PHYSICS

FOR SCIENTISTS AND ENGINEERS **A STRATEGIC APPROACH** 4/E

Chapter 11 Lecture

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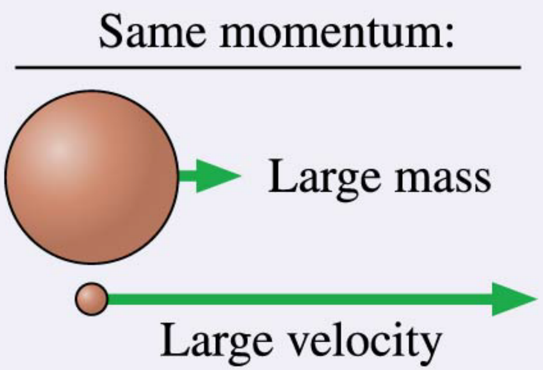
Chapter 11 Impulse and Momentum

IN THIS CHAPTER, you will learn to use the concepts of impulse and momentum.

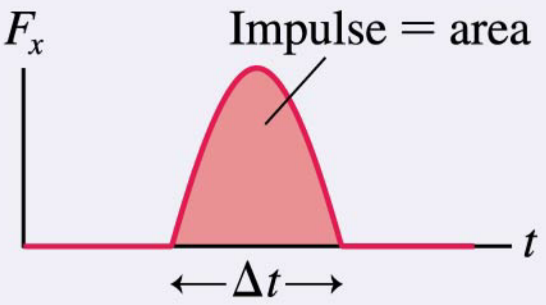
What is momentum?

An object's **momentum** is the product of its mass and velocity. An object can have a large momentum by having a large mass or a large velocity. **Momentum is a vector**, and it is especially important to pay attention to the *signs* of the components of momentum.



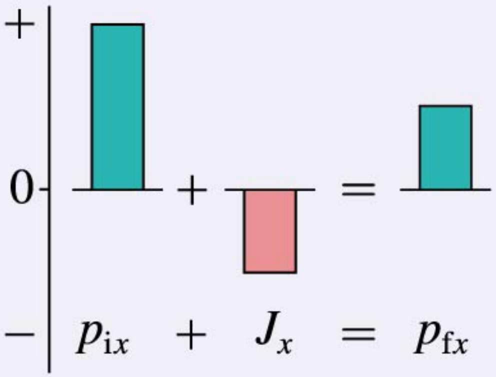
What is impulse?

A force of short duration is an impulsive force. The impulse J_x that this force delivers to an object is the area under the force-versus-time graph. For time-dependent forces, impulse and momentum are often more useful than Newton's laws.



How are impulse and momentum related?

Working with momentum is similar to working with energy. It's important to clearly define the system. The momentum principle says that a system's momentum changes when an impulse is delivered:

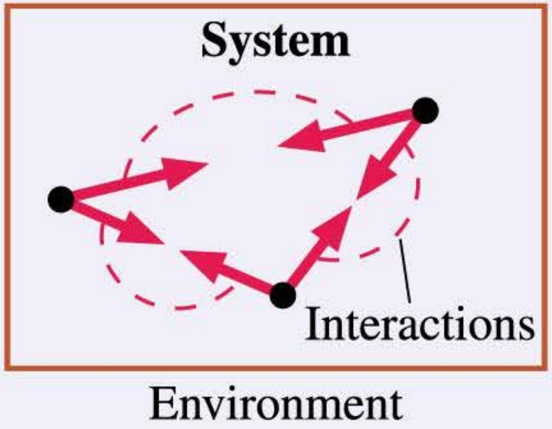


$$\Delta p_x = J_x$$

A momentum bar chart, similar to an energy bar chart, shows this principle graphically.

Is momentum conserved?

The total momentum of an isolated system is conserved. The particles of an isolated system interact with each other but not with the environment. Regardless of how intense the interactions are, the final momentum equals the initial momentum.

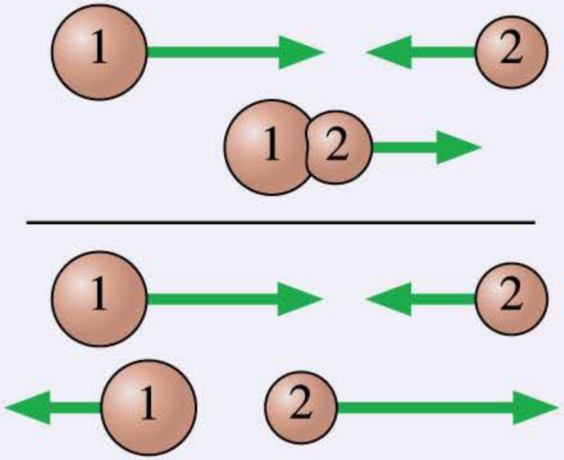


« LOOKING BACK Section 10.4 Energy conservation

How does momentum apply to collisions?

One important application of momentum conservation is the study of collisions.

- In a totally inelastic collision, the objects stick together. Momentum is conserved.
- In a perfectly elastic collision, the objects bounce apart. Both momentum and energy are conserved.

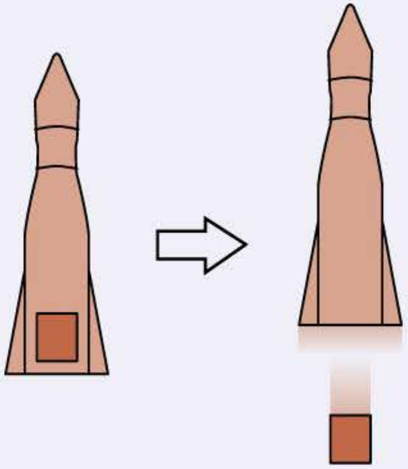


Where else is momentum used?

This chapter looks at two other important applications of momentum conservation:

- An explosion is a short interaction that drives two or more objects apart.

- In rocket propulsion the object's mass is changing continuously.



Chapter 11 Reading Questions

Reading Question 11.1

Momentum is

A. Mass times velocity.
B. ½ mass times speed-squared.
C. The area under the force curve in a force-versus-time graph.
D. Velocity per unit mass.

Reading Question 11.1

Momentum is

A. **Mass times velocity.**
B. ½ mass times speed-squared.
C. The area under the force curve in a force-versus-time graph.
D. Velocity per unit mass.

Reading Question 11.2

Impulse is

A. A force that is applied at a random time.
B. A force that is applied very suddenly.
C. The area under the force curve in a force-versus-time graph.
D. The time interval that a force lasts.

Reading Question 11.2

Impulse is

A. A force that is applied at a random time.
B. A force that is applied very suddenly.
C. The area under the force curve in a force-versus-time graph.
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Reading Question 11.3

A method for “momentum accounting,” introduced in this chapter, is

A. Credit-debit tables.
B. Impulse-versus-time graphs.
C. Momentum bar charts.
D. Momentum conservation pools.
E. Momentum spreadsheets.

Reading Question 11.3

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A. Credit-debit tables.
B. Impulse-versus-time graphs.
C. Momentum bar charts.
D. Momentum conservation pools.
E. Momentum spreadsheets.

Reading Question 11.4

The total momentum of a system is conserved

A. Always.
B. If the system is isolated.
C. If the forces are conservative.
D. Never; it's just an approximation.

Reading Question 11.4

The total momentum of a system is conserved

A. Always.
✔**B. If the system is isolated.**
C. If the forces are conservative.
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Reading Question 11.5

In an *inelastic collision*,

A. Impulse is conserved.
B. Momentum is conserved.
C. Force is conserved.
D. Energy is conserved.
E. Elasticity is conserved.

Reading Question 11.5

In an *inelastic collision*,

A. Impulse is conserved.
B. Momentum is conserved.
C. Force is conserved.
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Reading Question 11.6

A *perfectly elastic collision* is a collision

A. Between two springs.
B. That conserves thermal energy.
C. That conserves kinetic energy.
D. That conserves potential energy.
E. That conserves mechanical energy.

Reading Question 11.6

A *perfectly elastic collision* is a collision

A. Between two springs.
B. That conserves thermal energy.
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D. That conserves potential energy.
✔ **E. That conserves mechanical energy.**

Chapter 11 Content, Examples, and QuickCheck Questions

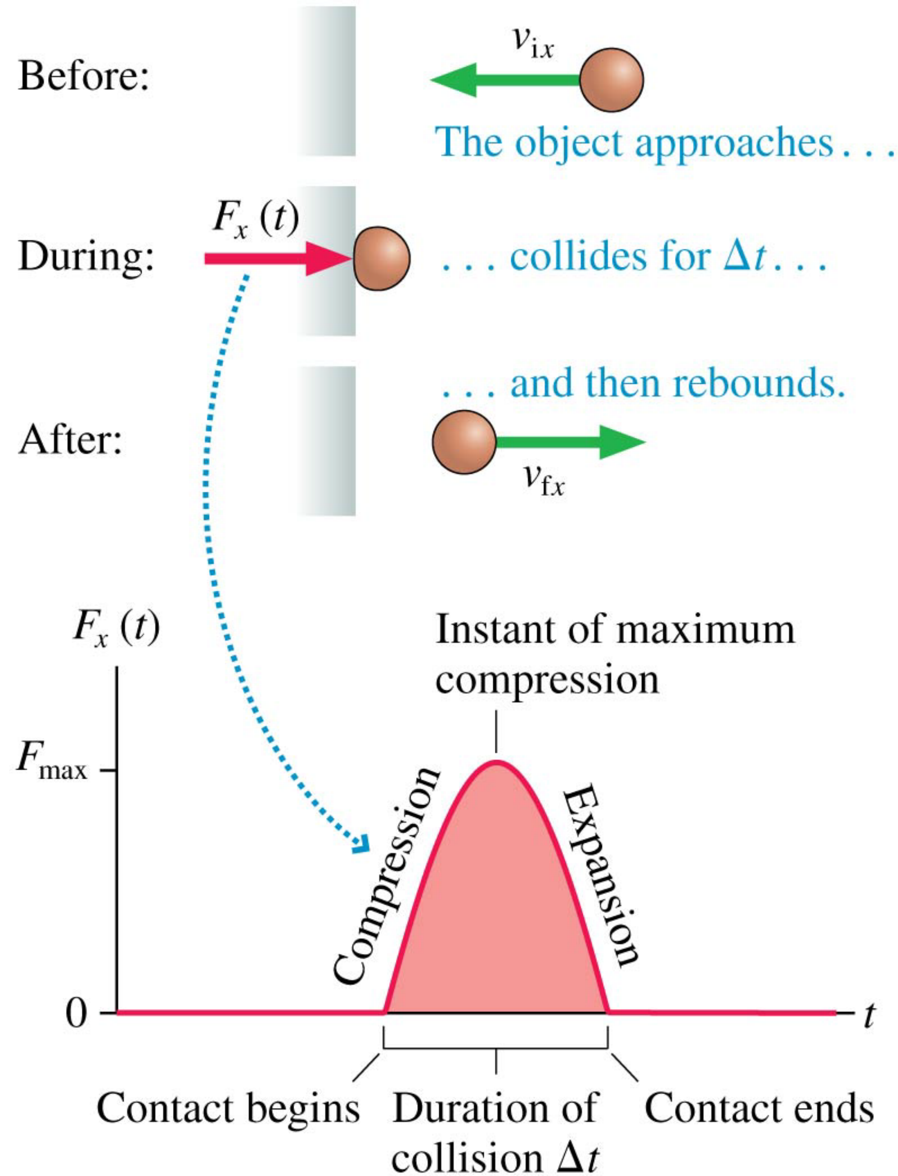
Collisions

- A **collision** is a short-duration interaction between two objects.
- The collision between a tennis ball and a racket is quick, but it is *not* instantaneous.

- Notice that the left side of the ball is flattened.
- It takes time to compress the ball, and more time for the ball to re-expand as it leaves the racket.

Impulse During a Collision

- A large force exerted for a small interval of time is called an **impulsive force**.
- The figure shows a particle with initial velocity in the −x-direction
- The particle experiences an impulsive force of short duration Δt.
- The particle leaves with final velocity in the +x-direction.

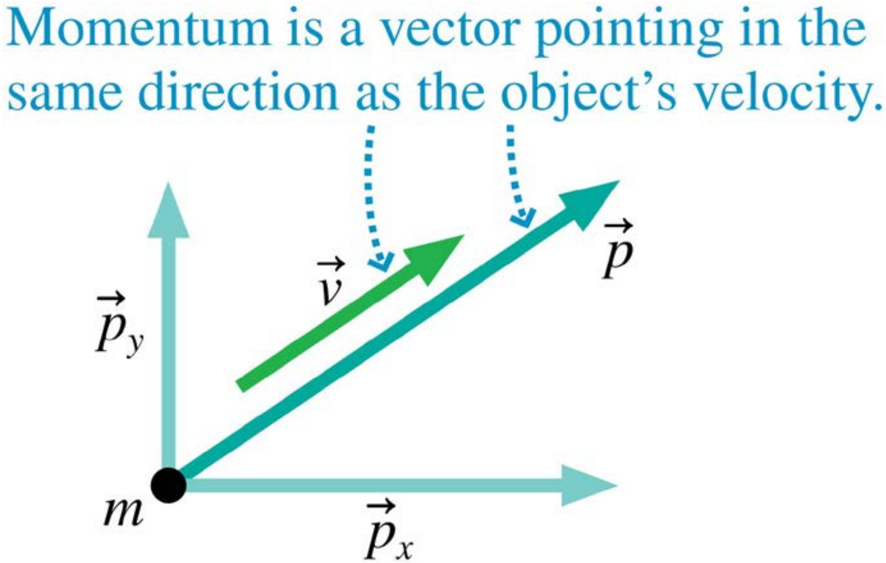


Momentum

- The product of a particle's mass and velocity is called the *momentum* of the particle:

$$\textbf{momentum} = \vec{p} \equiv m\vec{v}$$

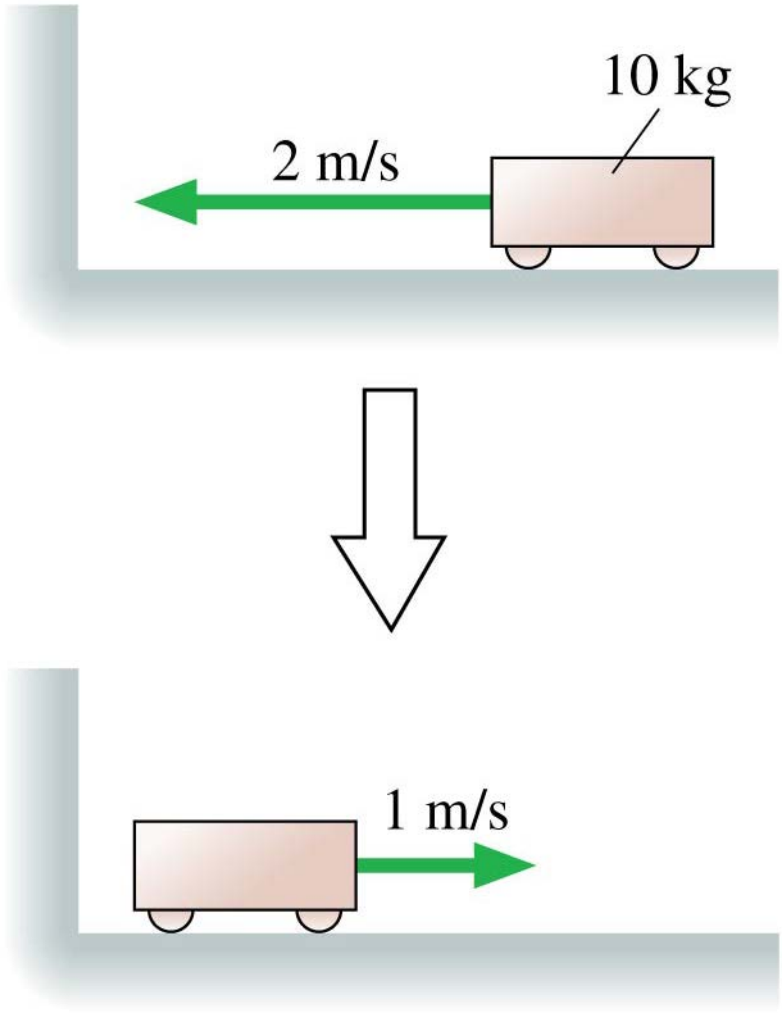
- Momentum is a vector, with units of kg m/s.
- A particle's momentum vector can be decomposed into x- and y-components.



