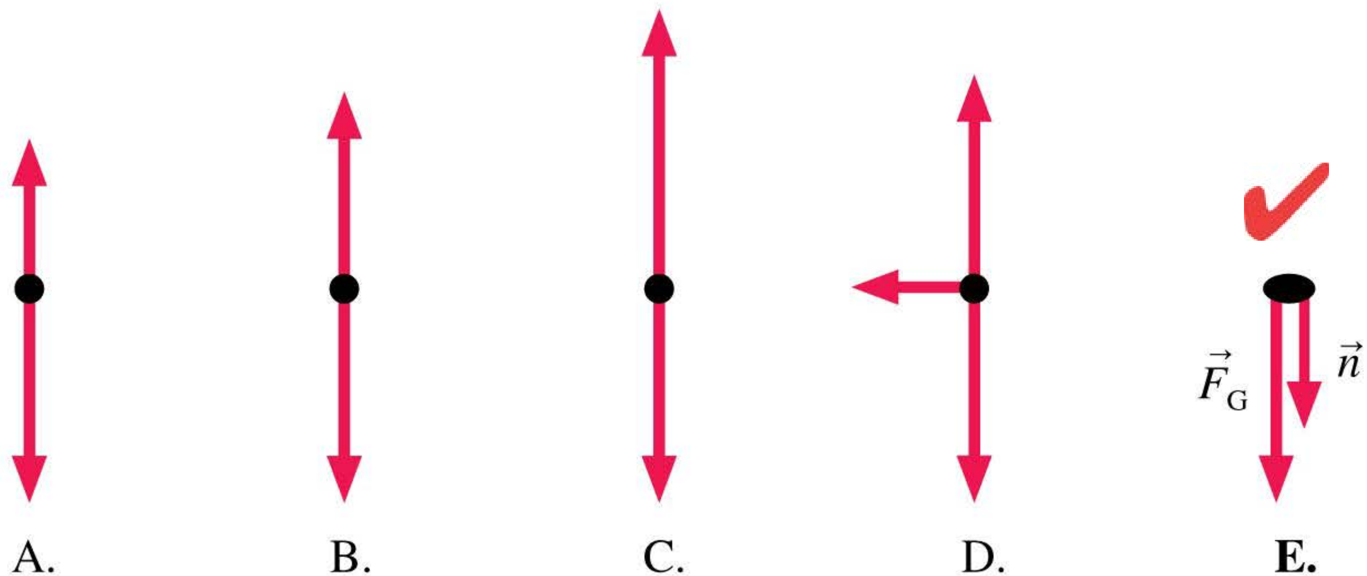
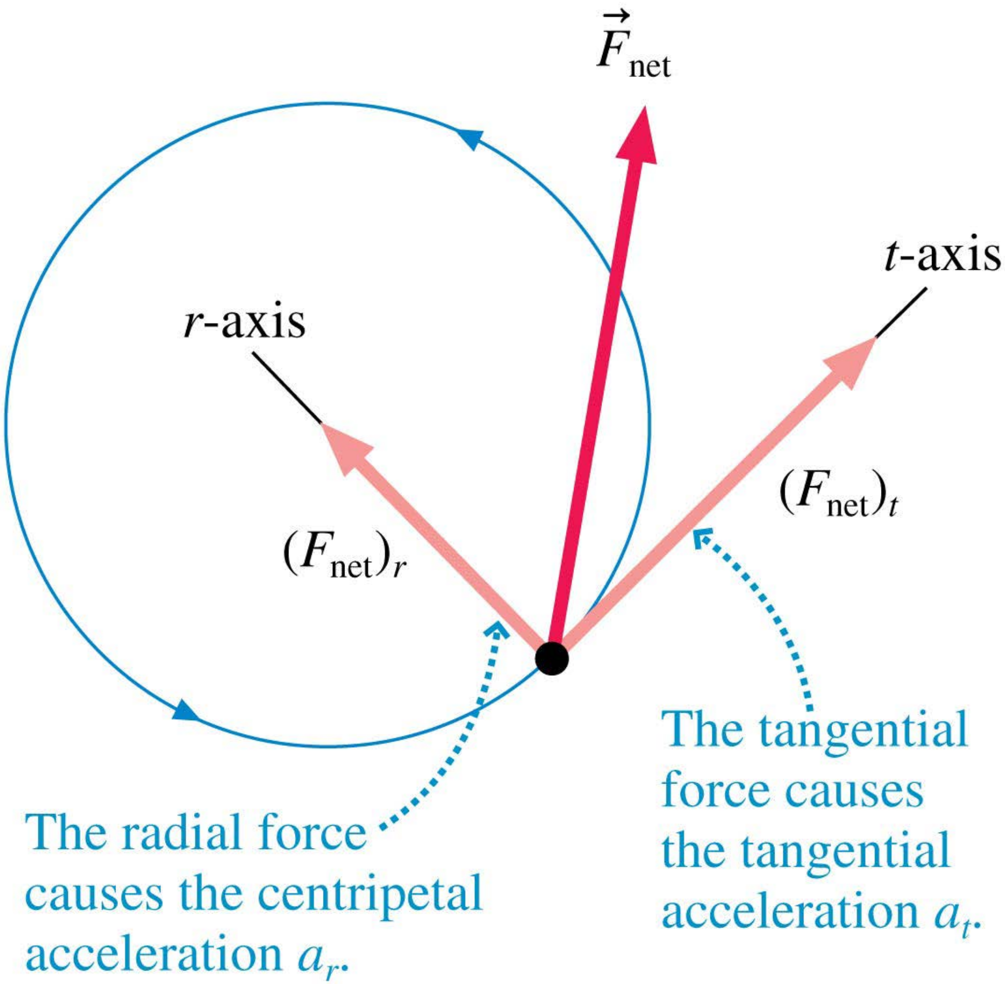


