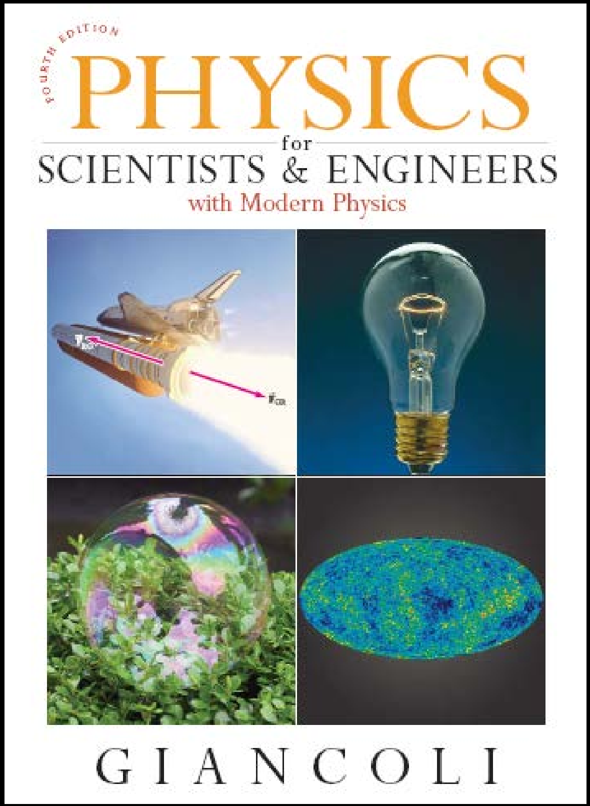


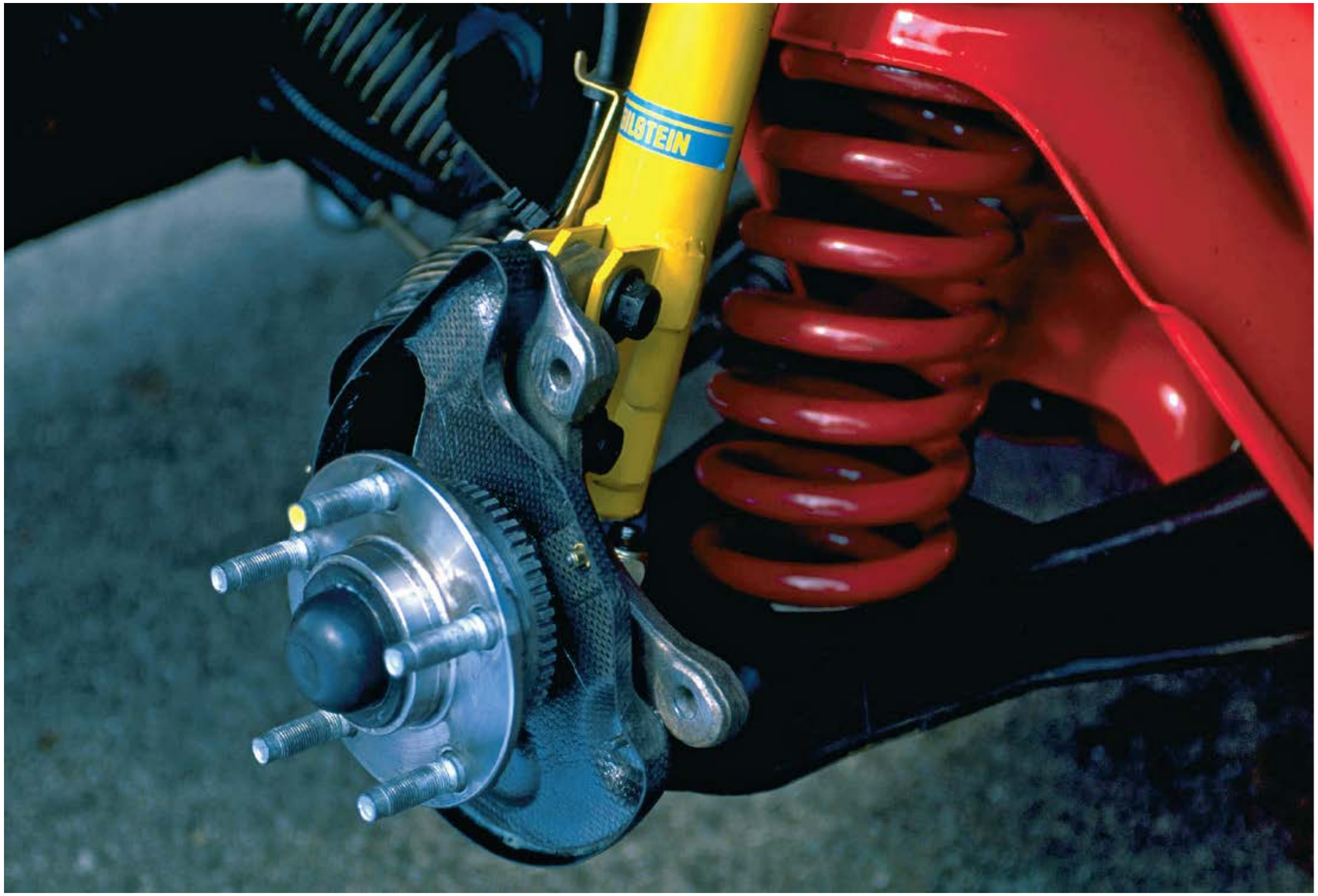
Lecture PowerPoints

Chapter 14

Physics for Scientists and Engineers, with Modern Physics, 4th edition

Giancoli

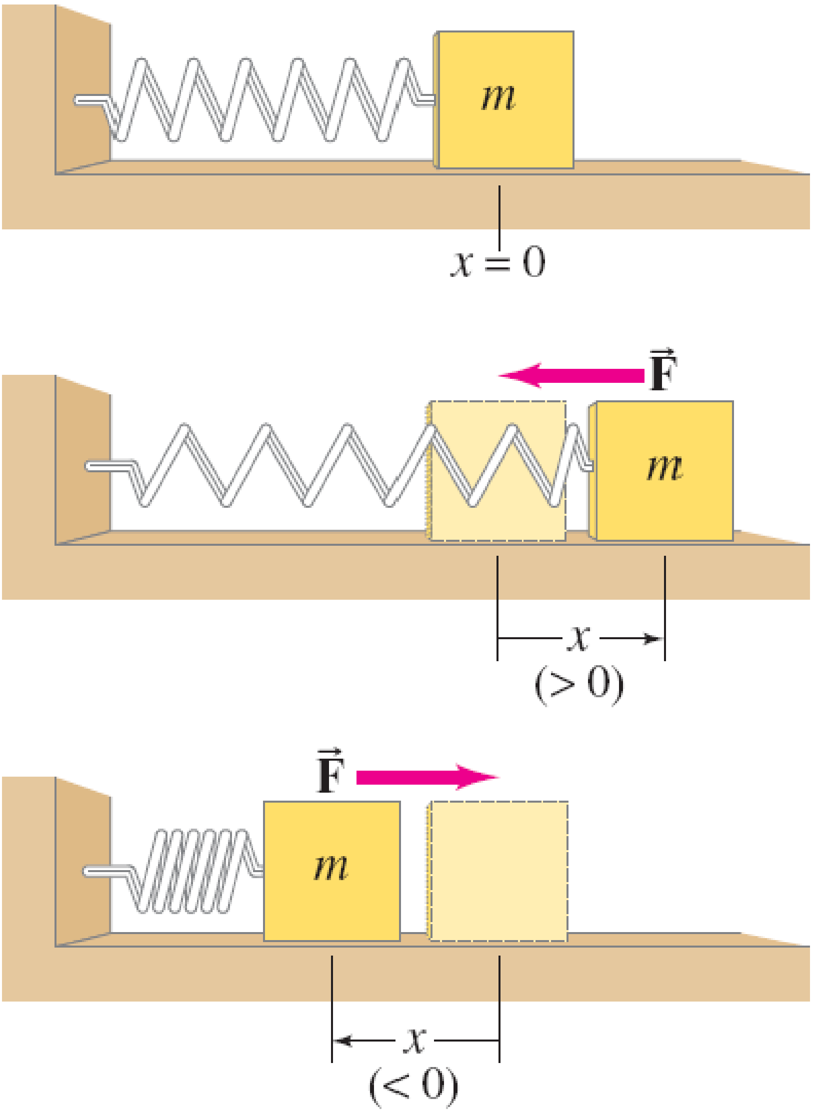
Chapter 14
Oscillations



Units of Chapter 14

- Oscillations of a Spring
- Simple Harmonic Motion
- Energy in the Simple Harmonic Oscillator