QuickCheck 11.1

The cart's change of momentum Δp_x is

A. -20 kg m/s
B. -10 kg m/s
C. 0 kg m/s
D. 10 kg m/s
E. 30 kg m/s



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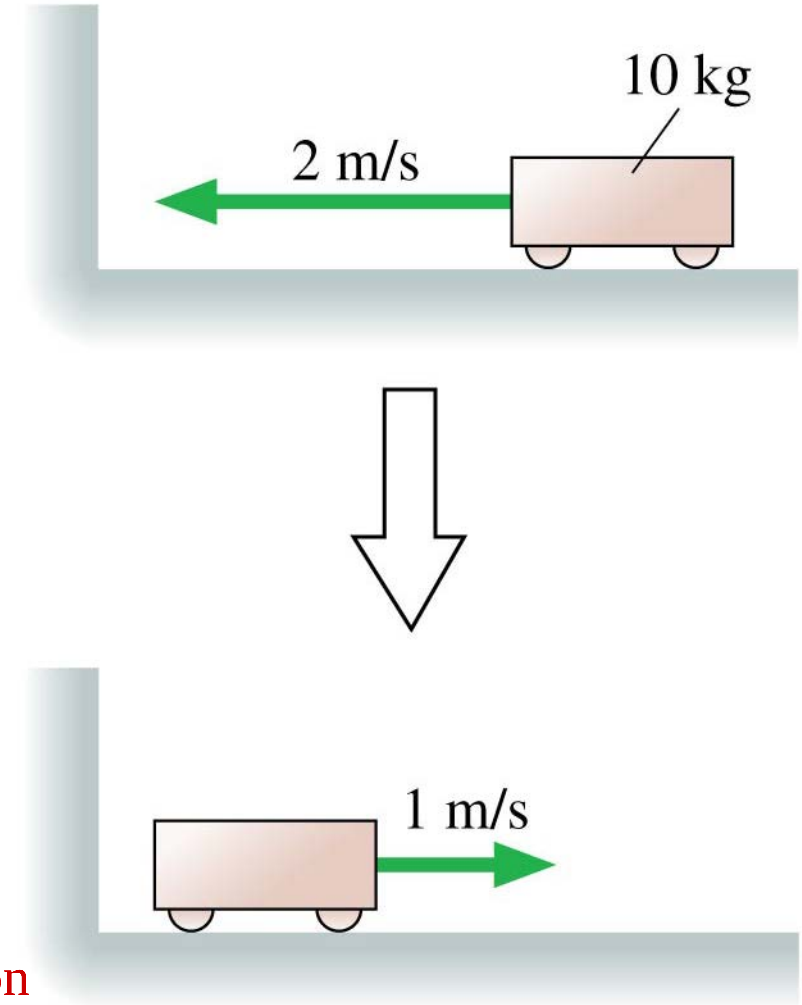


A. –20 kg m/s

B. –10 kg m/s

C. 0 kg m/s

D. 10 kg m/s

✔ **E. 30 kg m/s**

$\Delta p_x = 10 \text{ kg m/s} - (-20 \text{ kg m/s}) = 30 \text{ kg m/s}$

Negative initial momentum because motion is to the left and $v_x < 0$.

Impulse

- Newton's second law may be formulated in terms of momentum rather than acceleration:

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}$$

- Rearranging, and integrating over time, we have

$$\Delta p_x = p_{\mathrm{f}x} - p_{\mathrm{i}x} = \int_{t_\mathrm{i}}^{t_\mathrm{f}} F_x(t)\,dt$$

- We define the right-hand side to be the *impulse*:

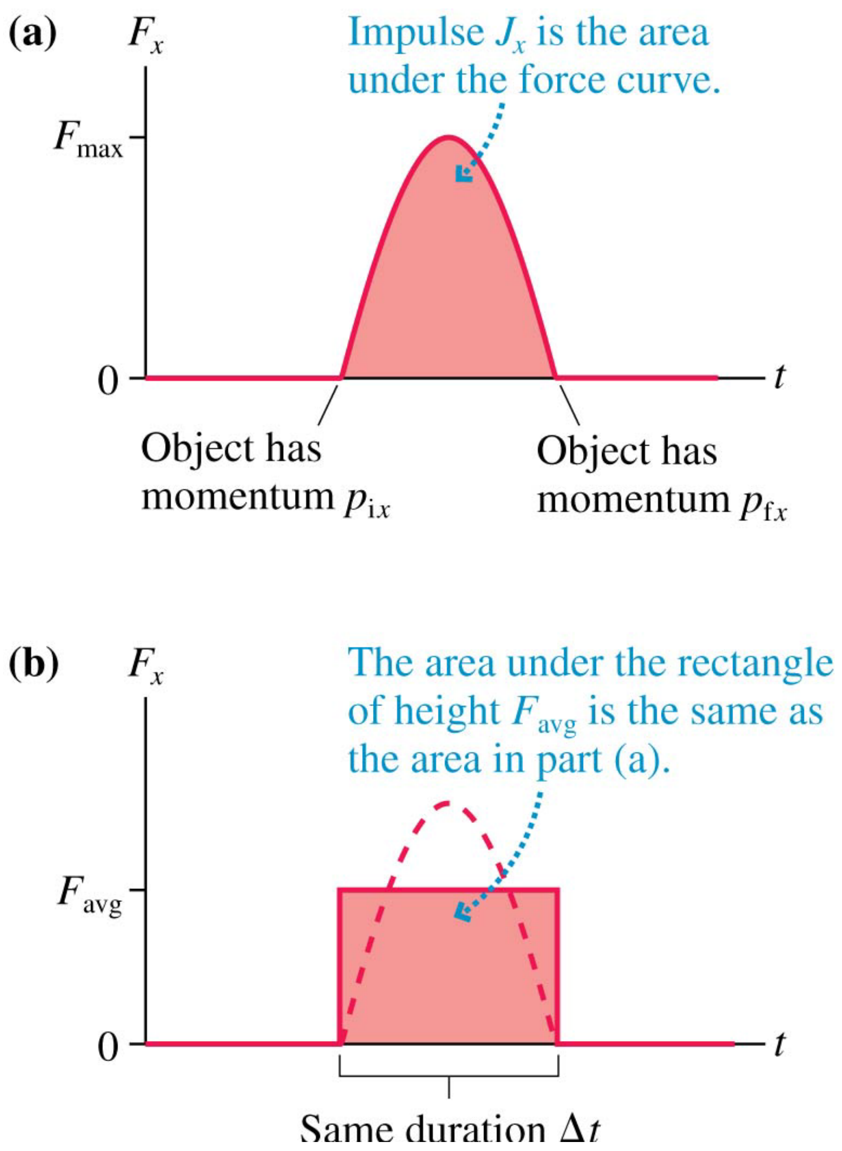
$$\textbf{impulse} = J_x \equiv \int_{t_\mathrm{i}}^{t_\mathrm{f}} F_x(t)\,dt$$
$$= \text{area under the } F_x(t) \text{ curve between } t_\mathrm{i} \text{ and } t_\mathrm{f}$$

- Impulse has units of N s, which are equivalent to kg m/s.

Impulse

- Figure (a) portrays the impulse graphically.
- Figure (b) shows that the average force F_{avg} is the height of a rectangle that has the same impulse as the real force curve.
- The impulse exerted during the collision is

$$J_x = F_{avg}\,\Delta t$$



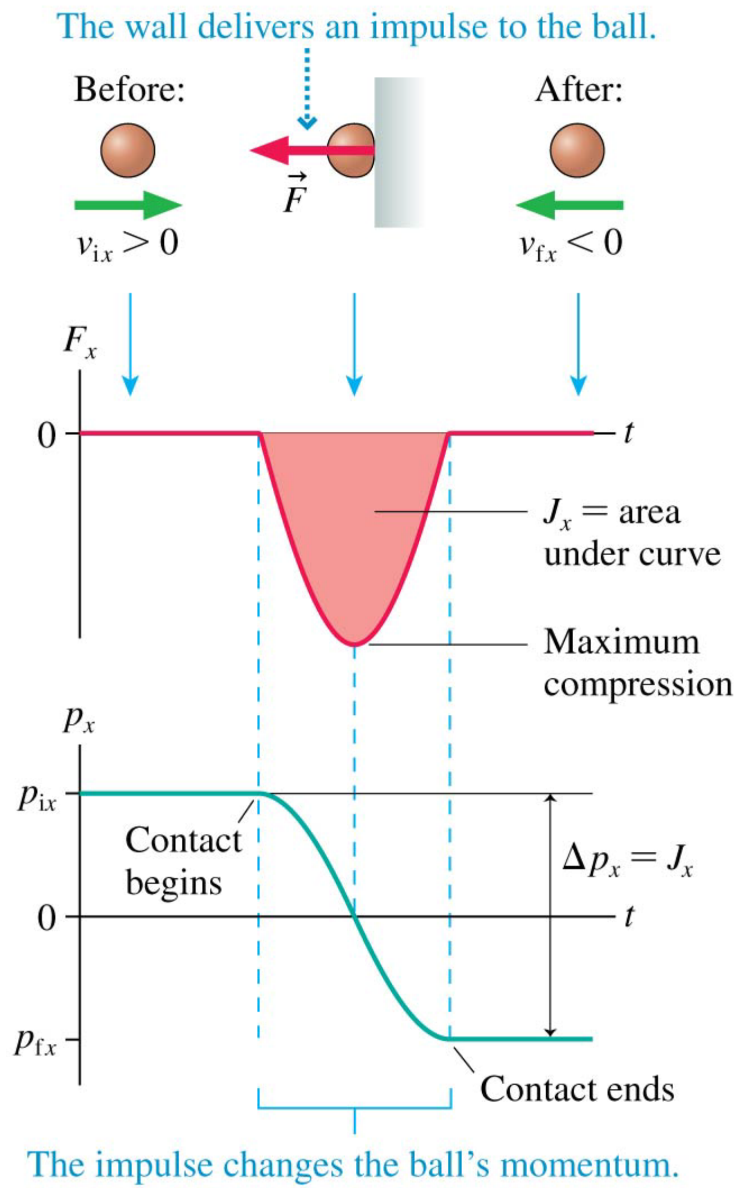
The Momentum Principle

- A particle experiences an impulsive force in the x-direction.
- The impulse delivered to the particle is equal to the change in the particle's momentum.

$$\Delta p_x = J_x \qquad \text{(momentum principle)}$$

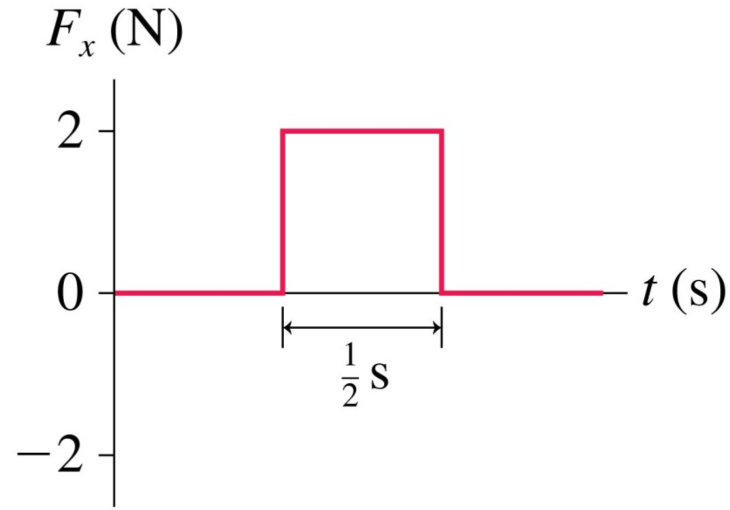
The Momentum Principle

- A rubber ball bounces off a wall.
- The ball is initially traveling toward the right, so v_{ix} and p_{ix} are positive.
- After the bounce, v_{fx} and p_{fx} are negative.
- The force *on the ball* is toward the left, so F_x is negative.
- In this example, the impulse, or area under the force curve, has a negative value.



QuickCheck 11.2

A 2.0 kg object moving to the right with speed 0.50 m/s experiences the force shown. What are the object's speed and direction after the force ends?



A. 0.50 m/s left

B. At rest

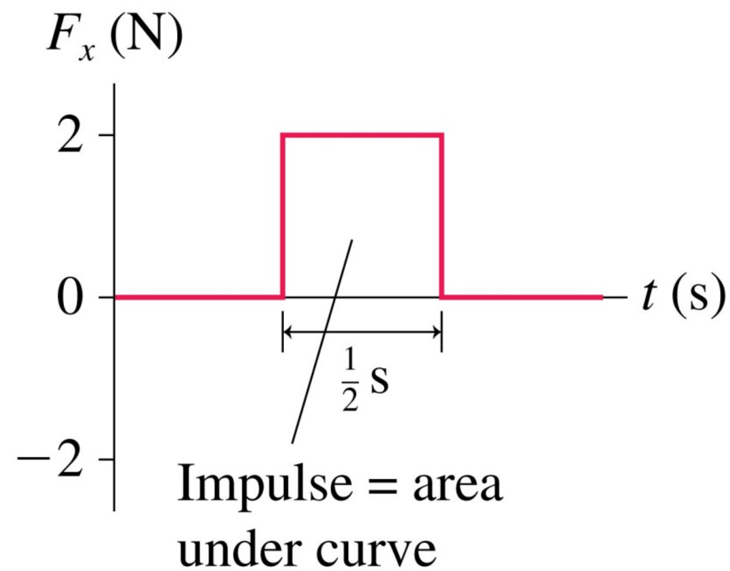
C. 0.50 m/s right

D. 1.0 m/s right

E. 2.0 m/s right

QuickCheck 11.2

A $2.0\ \mathrm{kg}$ object moving to the right with speed $0.50\ \mathrm{m/s}$ experiences the force shown. What are the object's speed and direction after the force ends?



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B. At rest

C. $0.50\ \mathrm{m/s}$ right

D. $1.0\ \mathrm{m/s}$ right

E. $2.0\ \mathrm{m/s}$ right

$\Delta p_x = J_x$ or $p_{\mathrm{fx}} = p_{\mathrm{ix}} + J_x$

QuickCheck 11.3

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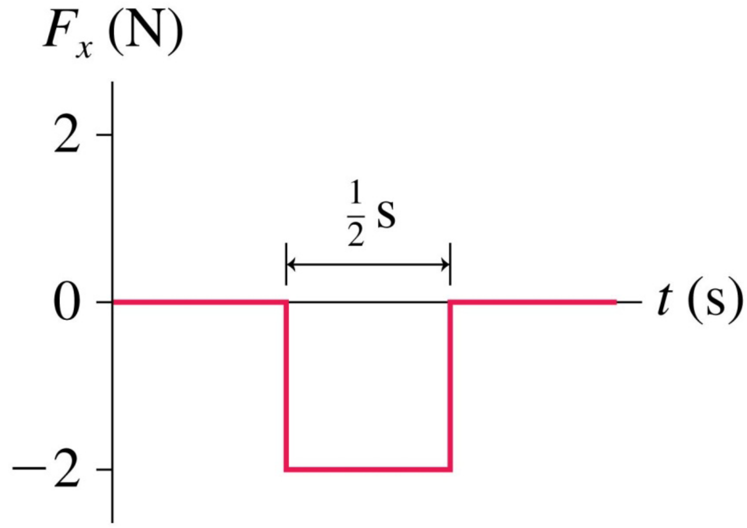

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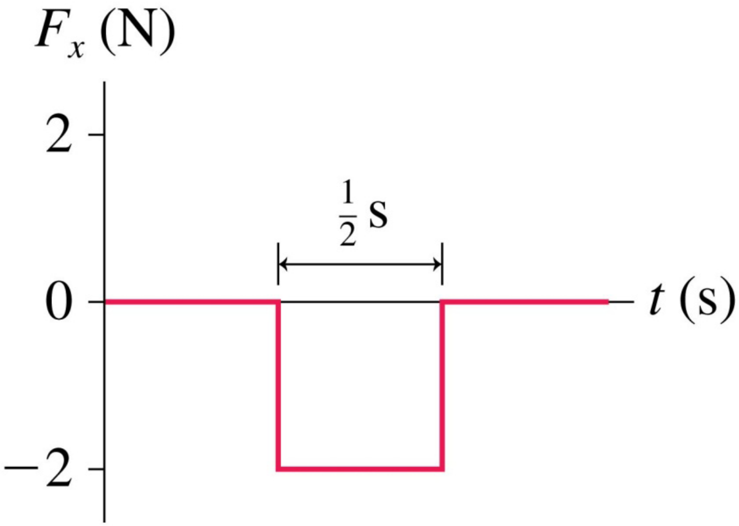

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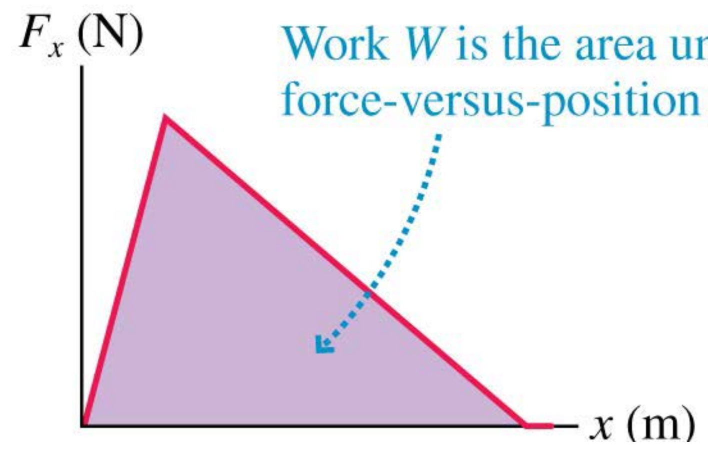
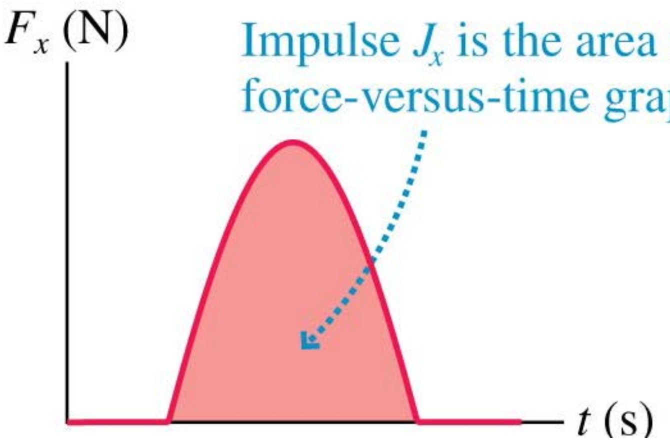
An Analogy with the Energy Principle





- When an object is acted on by a force, in general this force *both* does work on the object *and* creates an impulse on the object:

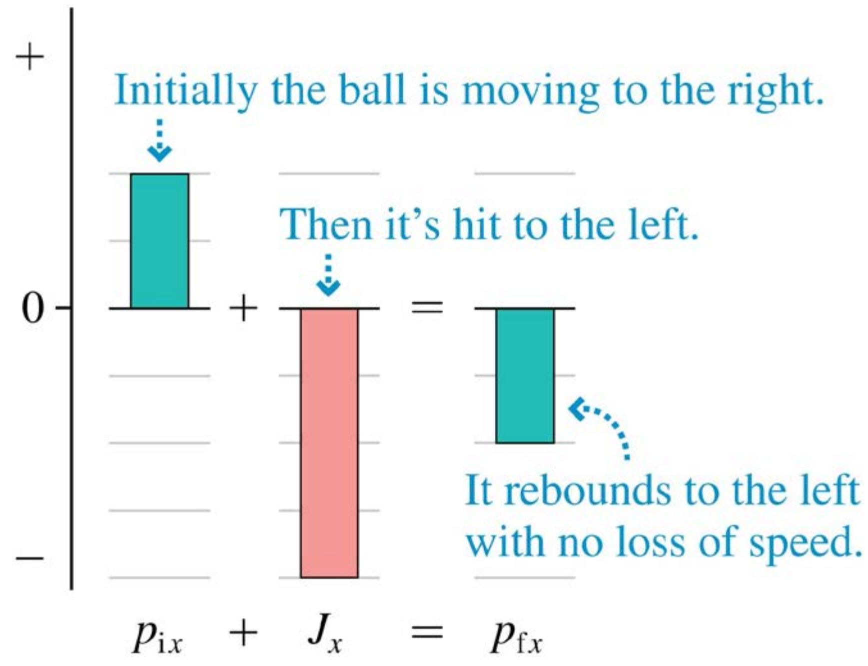
energy principle: $$\Delta K = W = \int_{x_i}^{x_f} F_x\,dx$$

momentum principle: $$\Delta p_x = J_x = \int_{t_i}^{t_f} F_x\,dt$$

- Whether you use the energy principle or the momentum principle depends on the question you are trying to answer.