The track is *above* the car, so the normal force of the track pushes <u>down</u>.

Nonuniform Circular Motion

- The figure shows a particle moving in a circle of radius *r*.
- In addition to a radial force component—required for all circular motion—this particle experiences a tangential force component and hence a tangential acceleration:

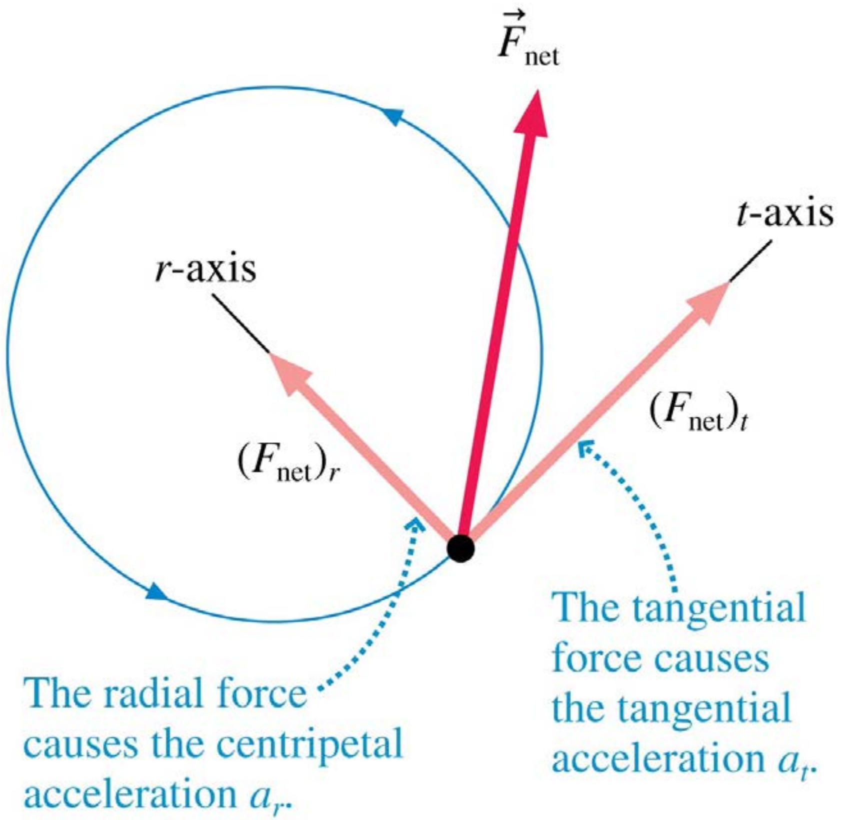
$$a_t = \frac{dv_t}{dt}$$



Dynamics of Nonuniform Circular Motion

- Force and acceleration are related through Newton's second law:

$$(F_{\text{net}})_r = \sum F_r = ma_r = \frac{mv_t^2}{r} = m\omega^2 r$$
$$(F_{\text{net}})_t = \sum F_t = ma_t$$
$$(F_{\text{net}})_z = \sum F_z = 0$$



Problem-Solving Strategy: Circular-Motion Problems

PROBLEM-SOLVING STRATEGY 8.1

MP

Circular-motion problems

MODEL Model the object as a particle and make other simplifying assumptions.

VISUALIZE Draw a pictorial representation. Use *rtz*-coordinates.