Momentum Bar Charts

- A rubber ball is initially moving to the right with $p_{ix} = +2$ kg m/s.
- It collides with a wall, which delivers an impulse of $J_x = -4$ N s.
- The figure shows the **momentum bar chart** to help analyze this collision.
- The final momentum is $p_{fx} = -2$ kg m/s.



QuickCheck 11.4

A force pushes the cart for $1\ \mathrm{s}$, starting from rest. To achieve the same speed with a force half as big, the force would need to push for

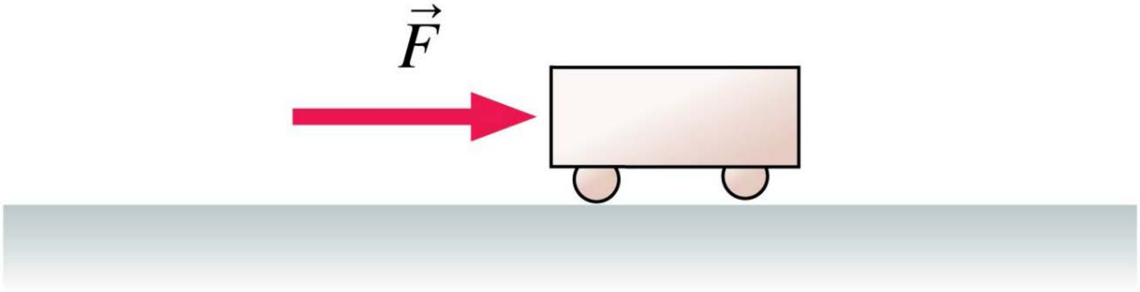
A. $\frac{1}{4}\ \mathrm{s}$

B. $\frac{1}{2}\ \mathrm{s}$

C. $1\ \mathrm{s}$

D. $2\ \mathrm{s}$

E. $4\ \mathrm{s}$



QuickCheck 11.4

A force pushes the cart for 1 s, starting from rest. To achieve the same speed with a force half as big, the force would need to push for

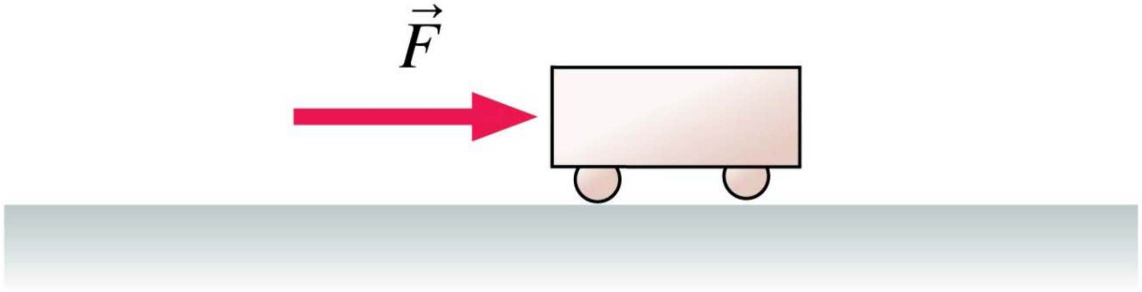
A. $\frac{1}{4}$ s

B. $\frac{1}{2}$ s

C. 1 s

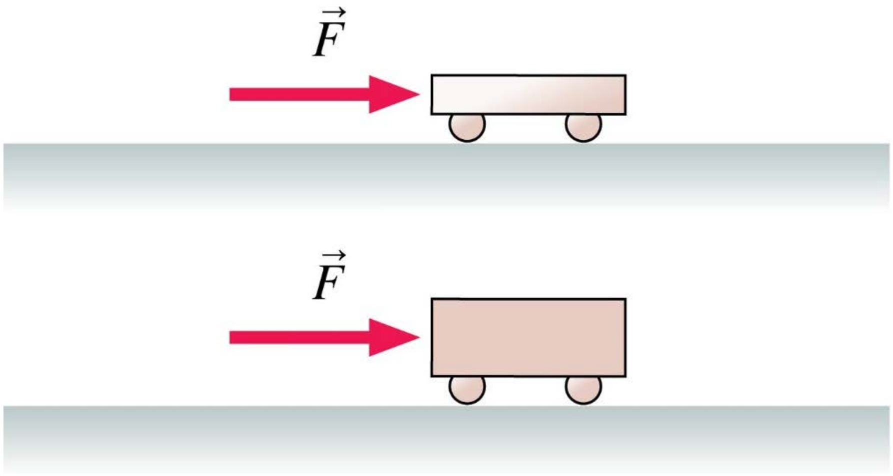
D. 2 s

E. 4 s



QuickCheck 11.5

A light plastic cart and a heavy steel cart are both pushed with the same force for $1.0\ \mathrm{s}$, starting from rest. After the force is removed, the momentum of the light plastic cart is _________ that of the heavy steel cart.



A. greater than

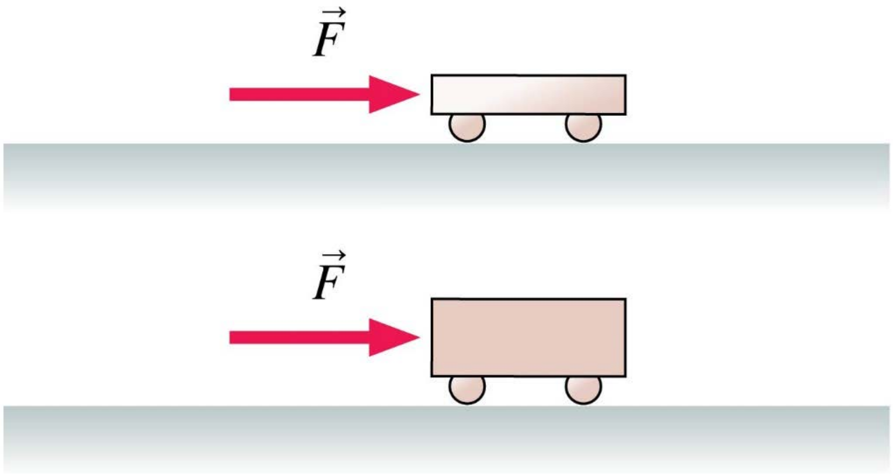
B. equal to

C. less than

D. Can’t say. It depends on how big the force is.

QuickCheck 11.5

A light plastic cart and a heavy steel cart are both pushed with the same force for $1.0\ \mathrm{s}$, starting from rest. After the force is removed, the momentum of the light plastic cart is _________ that of the heavy steel cart.



A. greater than

B. equal to

C. less than

D. Can't say. It depends on how big the force is.

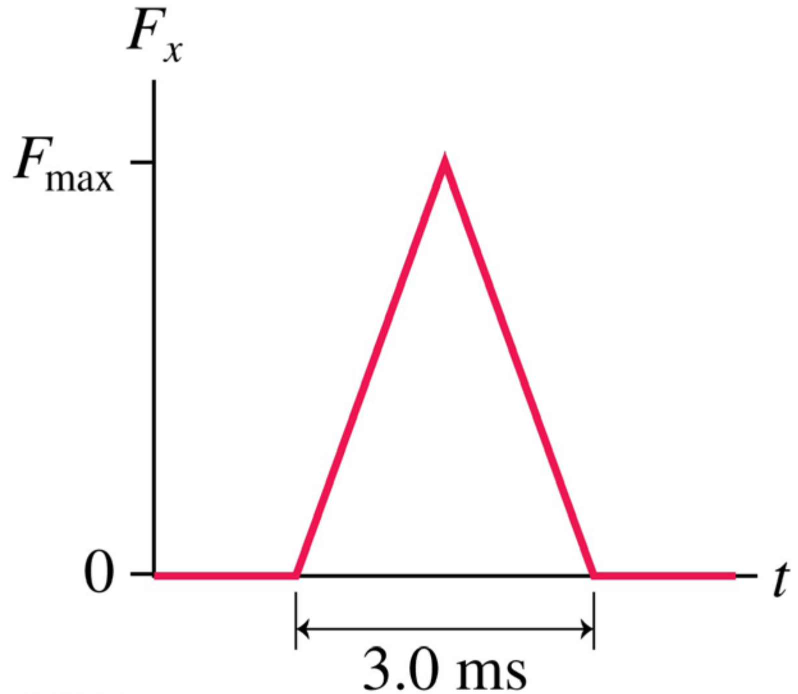
Same force, same time → same impulse
Same impulse → same change of momentum

Example 11.1 Hitting a Baseball

EXAMPLE 11.1 | **Hitting a baseball**

A 150 g baseball is thrown with a speed of It is hit straight back toward the pitcher at a speed of The interaction force between the ball and the bat is shown in **FIGURE 11.7**. What *maximum* force does the bat exert on the ball? What is the *average* force of the bat on the ball?

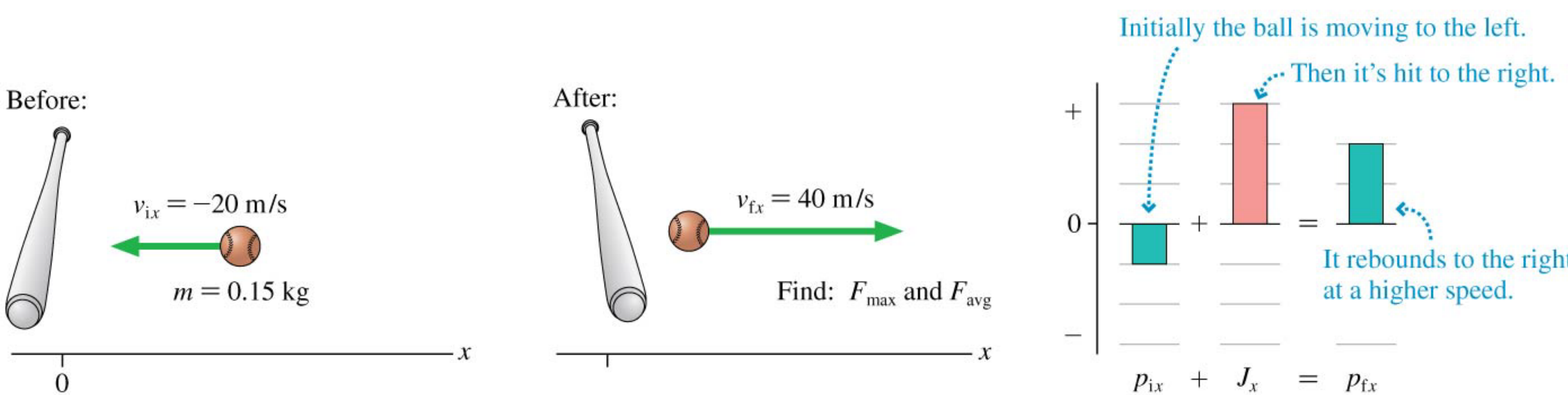
MODEL Model the baseball as an elastic object and the interaction as a collision.



Example 11.1 Hitting a Baseball

EXAMPLE 11.1 | **Hitting a baseball**

VISUALIZE **FIGURE 11.8** is a before-and-after pictorial representation. Because F_x is positive (a force to the right), we know the ball was initially moving toward the left and is hit back toward the right. Thus we converted the statements about *speeds* into information about *velocities*, with v_{ix} negative.



Example 11.1 Hitting a Baseball

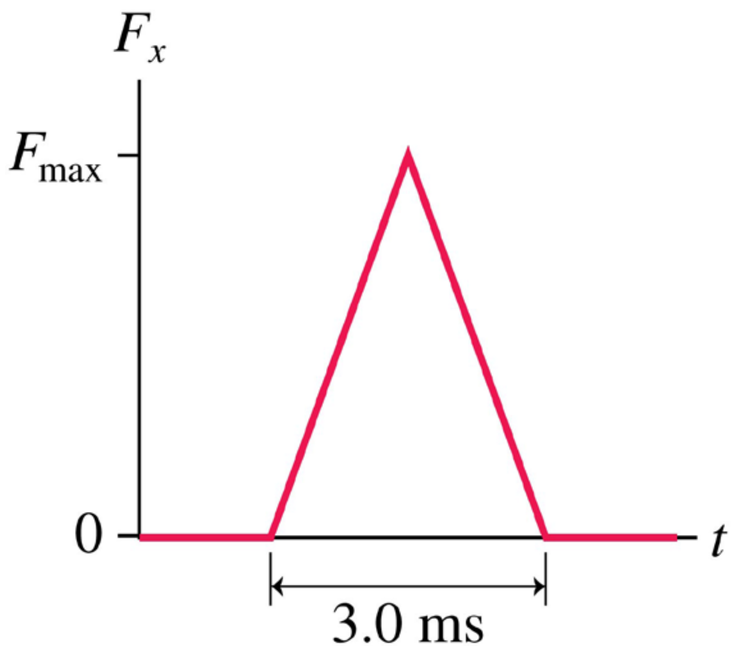
EXAMPLE 11.1 | **Hitting a baseball**

SOLVE The momentum principle is

$$\Delta p_x = J_x = \text{area under the force curve}$$

We know the velocities before and after the collision, so we can calculate the ball's momenta:

$$p_{ix} = mv_{ix} = (0.15\text{ kg})(-20\text{ m/s}) = -3.0\text{ kg m/s}$$
$$p_{fx} = mv_{fx} = (0.15\text{ kg})(40\text{ m/s}) = 6.0\text{ kg m/s}$$



Example 11.1 Hitting a Baseball

EXAMPLE 11.1 | **Hitting a baseball**

SOLVE Thus the *change* in momentum is

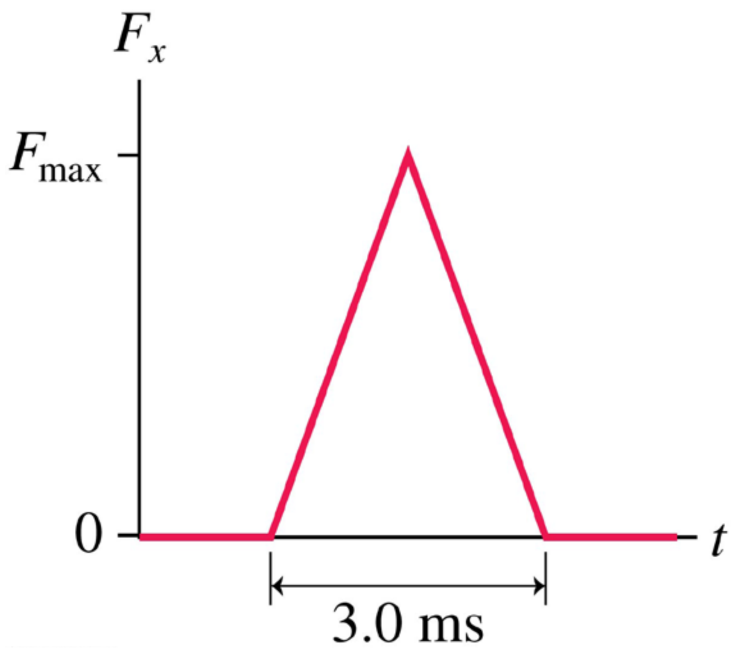
$$\Delta p_x = p_{fx} - p_{ix} = 9.0 \text{ kg m/s}$$

The force curve is a triangle with height F_{max} and width 3.0 ms. The area under the curve is

$$J_x = \text{area} = \tfrac{1}{2}(F_{max})(0.0030 \text{ s}) = (F_{max})(0.0015 \text{ s})$$