- Establish a coordinate system with the *r*-axis pointing toward the center of the circle.
- Show important points in the motion on a sketch. Define symbols and identify what the problem is trying to find.
- Identify the forces and show them on a free-body diagram.

Problem-Solving Strategy: Circular-Motion Problems

PROBLEM-SOLVING STRATEGY 8.1 MP

Circular-motion problems

SOLVE Newton's second law is

$$(F_{\text{net}})_r = \sum F_r = ma_r = \frac{mv_t^2}{r} = m\omega^2 r$$

$$(F_{\text{net}})_t = \sum F_t = ma_t$$

$$(F_{\text{net}})_z = \sum F_z = 0$$

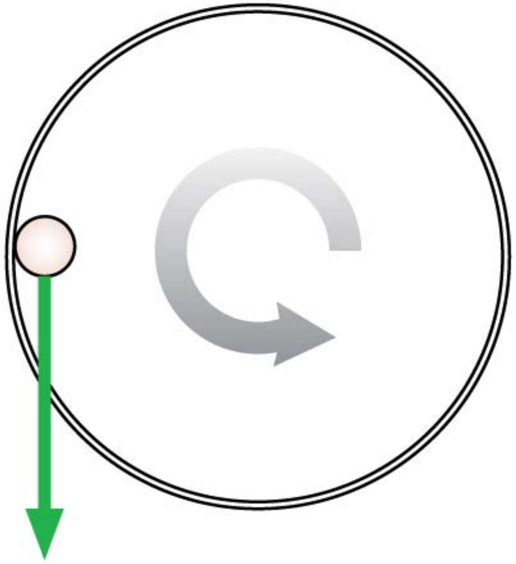
- Determine the force components from the free-body diagram. Be careful with signs.
- The tangential acceleration for uniform circular motion is $a_t = 0$.
- Solve for the acceleration, then use kinematics to find velocities and positions.

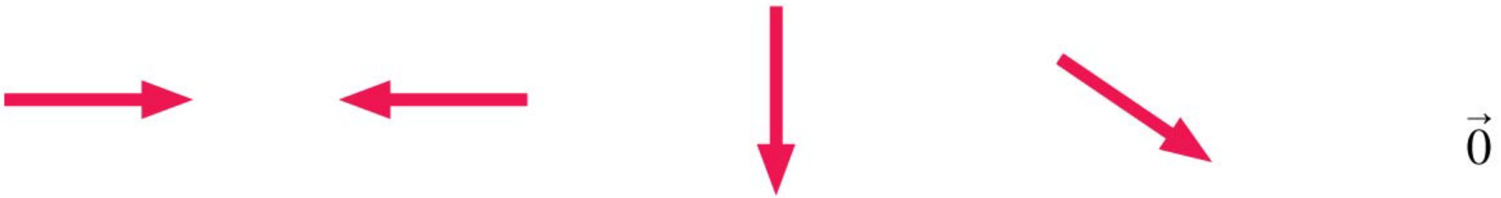
ASSESS Check that your result has the correct units and significant figures, is reasonable, and answers the question.

Exercise 11

QuickCheck 8.12

A ball rolls ccw around the inside of a horizontal pipe. The ball is fastest at the lowest point, slowest at the highest point. At the point shown, with the ball moving down, what is the direction of the net force on the ball?

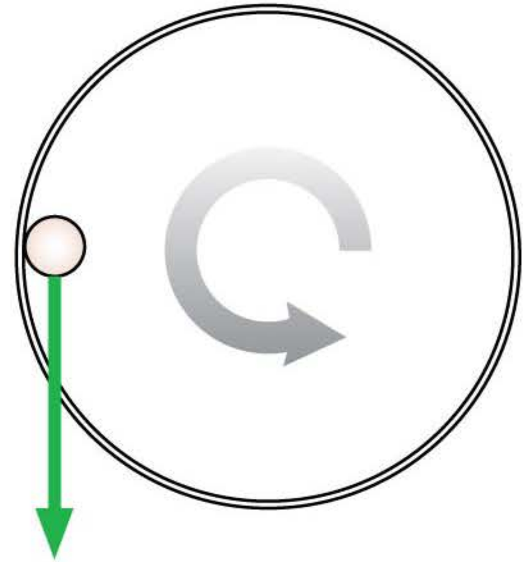


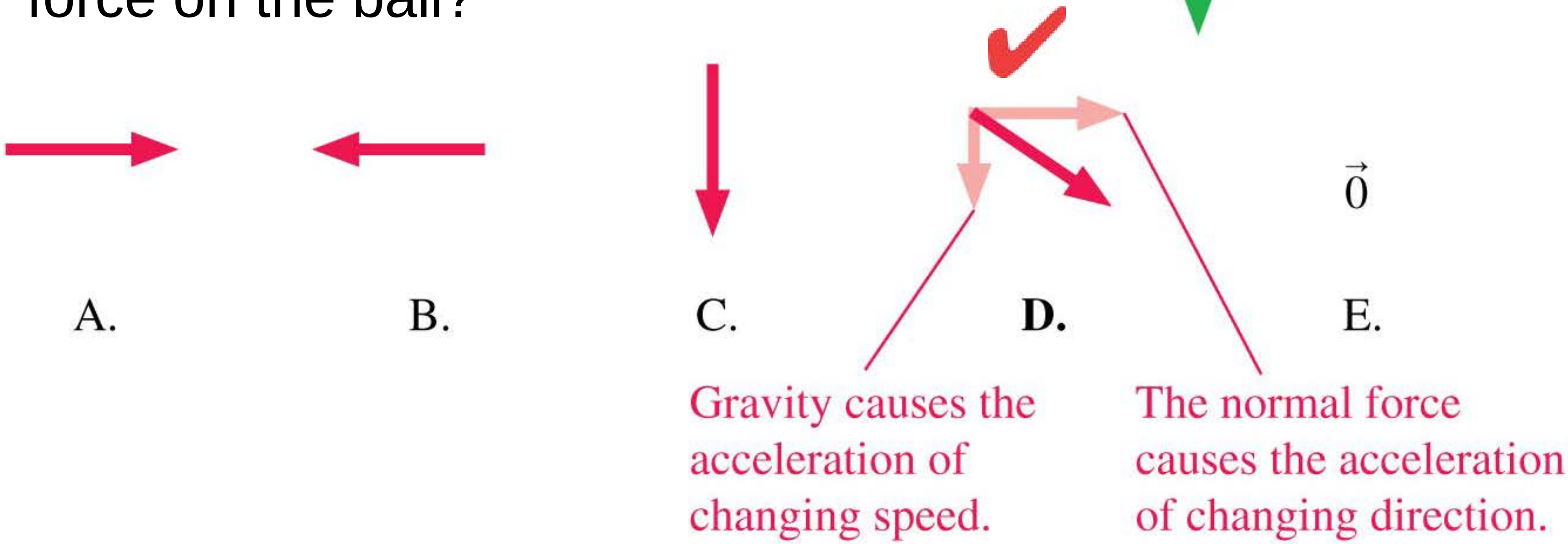


A. B. C. D. E.

QuickCheck 8.12

A ball rolls ccw around the inside of a horizontal pipe. The ball is fastest at the lowest point, slowest at the highest point. At the point shown, with the ball moving down, what is the direction of the net force on the ball?