Using this information in the momentum principle, we have

$$9.0 \text{ kg m/s} = (F_{max})(0.0015 \text{ s})$$



Example 11.1 Hitting a Baseball

EXAMPLE 11.1 Hitting a baseball

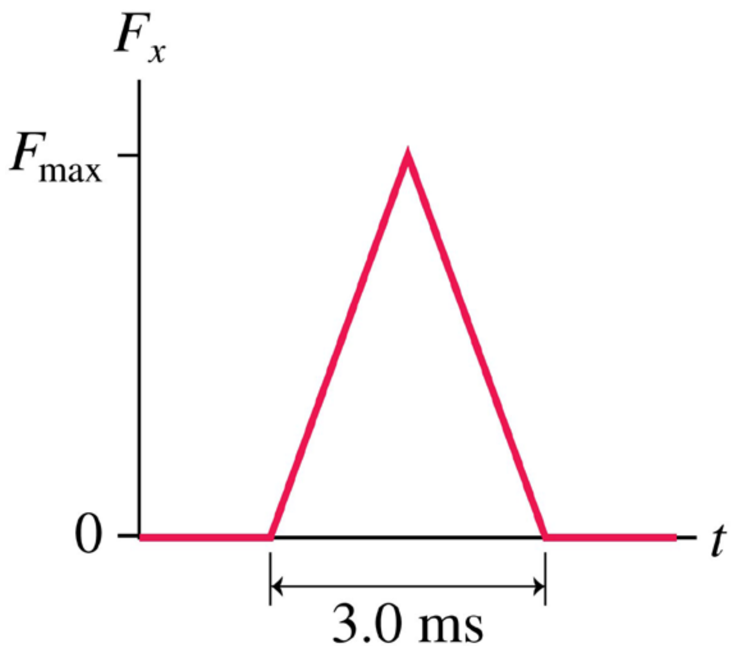
SOLVE Thus the *maximum* force is

$$F_{\text{max}} = \frac{9.0 \text{ kg m/s}}{0.0015 \text{ s}} = 6000 \text{ N}$$

The *average* force, which depends on the collision duration $\Delta t = 0.0030$ s, has the smaller value:

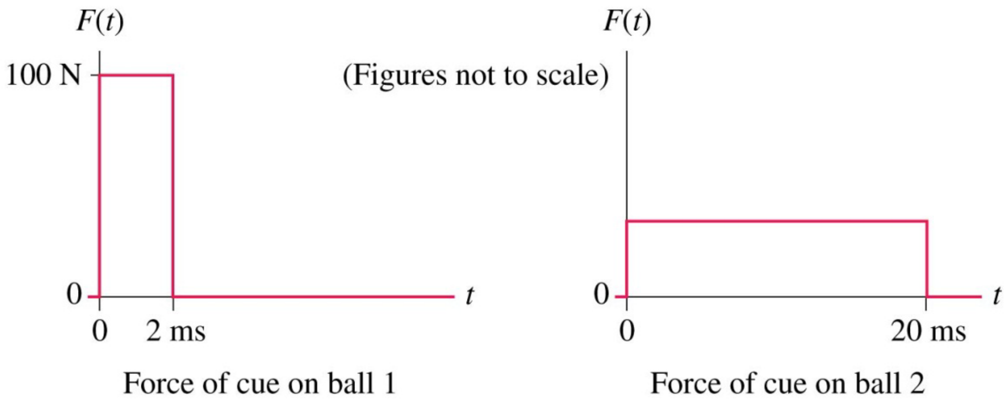
$$F_{\text{avg}} = \frac{J_x}{\Delta t} = \frac{\Delta p_x}{\Delta t} = \frac{9.0 \text{ kg m/s}}{0.0030 \text{ s}} = 3000 \text{ N}$$

ASSESS F_{max} is a large force, but quite typical of the impulsive forces during collisions. The main thing to focus on is our new perspective: An impulse changes the momentum of an object.



QuickCheck 11.6

Two $1.0\ \mathrm{kg}$ stationary cue balls are struck by cue sticks. The cues exert the forces shown. Which ball has the greater final speed?



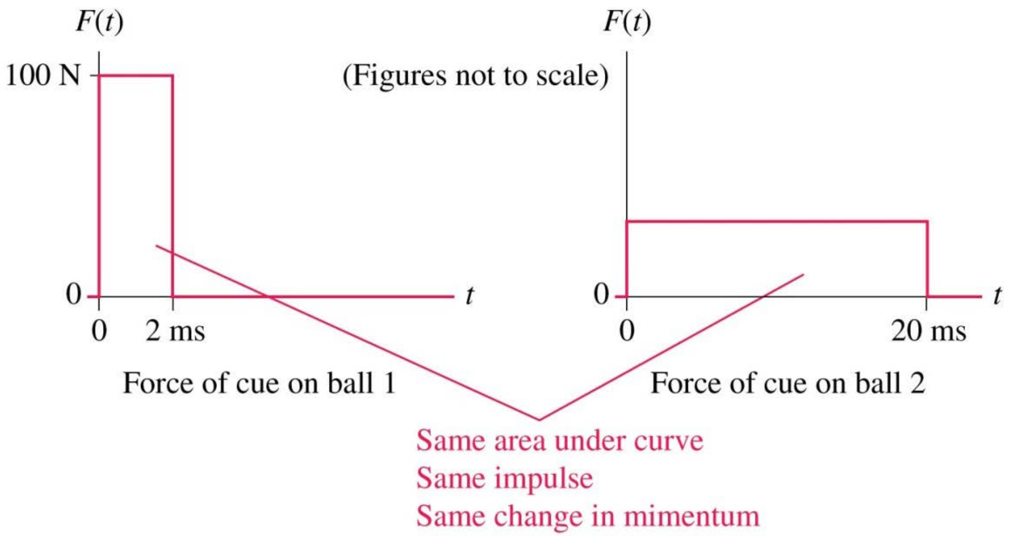
A. Ball 1

B. Ball 2

C. Both balls have the same final speed.

QuickCheck 11.6

Two 1.0 kg stationary cue balls are struck by cue sticks. The cues exert the forces shown. Which ball has the greater final speed?



A. Ball 1

B. Ball 2

✔ **C. Both balls have the same final speed.**

QuickCheck 11.7

You awake in the night to find that your living room is on fire. Your one chance to save yourself is to throw something that will hit the back of your bedroom door and close it, giving you a few seconds to escape out the window. You happen to have both a sticky ball of clay and a super-bouncy Superball next to your bed, both the same size and same mass. You've only time to throw one. Which will it be? Your life depends on making the right choice!

A. Throw the Superball.

B. Throw the ball of clay.

C. It doesn't matter. Throw either.

QuickCheck 11.7

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✔ **A. Throw the Superball.** Larger $\Delta p \rightarrow$ more impulse to door

B. Throw the ball of clay.

C. It doesn't matter. Throw either.

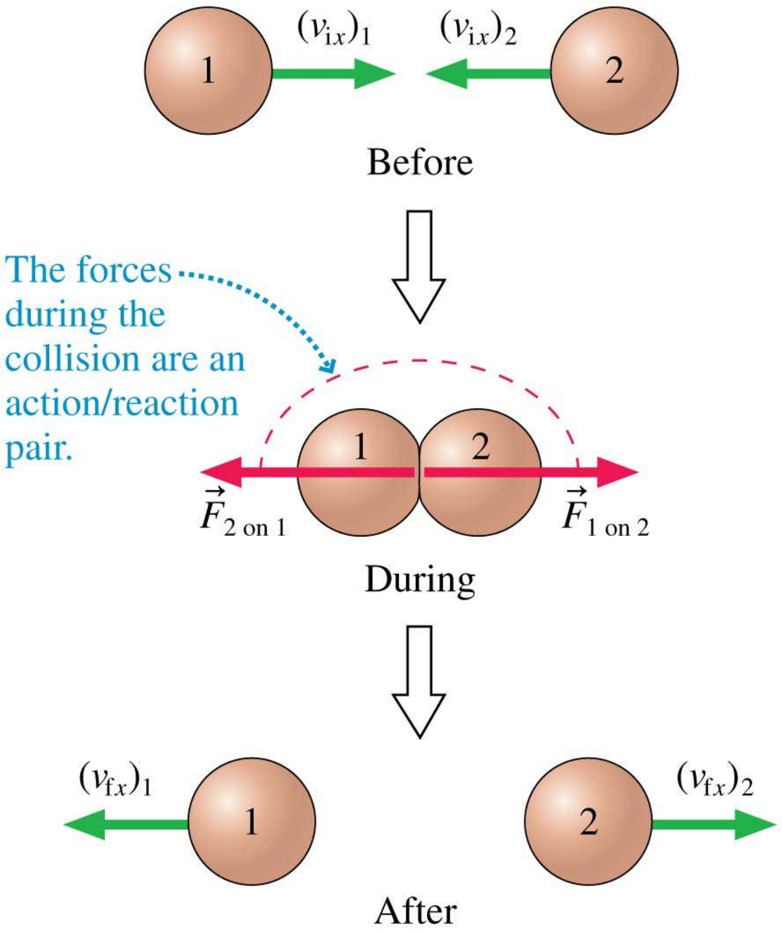
Conservation of Momentum

- Two objects collide, as shown.
- Neglect all outside forces on the objects.
- Due to the fact that the only forces on the objects are equal and opposite, the sum of their momenta is

$$(p_x)_1 + (p_x)_2 = \text{constant}$$

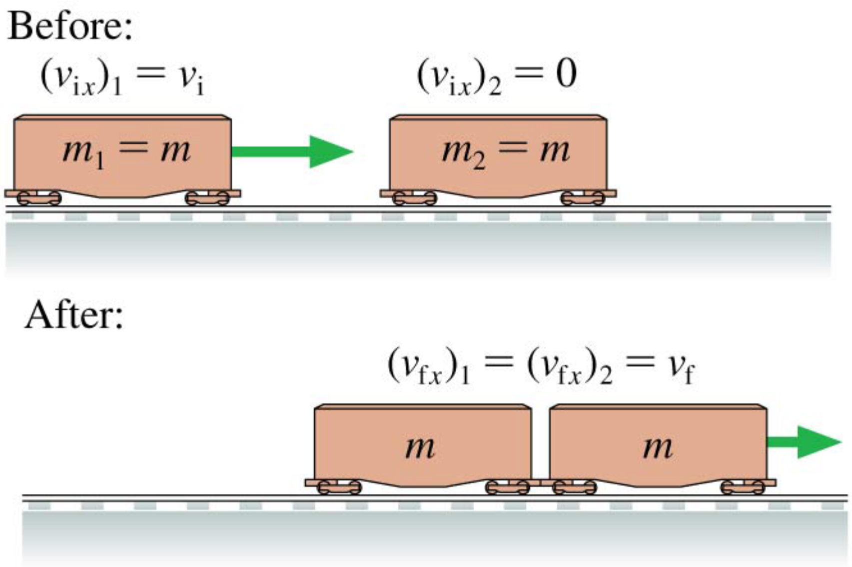
- This is a conservation law!
- The sum of the momenta *before* and *after* the collision are equal:

$$(p_{fx})_1 + (p_{fx})_2 = (p_{ix})_1 + (p_{ix})_2$$



Conservation of Momentum: Quick Example

A train car moves to the right with initial speed v_i. It collides with a stationary train car of equal mass. After the collision the two cars are stuck together. What is the train cars' final velocity?



- According to conservation of momentum, before and after the collision

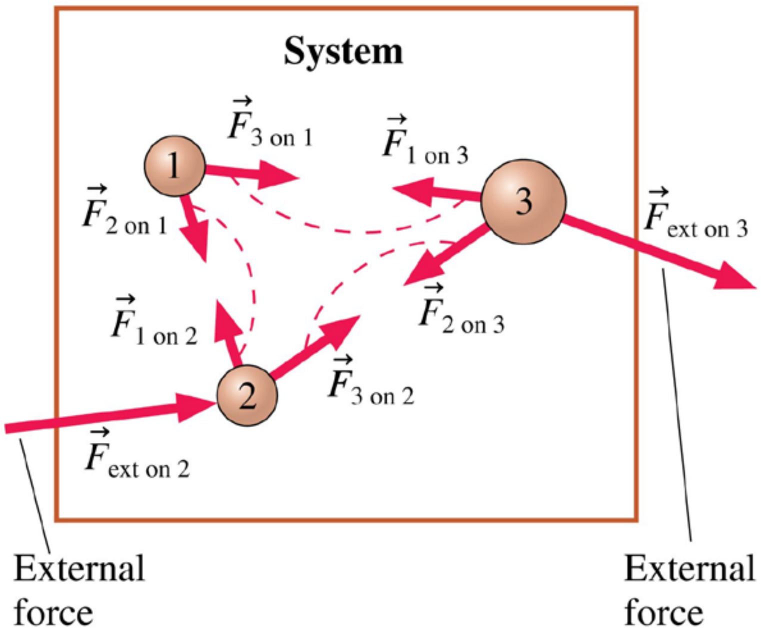
$$m_1\,(v_{fx})_1 + m_2\,(v_{fx})_2 = m_1\,(v_{ix})_1 + m_2\,(v_{ix})_2$$

$$mv_f + mv_f = 2mv_f = mv_i + 0$$

- The mass cancels, and we find that the final velocity is $v_f = \frac{1}{2}\,v_i$.

Systems of Particles

- Consider a system of N interacting particles.
- The figure shows a simple case where $N = 3$.
- The system has a **total momentum**:



$$\vec{P} = \text{total momentum} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots + \vec{p}_N = \sum_{k=1}^{N} \vec{p}_k$$

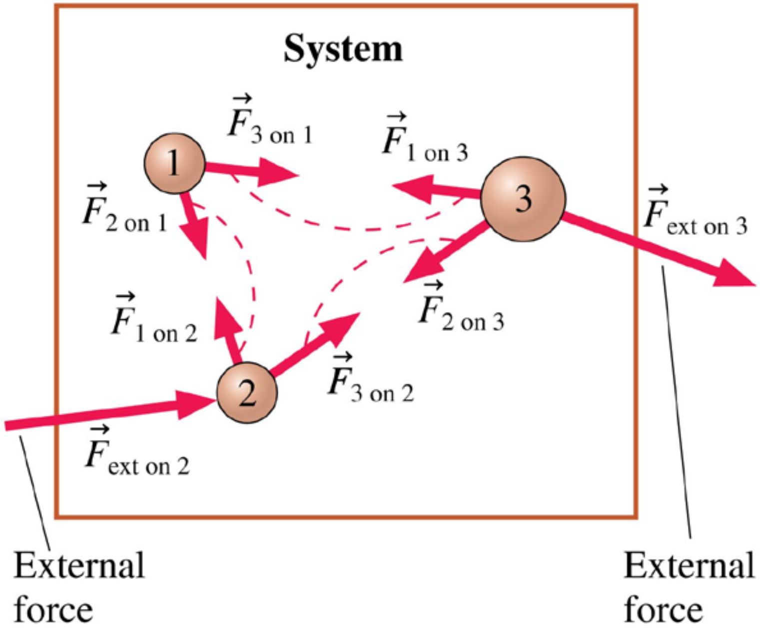
- Applying Newton's second law for each individual particle, we find the rate of change of the total momentum of the system is

$$\frac{d\vec{P}}{dt} = \sum_{k} \sum_{j \neq k} \vec{F}_{j \text{ on } k} + \sum_{k} \vec{F}_{\text{ext on } k}$$

Momentum of a System

- The interaction forces come in action/reaction pairs, with $\vec{F}_{k \text{ on } j} = -\vec{F}_{j \text{ on } k}$
- Consequently, the sum of all the interaction forces is zero.
- Therefore:

$$\frac{d\vec{P}}{dt} = \sum_k \vec{F}_{\text{ext on } k} = \vec{F}_{\text{net}}$$



- **The rate of change of the total momentum of the system is equal to the net force applied to the system.**
- This result *justifies* our particle model: Internal forces between atoms in an object do not affect the motion of the object as a whole.

Law of Conservation of Momentum

- For an isolated system:

$$\frac{d\vec{P}}{dt} = \vec{0} \qquad \text{(isolated system)}$$

- The total momentum of the rocket + gases system is conserved, so the rocket accelerates forward as the gases are expelled backward.

Law of Conservation of Momentum

- An **isolated system** is a system for which the *net* external force is zero:

$$\vec{F}_{\text{net}} = \vec{0}$$

- For an isolated system:

$$\frac{d\vec{P}}{dt} = \vec{0} \qquad \text{(isolated system)}$$

Law of conservation of momentum The total momentum $\vec{P}$ of an isolated system is a constant. Interactions within the system do not change the system's total momentum.

- Or, written mathematically:

$$\vec{P}_{\text{f}} = \vec{P}_{\text{i}}$$

Problem-Solving Strategy: Conservation of Momentum

PROBLEM-SOLVING STRATEGY 11.1

Conservation of momentum

MODEL Clearly define the *system*.

- If possible, choose a system that is isolated ($\vec{F}_{net} = \vec{0}$) or within which the interactions are sufficiently short and intense that you can ignore external forces for the duration of the interaction (the impulse approximation). Momentum is conserved.
- If it's not possible to choose an isolated system, try to divide the problem into parts such that momentum is conserved during one segment of the motion. Other segments of the motion can be analyzed using Newton's laws or conservation of energy.

Problem-Solving Strategy: Conservation of Momentum

PROBLEM-SOLVING STRATEGY 11.1

Conservation of momentum

VISUALIZE Draw a before-and-after pictorial representation. Define symbols that will be used in the problem, list known values, and identify what you're trying to find.

SOLVE The mathematical representation is based on the law of conservation of momentum: $\vec{P}_f = \vec{P}_i$. In component form, this is

$$(p_{fx})_1 + (p_{fx})_2 + (p_{fx})_3 + \cdots = (p_{ix})_1 + (p_{ix})_2 + (p_{ix})_3 + \cdots$$
$$(p_{fy})_1 + (p_{fy})_2 + (p_{fy})_3 + \cdots = (p_{iy})_1 + (p_{iy})_2 + (p_{iy})_3 + \cdots$$

ASSESS Check that your result has correct units and significant figures, is reasonable, and answers the question.

Exercise 17

QuickCheck 11.8

A mosquito and a truck have a head-on collision. Splat! Which has a larger change of momentum?

A. The mosquito
B. The truck
C. They have the same change of momentum.
D. Can’t say without knowing their initial velocities.

QuickCheck 11.8

A mosquito and a truck have a head-on collision. Splat! Which has a larger change of momentum?

A. The mosquito.

B. The truck.

✔ **C. They have the same change of momentum.**

D. Can't say without knowing their initial velocities.

Momentum is conserved, so $\Delta p_{\text{mosquito}} + \Delta p_{\text{truck}} = 0$.

Equal magnitude (but opposite sign) changes in momentum.

Example 11.3 Rolling Away

EXAMPLE 11.3 | Rolling away

Bob sees a stationary cart 8.0 m in front of him. He decides to run to the cart as fast as he can, jump on, and roll down the street. Bob has a mass of 75 kg and the cart's mass is 25 kg. If Bob accelerates at a steady 1.0 m/s^2, what is the cart's speed just after Bob jumps on?

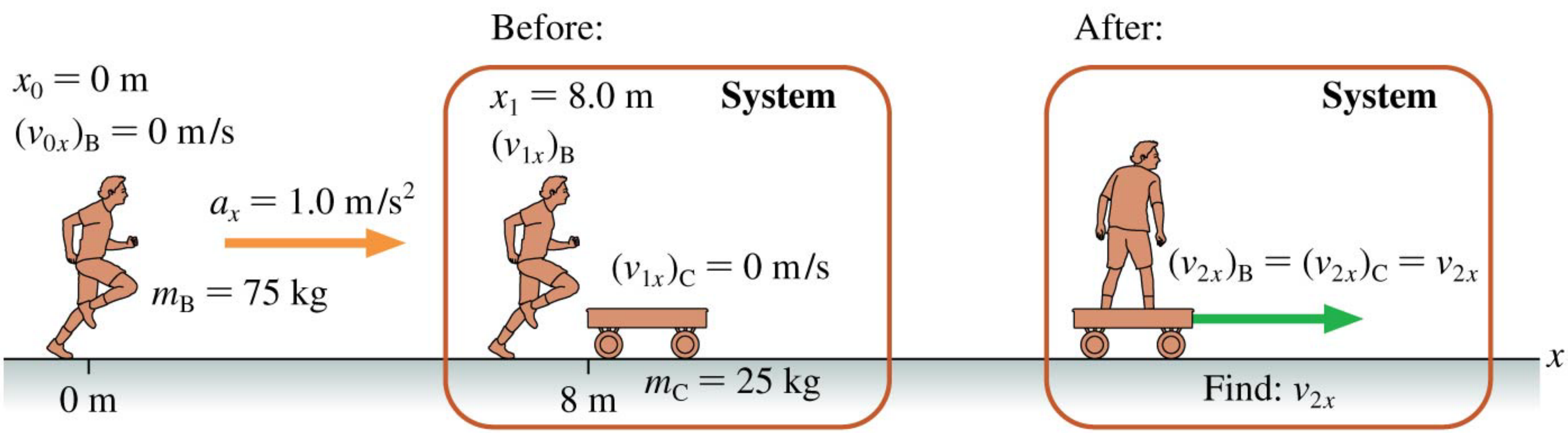
Example 11.3 Rolling Away

EXAMPLE 11.3 | **Rolling away**

MODEL This is a two-part problem. First Bob accelerates across the ground. Then Bob lands on and sticks to the cart, a "collision" between Bob and the cart. The interaction forces between Bob and the cart (i.e., friction) act only over the fraction of a second it takes Bob's feet to become stuck to the cart. Using the impulse approximation allows the system Bob + cart to be treated as an isolated system during the brief interval of the "collision," and thus the total momentum of Bob + cart is conserved during this interaction. But the system Bob + cart is *not* an isolated system for the entire problem because Bob's initial acceleration has nothing to do with the cart.

EXAMPLE 11.3 Rolling away

VISUALIZE Our strategy is to divide the problem into an *acceleration* part, which we can analyze using kinematics, and a *collision* part, which we can analyze with momentum conservation. The pictorial representation of **FIGURE 11.14** includes information about both parts. Notice that Bob's velocity $(v_{1x})_B$ at the end of his run is his "before" velocity for the collision.



Example 11.3 Rolling Away

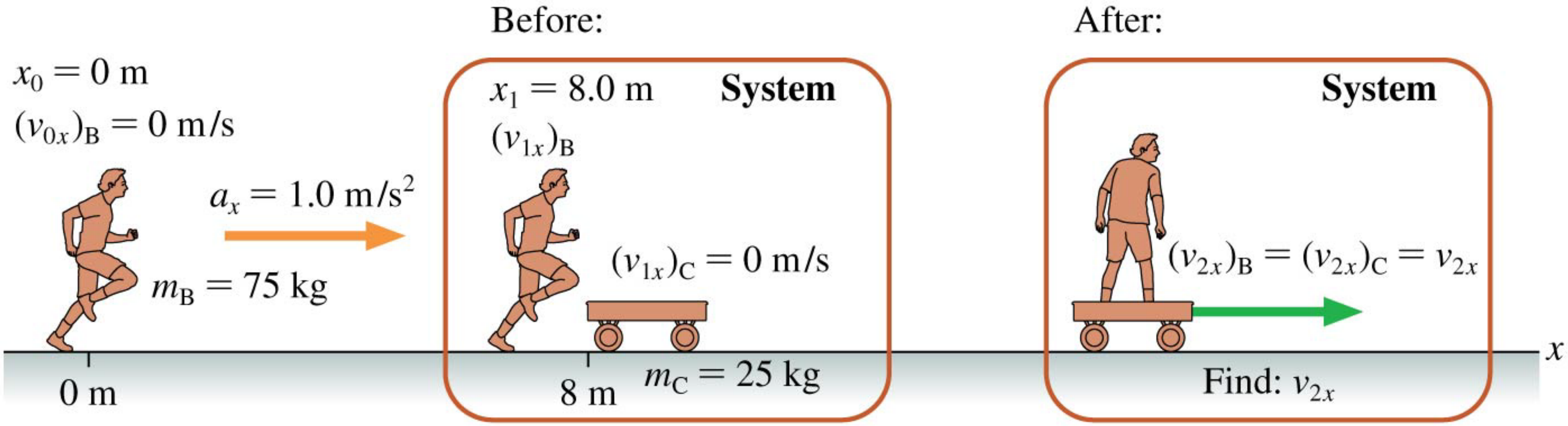
EXAMPLE 11.3 | **Rolling away**

SOLVE The first part of the mathematical representation is kinematics. We don't know how long Bob accelerates, but we do know his acceleration and the distance. Thus

$$(v_{1x})_B{}^2 = (v_{0x})_B{}^2 + 2a_x\,\Delta x = 2a_x x_1$$

His velocity after accelerating for 8.0 m is

$$(v_{1x})_B = \sqrt{2a_x x_1} = 4.0\text{ m/s}$$



Example 11.3 Rolling Away

EXAMPLE 11.3 Rolling away

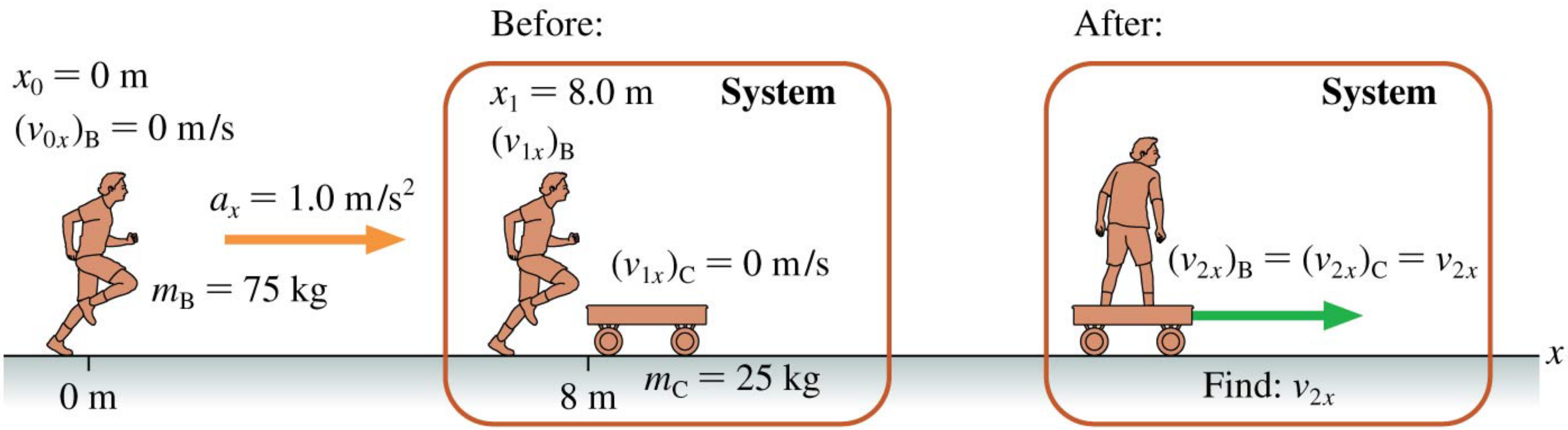
SOLVE The second part of the problem, the collision, uses conservation of momentum: $P_{2x} = P_{1x}$. Equation 11.24 is

$$m_B(v_{2x})_B + m_C(v_{2x})_C = m_B(v_{1x})_B + m_C(v_{1x})_C = m_B(v_{1x})_B$$

where we've used $(v_{1x})_C = 0$ m/s because the cart starts at rest. In this problem, Bob and the cart move together at the end with a common velocity, so we can replace both $(v_{2x})_B$ and $(v_{2x})_C$ with simply v_{2x}. Solving for v_{2x}, we find

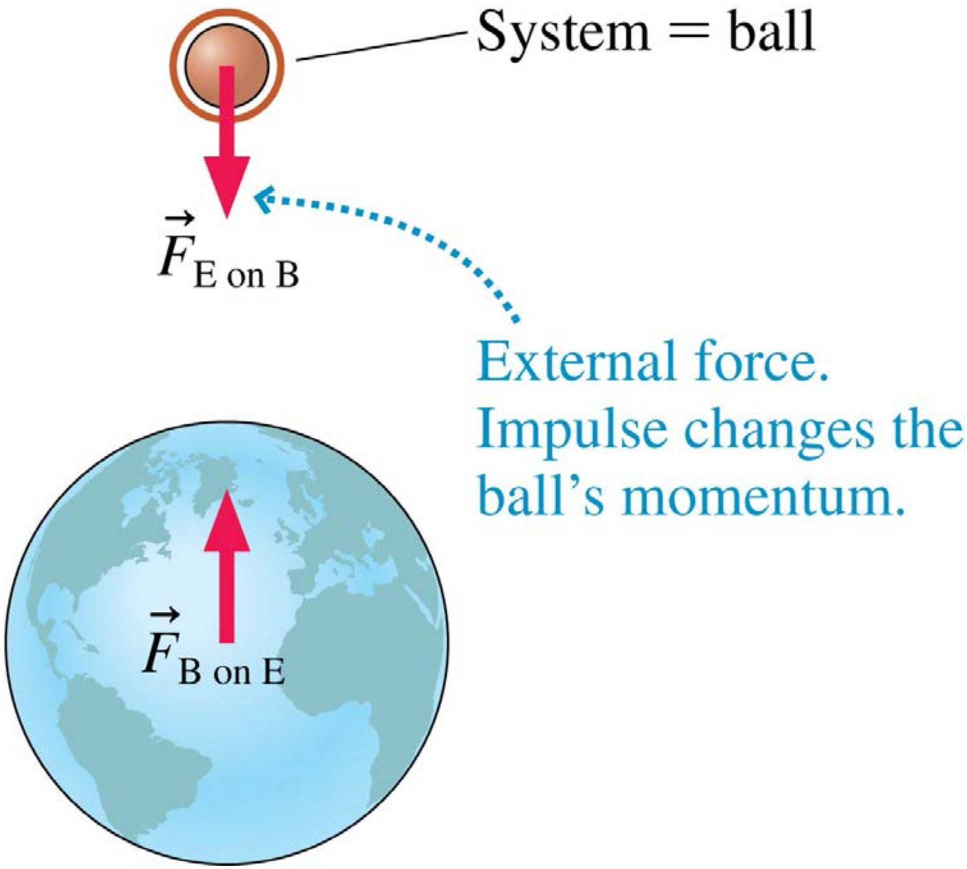
$$v_{2x} = \frac{m_B}{m_B + m_C}(v_{1x})_B = \frac{75\text{ kg}}{100\text{ kg}} \times 4.0\text{ m/s} = 3.0\text{ m/s}$$

The cart's speed is 3.0 m/s immediately after Bob jumps on.



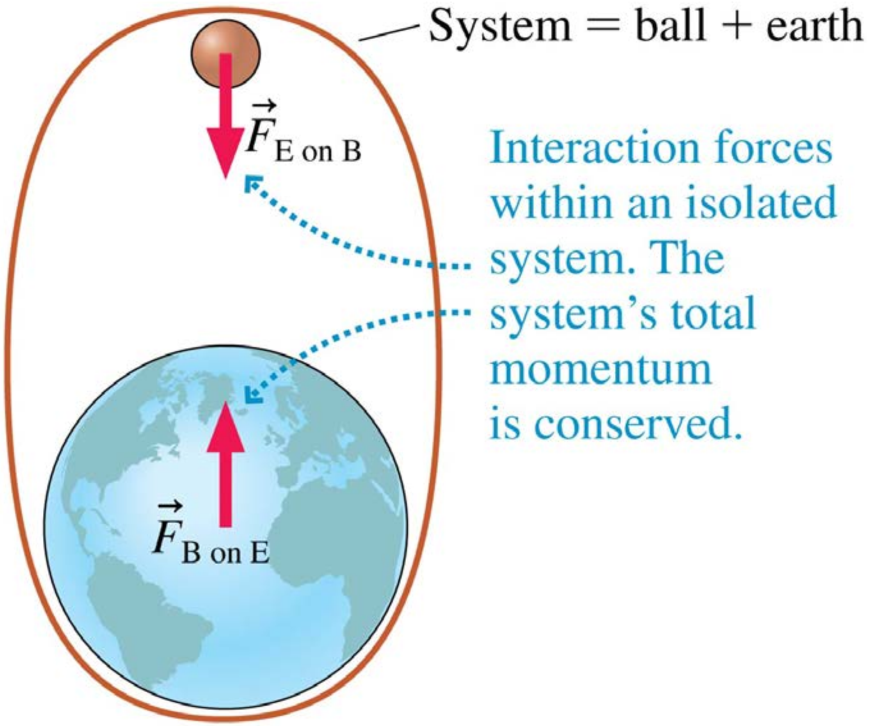
Conservation of Momentum Depends on the Choice of System

- A rubber ball is dropped and falls toward earth.
- Define the ball as the System.
- The gravitational force of the earth is an external force.
- The momentum of the system is *not* conserved.



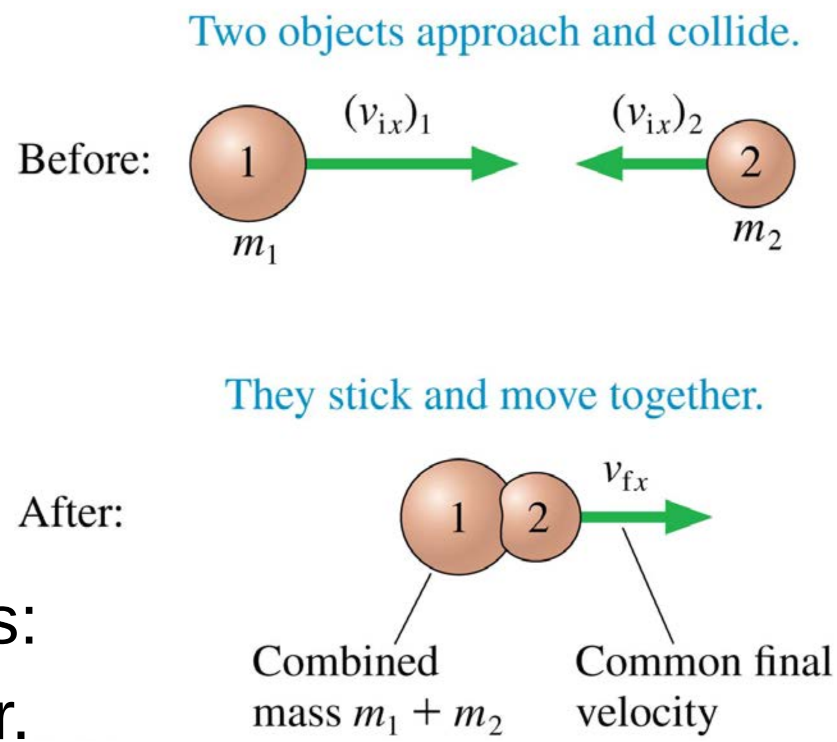
Conservation of Momentum Depends on the Choice of System

- A rubber ball is dropped, and falls toward earth.
- Define the ball + earth as the System.
- The gravitational forces are interactions *within* the system.
- This is an isolated system, so the total momentum $\vec{P} = \vec{p}_{\text{ball}} + \vec{p}_{\text{earth}}$ is conserved.



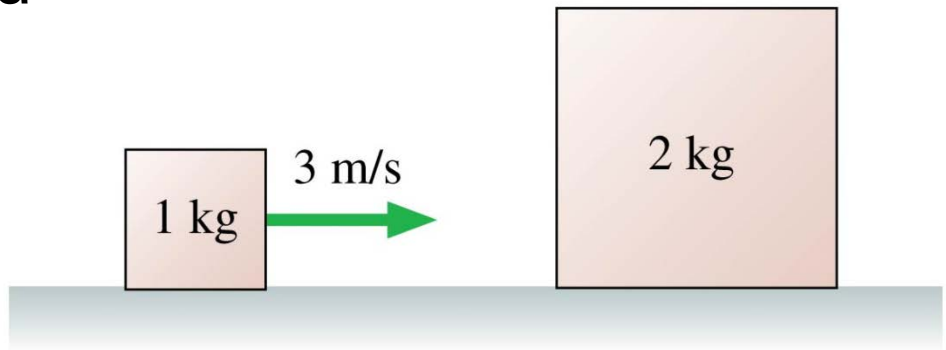
Inelastic Collisions

- A collision in which the two objects stick together and move with a common final velocity is called a **perfectly inelastic collision.**
- Examples of inelastic collisions:
  - A piece of clay hits the floor.
  - A bullet strikes a block of wood and embeds itself in the block.
  - Railroad cars coupling together upon impact.
  - A dart hitting a dart board.