Chapter 8 Summary Slides

General Principles

Newton's Second Law

Expressed in x- and y-component form:

$$(F_{\text{net}})_x = \sum F_x = ma_x$$

$$(F_{\text{net}})_y = \sum F_y = ma_y$$

Expressed in rtz-component form:

$$(F_{\text{net}})_r = \sum F_r = ma_r = \frac{mv_t^2}{r} = m\omega^2 r$$

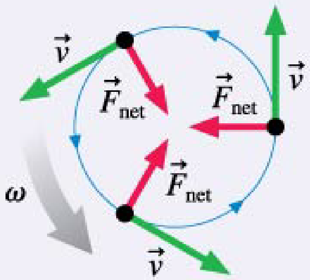
$$(F_{\text{net}})_t = \sum F_t = \begin{cases} 0 & \text{uniform circular motion} \\ ma_t & \text{nonuniform circular motion} \end{cases}$$

$$(F_{\text{net}})_z = \sum F_z = 0$$

General Principles

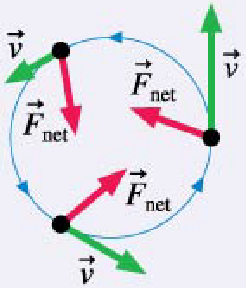
Uniform Circular Motion

- Speed is constant.
- $\vec{F}_{\text{net}}$ points toward the center of the circle.
- The **centripetal acceleration** $\vec{a}$ points toward the center of the circle. It changes the particle's direction but not its speed.



Nonuniform Circular Motion

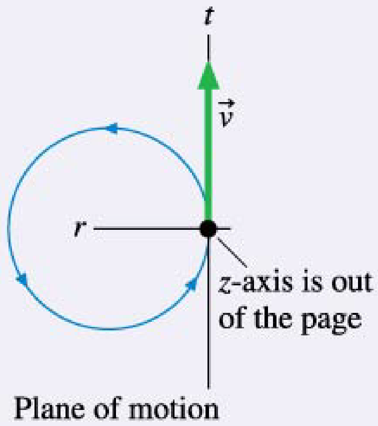
- Speed changes.
- $\vec{F}_{net}$ and $\vec{a}$ have both radial and tangential components.
- The radial component changes the particle's direction.
- The tangential component changes the particle's speed.



Important Concepts

rtz-coordinates

- The *r*-axis points toward the center of the circle.
- The *t*-axis is tangent, pointing counterclockwise.

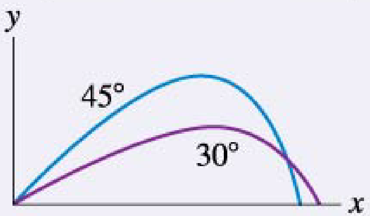


Important Concepts

Projectile motion

- With no drag, the *x*- and *y*-components of acceleration are independent. The trajectory is a parabola.
- With drag, the trajectory is not a parabola. Maximum range is achieved for an angle less than 45°.

Projectile motion with drag



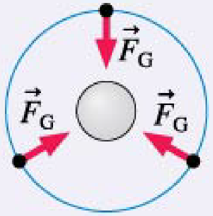
Applications

Orbits

An object acted on only by gravity has a circular orbit of radius r if its speed is

$$v = \sqrt{rg}$$

The object is in free fall.



Applications

Circular motion on surfaces

Circular motion requires a net force pointing to the center. n must be > 0 for the object to be in contact with a surface.