QuickCheck 11.9

The $1\ \mathrm{kg}$ box is sliding along a frictionless surface. It collides with and sticks to the $2\ \mathrm{kg}$ box. Afterward, the speed of the two boxes is



A. $0\ \mathrm{m/s}$

B. $1\ \mathrm{m/s}$

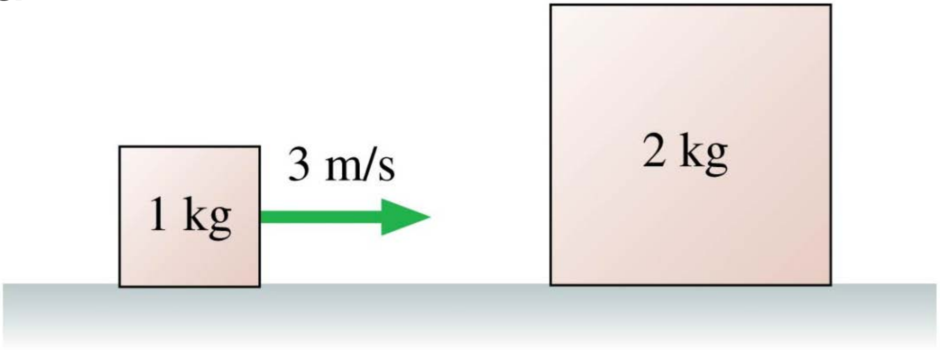
C. $2\ \mathrm{m/s}$

D. $3\ \mathrm{m/s}$

E. There's not enough information to tell.

QuickCheck 11.9

The $1\ \mathrm{kg}$ box is sliding along a frictionless surface. It collides with and sticks to the $2\ \mathrm{kg}$ box. Afterward, the speed of the two boxes is



A. 0 m/s

✔ **B. 1 m/s**

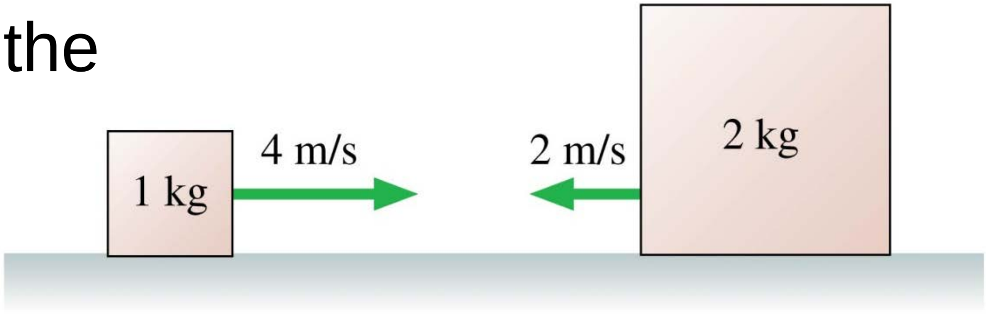
C. 2 m/s

D. 3 m/s

E. There's not enough information to tell.

QuickCheck 11.10

The two boxes are sliding along a frictionless surface. They collide and stick together. Afterward, the velocity of the two boxes is



A. 2 m/s to the left.

B. 1 m/s to the left.

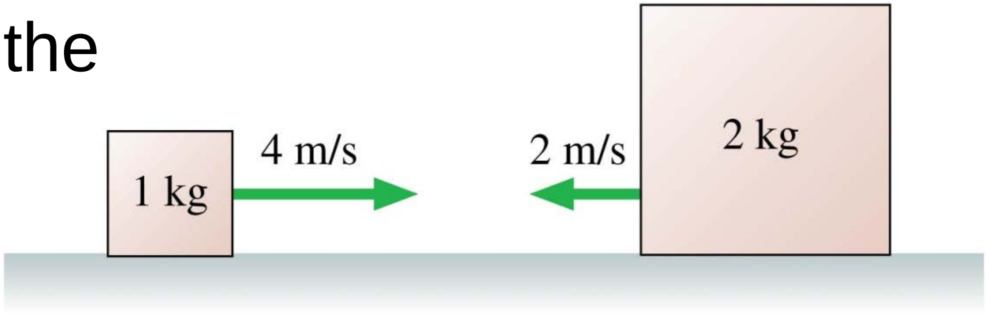
C. 0 m/s, at rest.

D. 1 m/s to the right.

E. 2 m/s to the right.

QuickCheck 11.10

The two boxes are sliding along a frictionless surface. They collide and stick together. Afterward, the velocity of the two boxes is



A. 2 m/s to the left.

B. 1 m/s to the left.

C. **0 m/s, at rest.**

D. 1 m/s to the right.

E. 2 m/s to the right.

Example 11.4 An Inelastic Glider Collision

EXAMPLE 11.4 | **An inelastic glider collision**

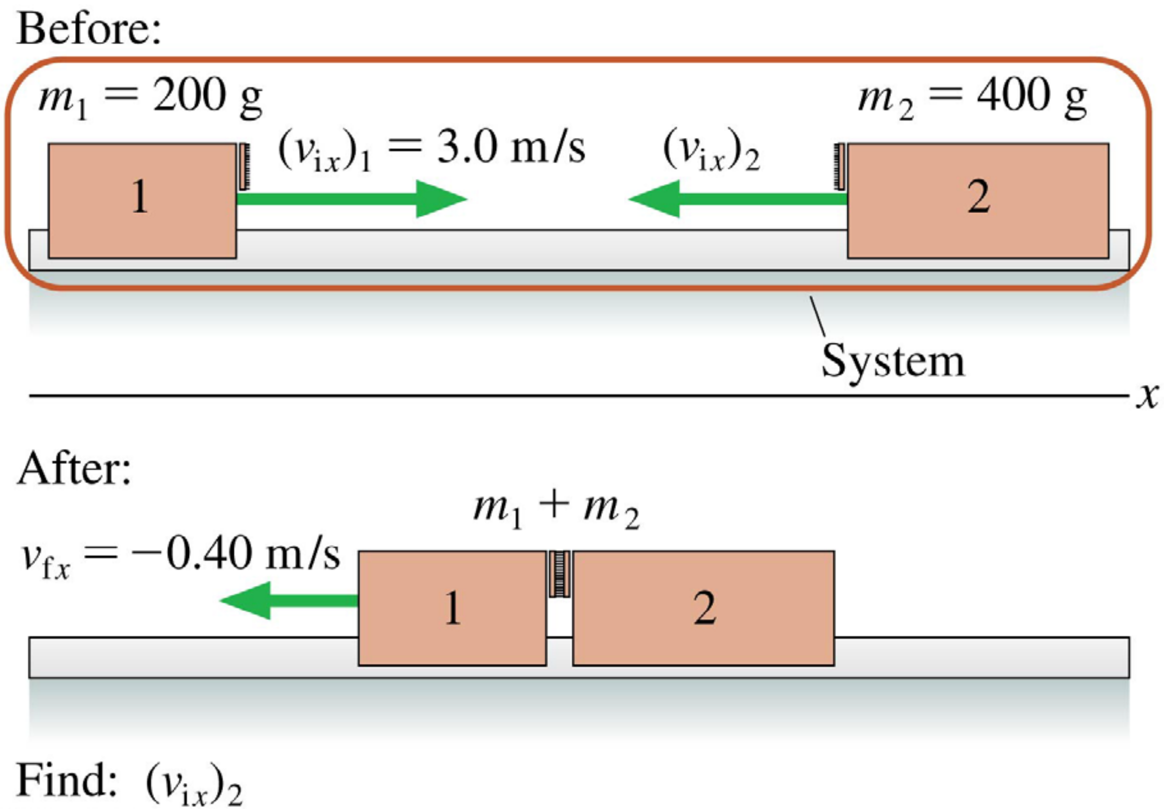
In a laboratory experiment, a 200 g air-track glider and a 400 g air-track glider are pushed toward each other from opposite ends of the track. The gliders have Velcro tabs on the front and will stick together when they collide. The 200 g glider is pushed with an initial speed of 3.0 m/s. The collision causes it to reverse direction at 0.40 m/s. What was the initial speed of the 400 g glider?

MODEL Define the system to be the two gliders. This is an isolated system, so its total momentum is conserved in the collision. The gliders stick together, so this is a perfectly inelastic collision.

Example 11.4 An Inelastic Glider Collision

EXAMPLE 11.4 | **An inelastic glider collision**

VISUALIZE **FIGURE 11.17** shows a pictorial representation. We've chosen to let the 200 g glider (glider 1) start out moving to the right, so $(v_{ix})_1$ is a positive 3.0 m/s. The gliders move to the left after the collision, so their common final velocity is $v_{fx} = -0.40$ m/s.



Example 11.4 An Inelastic Glider Collision

EXAMPLE 11.4 **An inelastic glider collision**

SOLVE The law of conservation of momentum, $P_{fx} = P_{ix}$, is

$$(m_1 + m_2)v_{fx} = m_1(v_{ix})_1 + m_2(v_{ix})_2$$

where we made use of the fact that the combined mass $m_1 + m_2$ moves together after the collision. We can easily solve for the initial velocity of the 400 g glider:

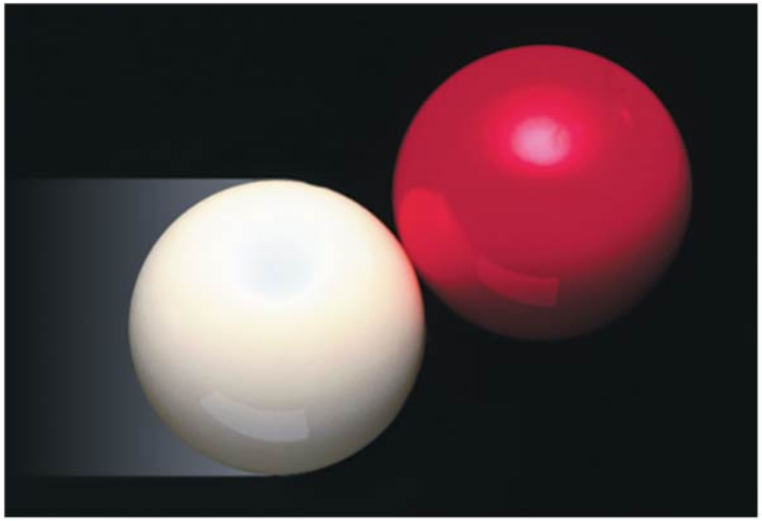
$$(v_{ix})_2 = \frac{(m_1 + m_2)v_{fx} - m_1(v_{ix})_1}{m_2}$$

$$= \frac{(0.60\ \text{kg})(-0.40\ \text{m/s}) - (0.20\ \text{kg})(3.0\ \text{m/s})}{0.40\ \text{kg}} = -2.1\ \text{m/s}$$

The negative sign indicates that the 400 g glider started out moving to the left. The initial *speed* of the glider, which we were asked to find, is 2.1 m/s.

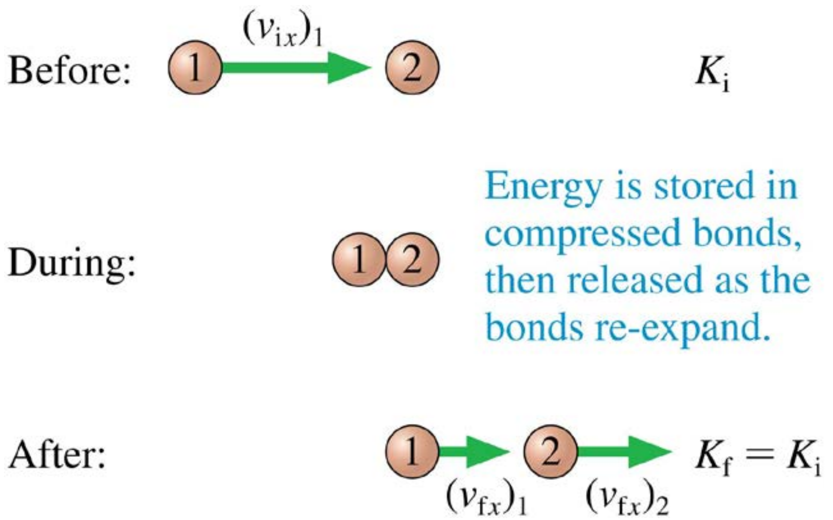
Elastic Collisions

- During an inelastic collision of two objects, some of the mechanical energy is dissipated inside the objects as thermal energy.
- A collision in which mechanical energy is conserved is called a **perfectly elastic collision**.
- Collisions between two very hard objects, such as two billiard balls or two steel balls, come close to being perfectly elastic.



A Perfectly Elastic Collision

- Consider a head-on, perfectly elastic collision of a ball of mass m_1 and initial velocity $(v_{ix})_1$, with a ball of mass m_2 initially at rest.
- The balls' velocities after the collision are $(v_{fx})_1$ and $(v_{fx})_2$.



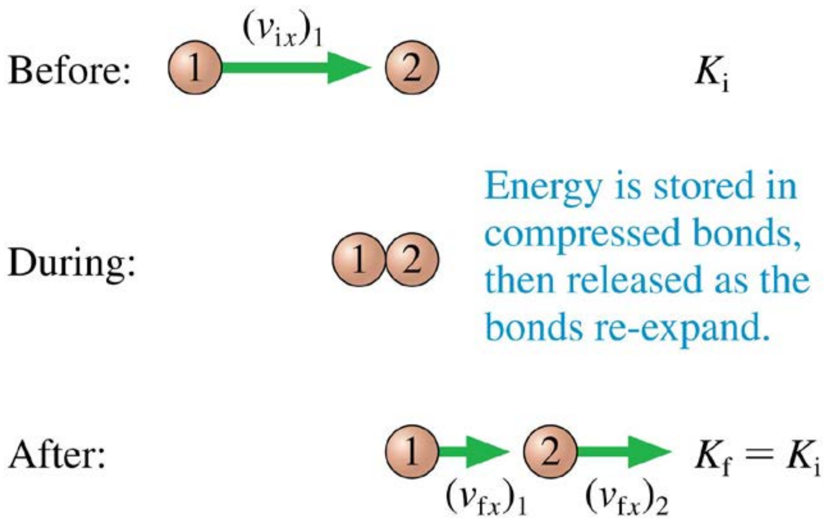
- Momentum is conserved in all isolated collisions.
- In a perfectly elastic collision in which potential energy is not changing, the kinetic energy must also be conserved.

momentum conservation: $$m_1(v_{fx})_1 + m_2(v_{fx})_2 = m_1(v_{ix})_1$$

energy conservation: $$\frac{1}{2}m_1(v_{fx})_1^2 + \frac{1}{2}m_2(v_{fx})_2^2 = \frac{1}{2}m_1(v_{ix})_1^2$$

A Perfectly Elastic Collision

- Simultaneously solving the conservation of momentum equation and the conservation of kinetic energy equations allows us to find the two unknown final velocities.



- The result is

$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1 \qquad (v_{fx})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1$$

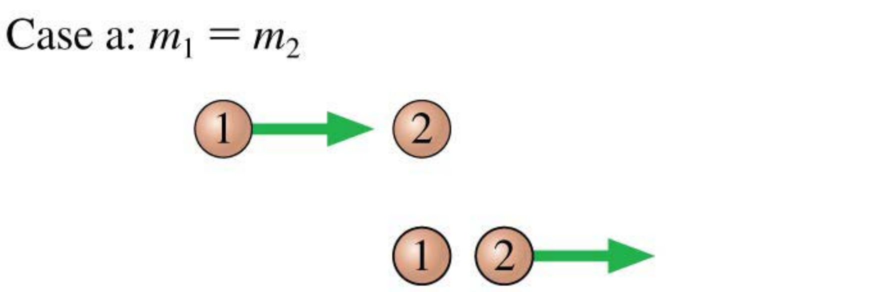
(perfectly elastic collision with ball 2 initially at rest)

A Perfectly Elastic Collision: Special Case a

$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1 \qquad (v_{fx})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1$$

(perfectly elastic collision with ball 2 initially at rest)

- Consider a head-on, perfectly elastic collision of a ball of mass m_1 and initial velocity $(v_{ix})_1$, with a ball of mass m_2 initially at rest.
- Case a: $m_1 = m_2$
- Equations 11.29 give $v_{f1} = 0$ and $v_{f2} = v_{i1}$
- The first ball stops and transfers all its momentum to the second ball.



Ball 1 stops. Ball 2 goes forward with $v_{f2} = v_{i1}$.

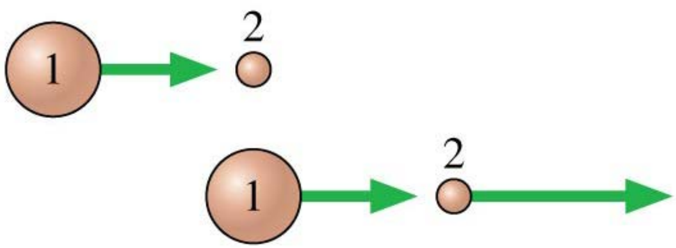
A Perfectly Elastic Collision: Special Case b

$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1 \qquad (v_{fx})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1$$

(perfectly elastic collision with ball 2 initially at rest)

- Consider a head-on, perfectly elastic collision of a ball of mass m_1 and initial velocity $(v_{ix})_1$, with a ball of mass m_2 initially at rest.
- Case b: $m_1 >> m_2$
- Equations 11.29 give $v_{f1} \approx v_{i1}$ and $v_{f2} \approx 2v_{i1}$
- The big first ball keeps going with about the same speed, and the little second ball flies off with about twice the speed of the first ball.

Case b: $m_1 \gg m_2$



Ball 1 hardly slows down. Ball 2 is knocked forward at $v_{f2} \approx 2v_{i1}$.

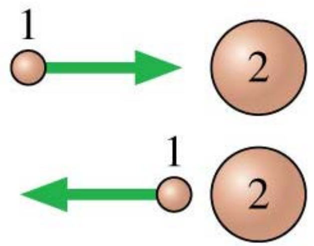
A Perfectly Elastic Collision: Special Case c

$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1 \qquad (v_{fx})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1$$

(perfectly elastic collision with ball 2 initially at rest)

- Consider a head-on, perfectly elastic collision of a ball of mass m_1 and initial velocity $(v_{ix})_1$, with a ball of mass m_2 initially at rest.
- Case c: $m_1 << m_2$

Case c: $m_1 \ll m_2$



Ball 1 bounces off ball 2 with almost no loss of speed. Ball 2 hardly moves.

- Equations 11.29 give $v_{f1} \approx -v_{i1}$ and $v_{f2} \approx 0$
- The little first ball rebounds with about the same speed, and the big second ball hardly moves at all.

Perfectly Elastic Collisions: Using Reference Frames

- Equations 11.29 assume ball 2 is at rest.
- What if you need to analyze a head-on collision when both balls are moving before the collision?
- You could solve the simultaneous momentum and energy equations, but there is an easier way.

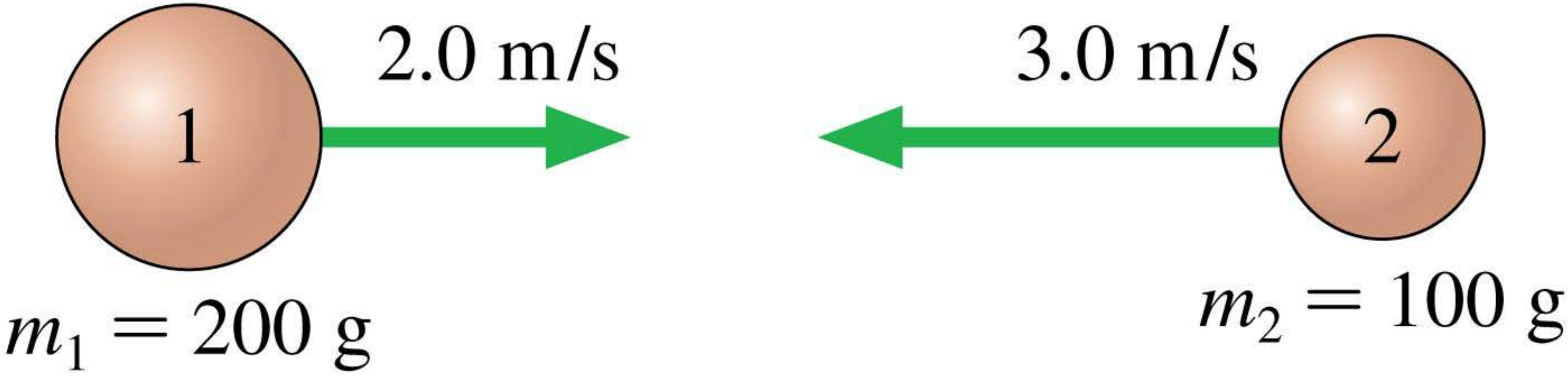
TACTICS BOX 11.1 MP

Analyzing elastic collisions

1. Use the Galilean transformation to transform the initial velocities of balls 1 and 2 from the "lab frame" to a reference frame in which ball 2 is at rest.
2. Use Equations 11.29 to determine the outcome of the collision in the frame where ball 2 is initially at rest.
3. Transform the final velocities back to the "lab frame."

Using Reference Frames: Quick Example

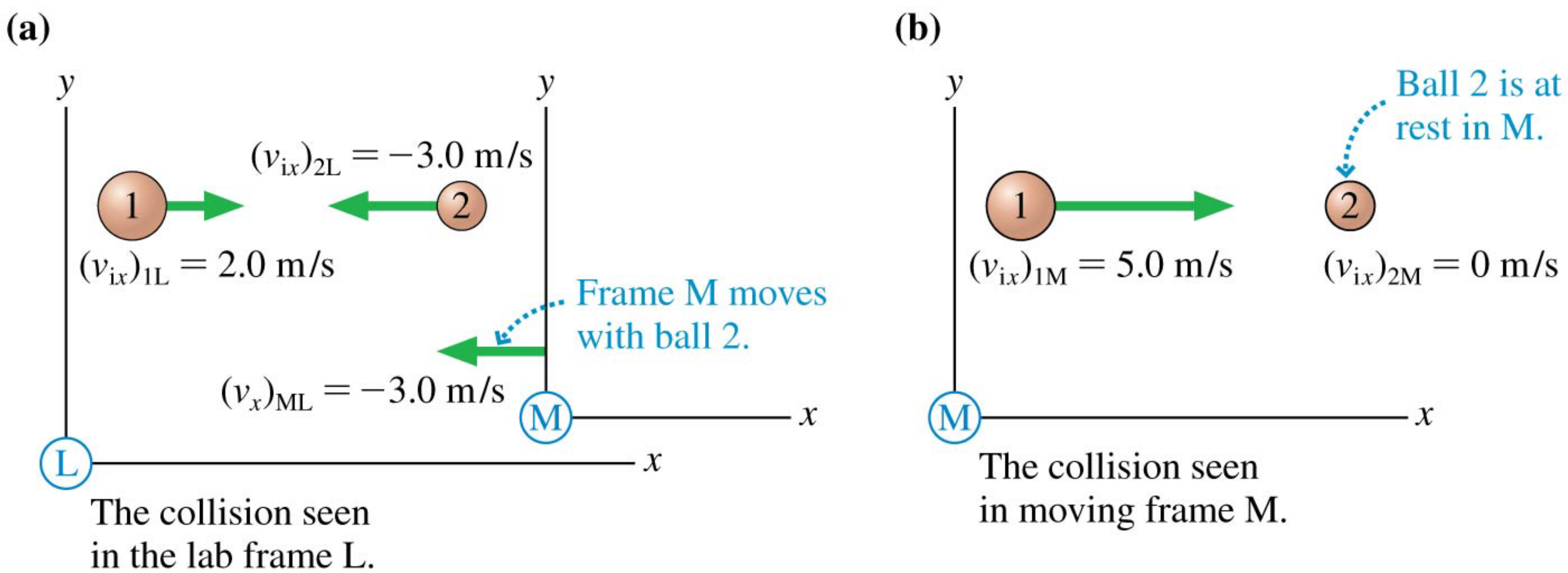
A 200 g ball moves to the right at 2.0 m/s. It has a head-on, perfectly elastic collision with a 100 g ball that is moving toward it at 3.0 m/s. What are the final velocities of both balls?



Using Reference Frames: Quick Example

- Figure (a) shows the situation just before the collision in the lab frame L.
- Figure (b) shows the situation just before the collision in the frame M that is moving along with ball 2.

$$(v_{ix})_{1M} = (v_{ix})_{1L} + (v_x)_{LM} = 2.0\text{ m/s} + 3.0\text{ m/s} = 5.0\text{ m/s}$$
$$(v_{ix})_{2M} = (v_{ix})_{2L} + (v_x)_{LM} = -3.0\text{ m/s} + 3.0\text{ m/s} = 0\text{ m/s}$$



Using Reference Frames: Quick Example

- We can use these equations to find the post-collision velocities in the moving frame M:

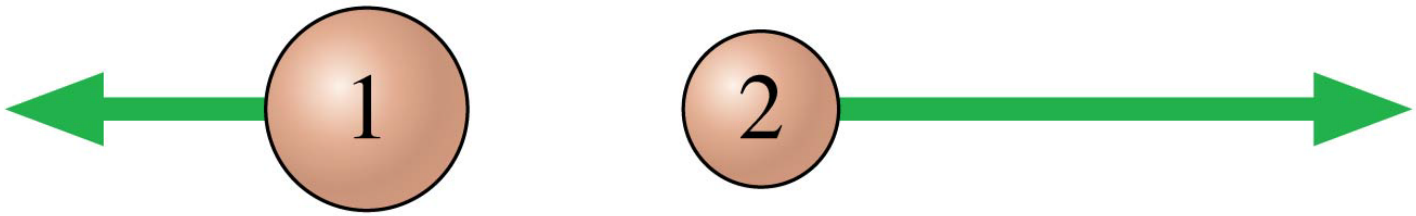
$$(v_{fx})_{1M} = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_{1M} = 1.7 \text{ m/s}$$

$$(v_{fx})_{2M} = \frac{2m_1}{m_1 + m_2}(v_{ix})_{1M} = 6.7 \text{ m/s}$$

- Transforming back to the lab frame L:

$$(v_{fx})_{1L} = (v_{fx})_{1M} + (v_x)_{ML} = 1.7 \text{ m/s} + (-3.0 \text{ m/s}) = -1.3 \text{ m/s}$$

$$(v_{fx})_{2L} = (v_{fx})_{2M} + (v_x)_{ML} = 6.7 \text{ m/s} + (-3.0 \text{ m/s}) = 3.7 \text{ m/s}$$



$(v_{fx})_{1L} = -1.3 \text{ m/s}$ $\quad (v_{fx})_{2L} = 3.7 \text{ m/s}$

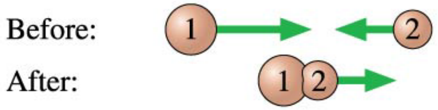
Two Collision Models

MODEL 11.1

Collisions

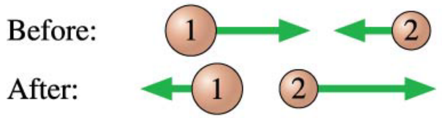
For two colliding objects.

- Represent the objects as elastic objects moving in a straight line.
- In a **perfectly inelastic collision,** the objects stick and move together. Kinetic energy is transformed into thermal energy.
  Mathematically:



$$(m_1 + m_2)v_{fx} = m_1(v_{ix})_1 + m_2(v_{ix})_2$$

- In a **perfectly elastic collision,** the objects bounce apart with no loss of energy.
  Mathematically:



  - If object 2 is initially at rest, then

$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1 \qquad (v_{ix})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1$$

  - If both objects are moving, use the Galilean transformation to transform the velocities to a reference frame in which object 2 is at rest.
- Limitations: Model fails if the collision is not head-on or cannot reasonably be approximated as a "thud" or as a "perfect bounce."

Exercise 22

Explosions

- An **explosion** is the opposite of a collision.
- The particles first have a brief, intense interaction, then they move apart from each other.

- The explosive forces are *internal* forces.
- If the system is isolated, its total momentum during the explosion will be conserved.

Example 11.6 Recoil

EXAMPLE 11.6 | **Recoil**

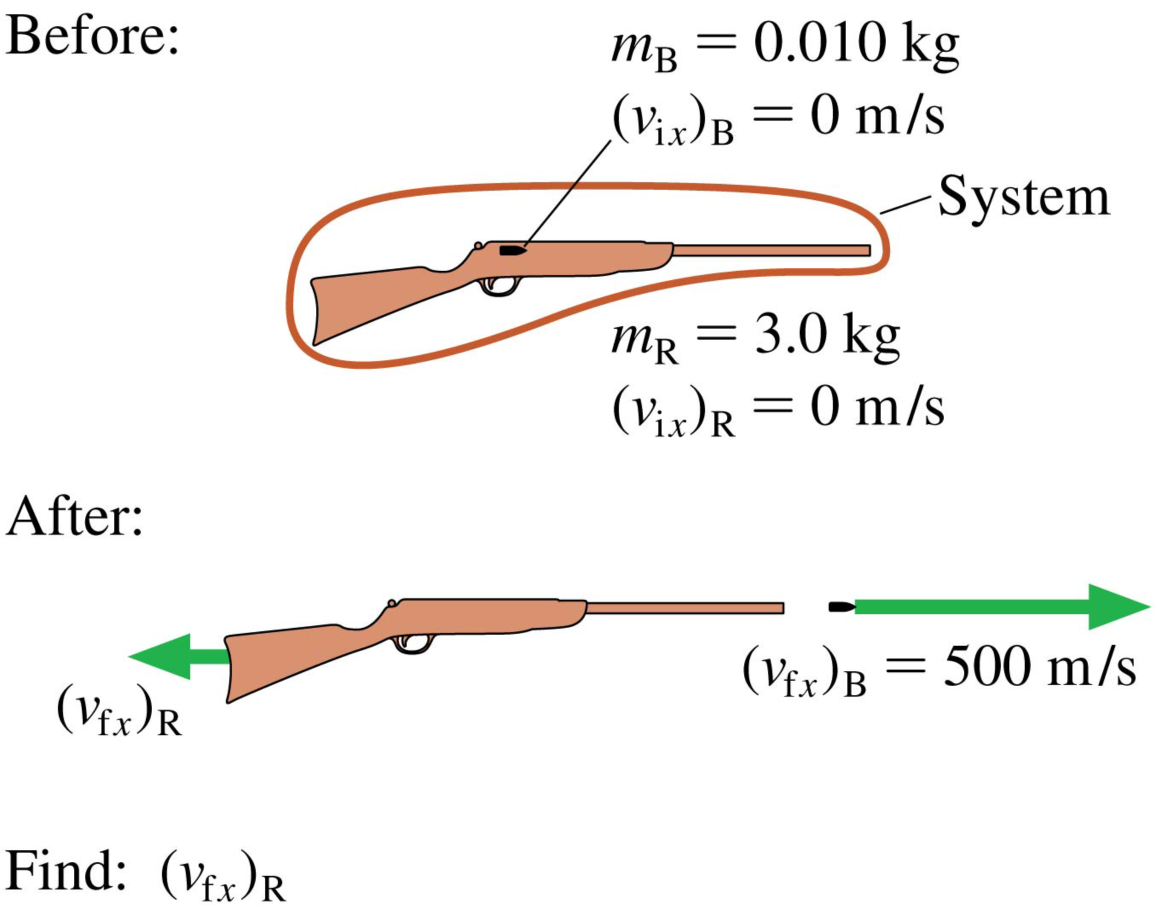
A 10 g bullet is fired from a 3.0 kg rifle with a speed of 500 m/s. What is the recoil speed of the rifle?

MODEL The rifle causes a small mass of gunpowder to explode, and the expanding gas then exerts forces on *both* the bullet and the rifle. Let's define the system to be bullet + gas + rifle. The forces due to the expanding gas during the explosion are internal forces, within the system. Any friction forces between the bullet and the rifle as the bullet travels down the barrel are also internal forces. Gravity is balanced by the upward force of the person holding the rifle, so $\vec{F}_{\text{net}} = \vec{0}$. This is an isolated system and the law of conservation of momentum applies.

Example 11.6 Recoil

EXAMPLE 11.6 | **Recoil**

VISUALIZE FIGURE 11.24 shows a pictorial representation before and after the bullet is fired.



Example 11.6 Recoil

EXAMPLE 11.6 **Recoil**

SOLVE The x-component of the total momentum is $P_x = (p_x)_B + (p_x)_R + (p_x)_{gas}$. Everything is at rest before the trigger is pulled, so the initial momentum is zero. After the trigger is pulled, the momentum of the expanding gas is the sum of the momenta of all the molecules in the gas. For every molecule moving in the forward direction with velocity v and momentum mv there is, on average, another molecule moving in the opposite direction with velocity $-v$ and thus momentum $-mv$. When the values are summed over the enormous number of molecules in the gas, we will be left with $p_{gas} \approx 0$. In addition, the mass of the gas is much less than that of the rifle or bullet. For both reasons, we can reasonably neglect the momentum of the gas. The law of conservation of momentum is thus

$$P_{fx} = m_B(v_{fx})_B + m_R(v_{fx})_R = P_{ix} = 0$$

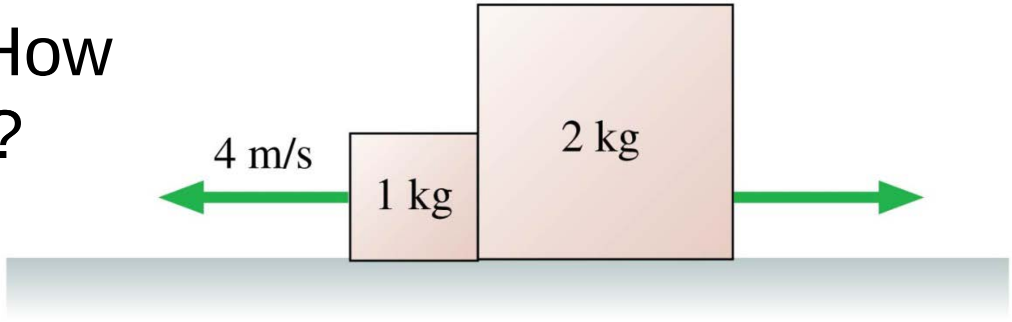
Solving for the rifle's velocity, we find

$$(v_{fx})_R = -\frac{m_B}{m_R}(v_{fx})_B = -\frac{0.010\text{ kg}}{3.0\text{ kg}} \times 500\text{ m/s} = -1.7\text{ m/s}$$

The minus sign indicates that the rifle's recoil is to the left. The recoil *speed* is 1.7 m/s.

QuickCheck 11.11

The two boxes are on a frictionless surface. They had been sitting together at rest, but an explosion between them has just pushed them apart. How fast is the 2 kg box going?



A. 1 m/s

B. 2 m/s

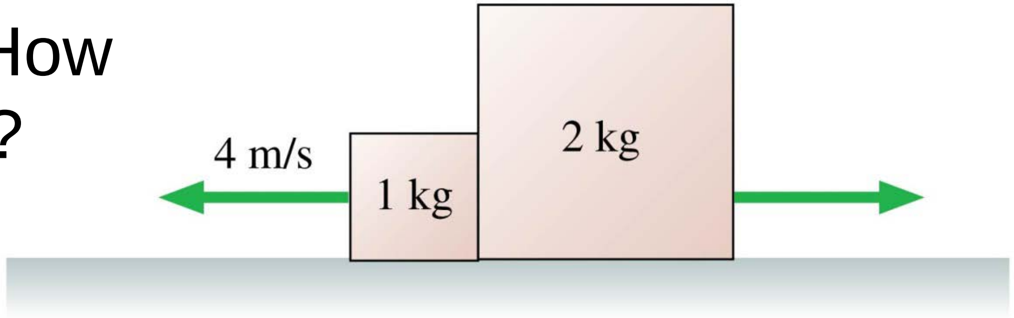
C. 4 m/s

D. 8 m/s

E. There's not enough information to tell.

QuickCheck 11.11

The two boxes are on a frictionless surface. They had been sitting together at rest, but an explosion between them has just pushed them apart. How fast is the 2 kg box going?



A. 1 m/s

B. 2 m/s

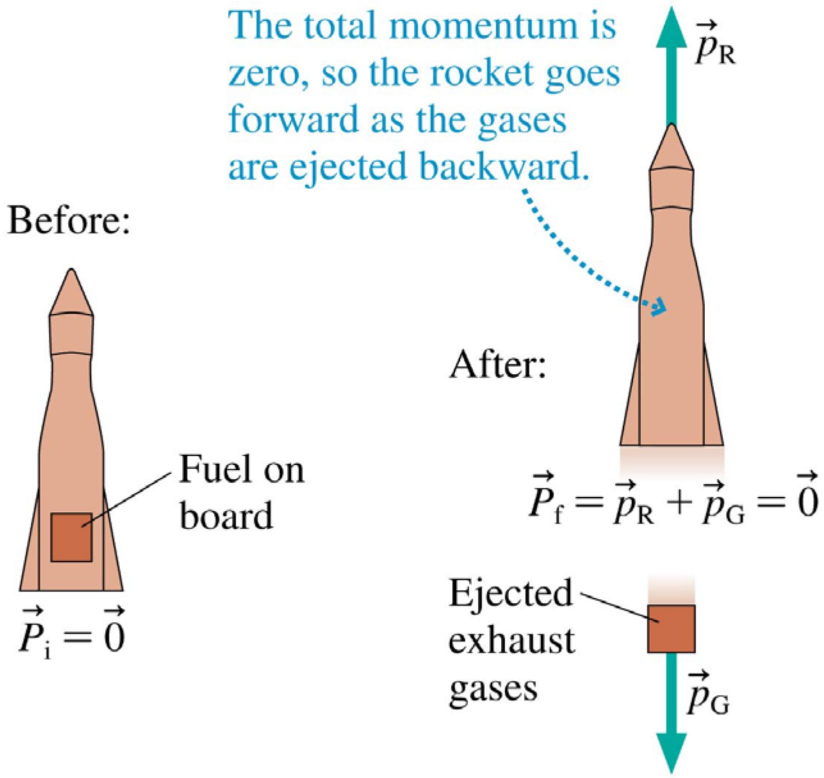
C. 4 m/s

D. 8 m/s

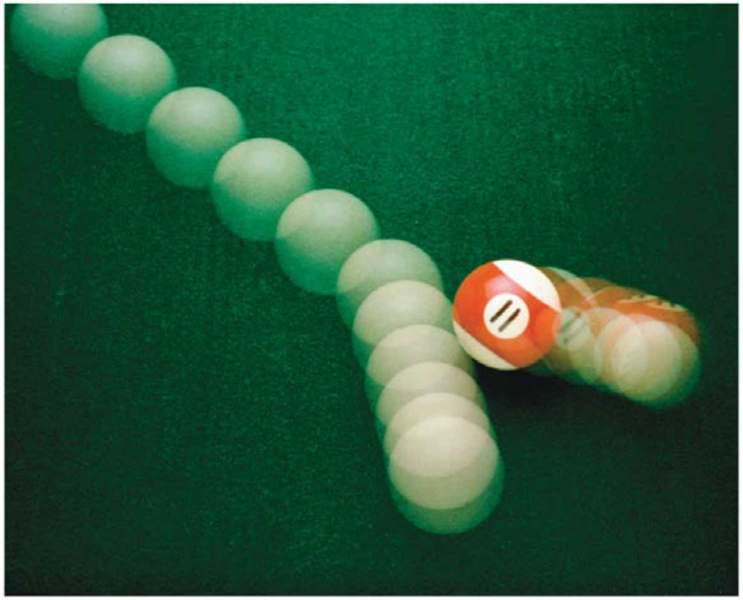
E. There's not enough information to tell.

Rocket or Jet Propulsion

- The figure shows a rocket with a parcel of fuel on board.
- If we choose rocket + gases to be the system, the burning and expulsion are both internal forces.
- The exhaust gases gain backward momentum as they are shot out the back.
- The *total* momentum of the system remains zero.
- Therefore, the rocket gains forward momentum.



Momentum in Two Dimensions



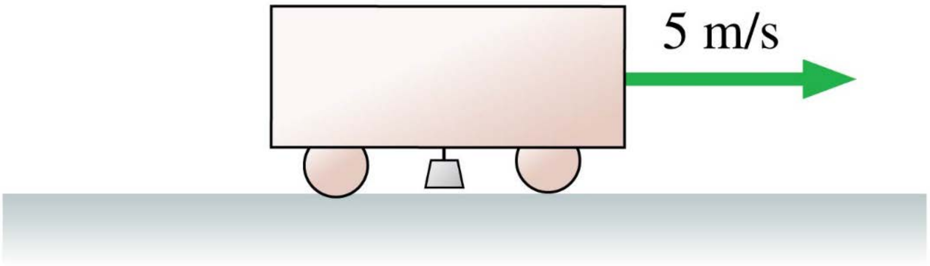
- The total momentum $\vec{P}$ is a vector sum of the momenta $\vec{p} = m\vec{v}$ of the individual particles.
- Momentum is conserved only if each component of $\vec{P}$ is conserved:

$$(p_{fx})_1 + (p_{fx})_2 + (p_{fx})_3 + \cdots = (p_{ix})_1 + (p_{ix})_2 + (p_{ix})_3 + \cdots$$

$$(p_{fy})_1 + (p_{fy})_2 + (p_{fy})_3 + \cdots = (p_{iy})_1 + (p_{iy})_2 + (p_{iy})_3 + \cdots$$

QuickCheck 11.12

A cart is rolling at 5 m/s. A heavy lead weight is suspended by a thread beneath the cart. Suddenly the thread breaks and the weight falls. Immediately afterward, the speed of the cart is



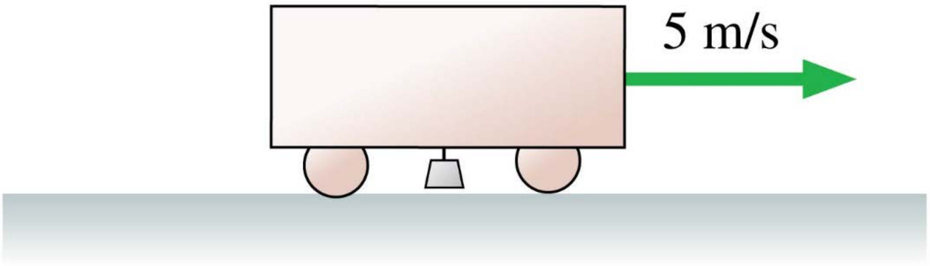
A. Less than 5 m/s

B. Still 5 m/s

C. More than 5 m/s

QuickCheck 11.12

A cart is rolling at 5 m/s. A heavy lead weight is suspended by a thread beneath the cart. Suddenly the thread breaks and the weight falls. Immediately afterward, the speed of the cart is



A. Less than 5 m/s

B. Still 5 m/s

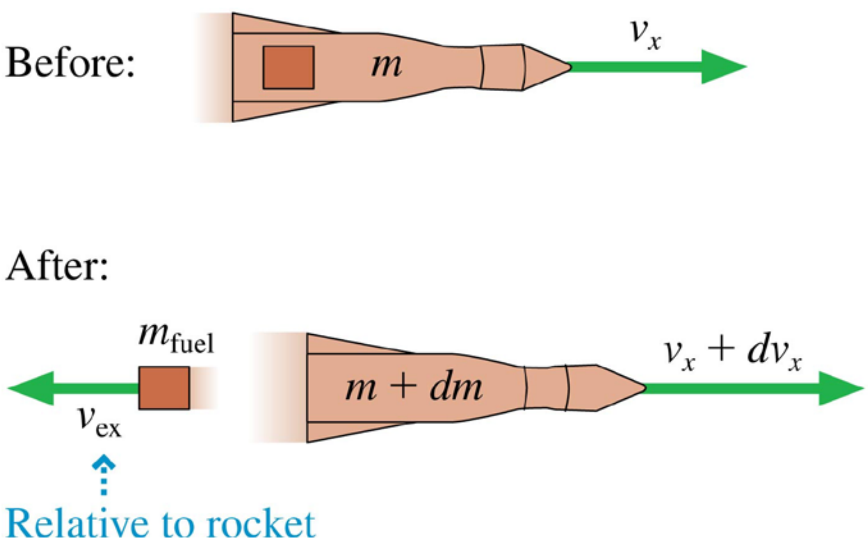
C. More than 5 m/s

No external forces to exert an impulse. The falling weight still has forward momentum.

Advanced Topic: Rocket Propulsion

- Newton's second law $\vec{F} = m\vec{a}$ applies to objects whose mass does not change.
- That's an excellent assumption for balls and bicycles, but what about something like a rocket that loses a significant amount of mass as its fuel is burned?
- Problems of varying mass are solved with momentum rather than acceleration.
- As exhaust gases are shot out the back, the rocket "recoils" in the opposite direction.

Advanced Topic: Rocket Propulsion



- The figure shows a rocket being propelled by the thrust of burning fuel but *not* influenced by gravity or drag.

- The "Before" state is a rocket of mass m (including all onboard fuel) moving with velocity v_x and having initial momentum $P_{ix} = mv_x$.
- During a small interval of time dt, the rocket burns a small mass of fuel m_{fuel} and expels the resulting gases from the back of the rocket at an exhaust speed v_{ex} *relative to the rocket.*

Advanced Topic: Rocket Propulsion

- After the gas has been ejected, both the rocket and the gas have momentum.
- Conservation of momentum tells us that

$$P_{fx} = m_{rocket}(v_x)_{rocket} + m_{fuel}(v_x)_{fuel} = P_{ix} = mv_x$$

- The mass of this little packet of burned fuel is the mass lost by the rocket: $m_{fuel} = -dm$
- Mathematically, the minus sign tells us that the mass of the burned fuel (the gases) and the rocket mass are changing in opposite directions.
- The gases are ejected toward the left at speed v_{ex} relative to the rocket.
- The infinitesimal change in the rocket's velocity is

$$dv_x = -v_{ex}\frac{dm}{m}$$

Advanced Topic: Rocket Propulsion

- Integrating the rocket's velocity between Before and After, we find that the rocket's velocity when its mass has decreased to m is

$$v = v_{ex} \ln\left(\frac{m_0}{m}\right)$$

- Initially, when $m = m_0$, $v = 0$ because $\ln 1 = 0$
- The maximum speed occurs when the fuel is completely gone and $m = m_R$
- This is

$$v_{max} = v_{ex} \ln\left(\frac{m_R + m_{F0}}{m_R}\right)$$

Example 11.10 Firing a Rocket

EXAMPLE 11.10 | Firing a rocket

Sounding rockets are small rockets used to gather weather data and do atmospheric research. One of the most popular sounding rockets has been the fairly small (10-in-diameter, 16-ft-long) Black Brant III. It is loaded with 210 kg of fuel, has a launch mass of 290 kg, and generates 49 kN of thrust for 9.0 s. What would be the maximum speed of a Black Brant III if launched from rest in deep space?

MODEL We define the system to be the rocket and its exhaust gases. This is an isolated system, its total momentum is conserved, and the rocket's maximum speed is given by Equation 11.44.

Example 11.10 Firing a Rocket

EXAMPLE 11.10 | **Firing a rocket**

SOLVE We're given $m_{F0} = 210$ kg. Knowing that the launch mass is 290 kg, we can deduce that the mass of the empty rocket is $m_R = 80$ kg. Because the rocket burns 210 kg of fuel in 9.0 s, the fuel burn rate is

$$R = \frac{210\text{ kg}}{9.0\text{ s}} = 23.3\text{ kg/s}$$

Knowing the burn rate and the thrust, we can use Equation 11.40 to calculate the exhaust velocity:

$$v_{ex} = \frac{F_{thrust}}{R} = \frac{49{,}000\text{ N}}{23.3\text{ kg/s}} = 2100\text{ m/s}$$

Thus the rocket's maximum speed in deep space would be

$$v_{max} = v_{ex}\ln\left(\frac{m_R + m_{F0}}{m_R}\right) = (2100\text{ m/s})\ln\left(\frac{290\text{ kg}}{80\text{ kg}}\right) = 2700\text{ m/s}$$

Example 11.10 Firing a Rocket

EXAMPLE 11.10 | **Firing a rocket**

ASSESS An actual sounding rocket doesn't reach this speed because it's affected both by gravity and by drag. Even so, the rocket's acceleration is so large that gravity plays a fairly minor role. A Black Brant III launched into the earth's atmosphere achieves a maximum speed of 2100 m/s and, because it continues to coast upward long after the fuel is exhausted, reaches a maximum altitude of 175 km (105 mi).

Chapter 11 Summary Slides

General Principles

Law of Conservation of Momentum

The total momentum $\vec{P} = \vec{p}_1 + \vec{p}_2 + \cdots$ of an isolated system is a constant. Thus

$$\vec{P}_\mathrm{f} = \vec{P}_\mathrm{i}$$

Newton's Second Law

In terms of momentum, Newton's second law is

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Solving Momentum Conservation Problems

MODEL Choose an isolated system or a system that is isolated during at least part of the problem.

VISUALIZE Draw a pictorial representation of the system before and after the interaction.

SOLVE Write the law of conservation of momentum in terms of vector components:

$$(p_{fx})_1 + (p_{fx})_2 + \cdots = (p_{ix})_1 + (p_{ix})_2 + \cdots$$
$$(p_{fy})_1 + (p_{fy})_2 + \cdots = (p_{iy})_1 + (p_{iy})_2 + \cdots$$

ASSESS Is the result reasonable?

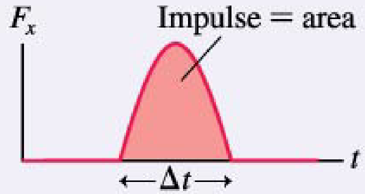
Important Concepts

Momentum $\vec{p} = m\vec{v}$

Impulse $J_x = \int_{t_i}^{t_f} F_x(t)\,dt$ = area under force curve

Impulse and momentum are related by the **momentum principle**

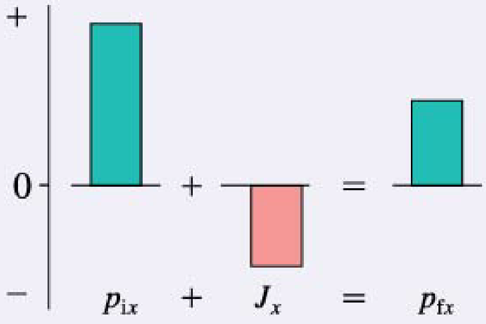
$$\Delta p_x = J_x$$



The impulse delivered to an object causes the object's momentum to change. This is an alternative statement of Newton's second law.

Important Concepts

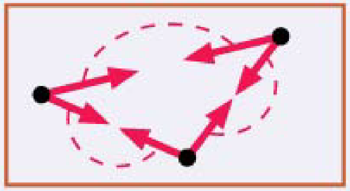
Momentum bar charts display the momentum principle $p_{fx} = p_{ix} + J_x$ in graphical form.



Important Concepts

System A group of interacting particles.

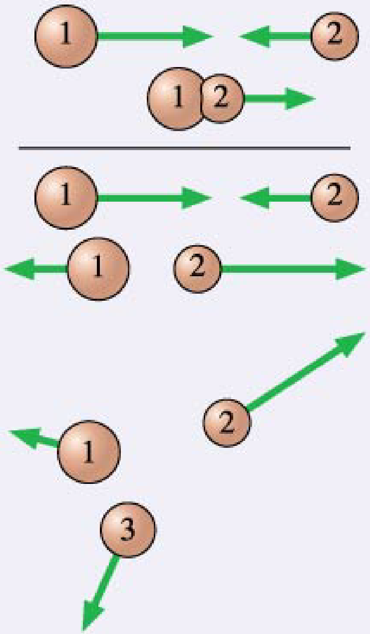
Isolated system A system on which there are no external forces or the net external force is zero.



Applications

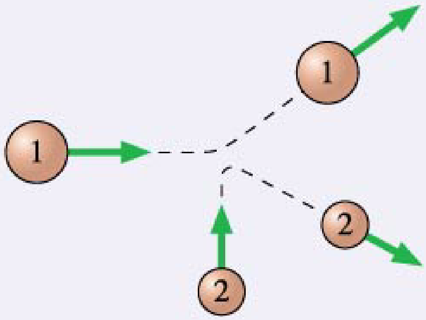
Collisions In a **perfectly inelastic collision,** two objects stick together and move with a common final velocity. In a **perfectly elastic collision,** they bounce apart and conserve mechanical energy as well as momentum.

Explosions Two or more objects fly apart from each other. Their total momentum is conserved.



Applications

Two dimensions The same ideas apply in two dimensions. Both the x- and y-components of $\vec{P}$ must be conserved. This gives two simultaneous equations to solve.



Rockets The momentum of the exhaust-gas + rocket system is conserved. **Thrust** is the product of the exhaust speed and the rate at which fuel is burned.