- Simple Harmonic Motion Related to Uniform Circular Motion
- The Simple Pendulum

14-1 Oscillations of a Spring



If an object **vibrates** or **oscillates** back and forth over the same path, each cycle taking the same amount of time, the motion is called **periodic**. The **mass** and **spring** system is a useful model for a periodic system.

14-1 Oscillations of a Spring

We assume that the surface is **frictionless**. There is a point where the spring is neither stretched nor compressed; this is the **equilibrium** position. We measure **displacement** from that point (x = 0 on the previous figure).

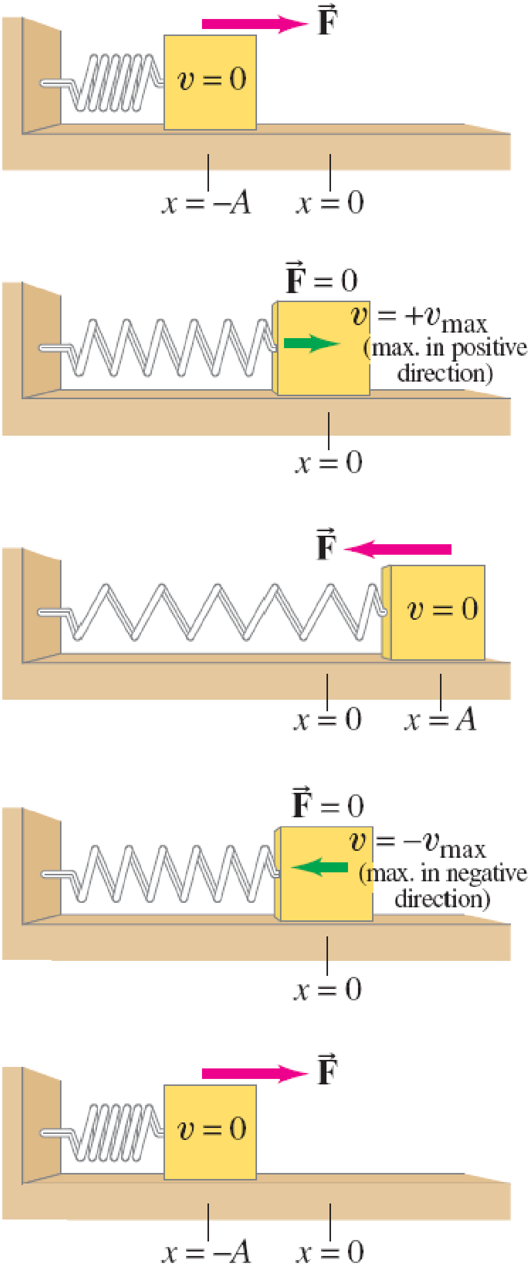
The **force** exerted by the spring depends on the **displacement**:

$$F = -kx.$$

14-1 Oscillations of a Spring

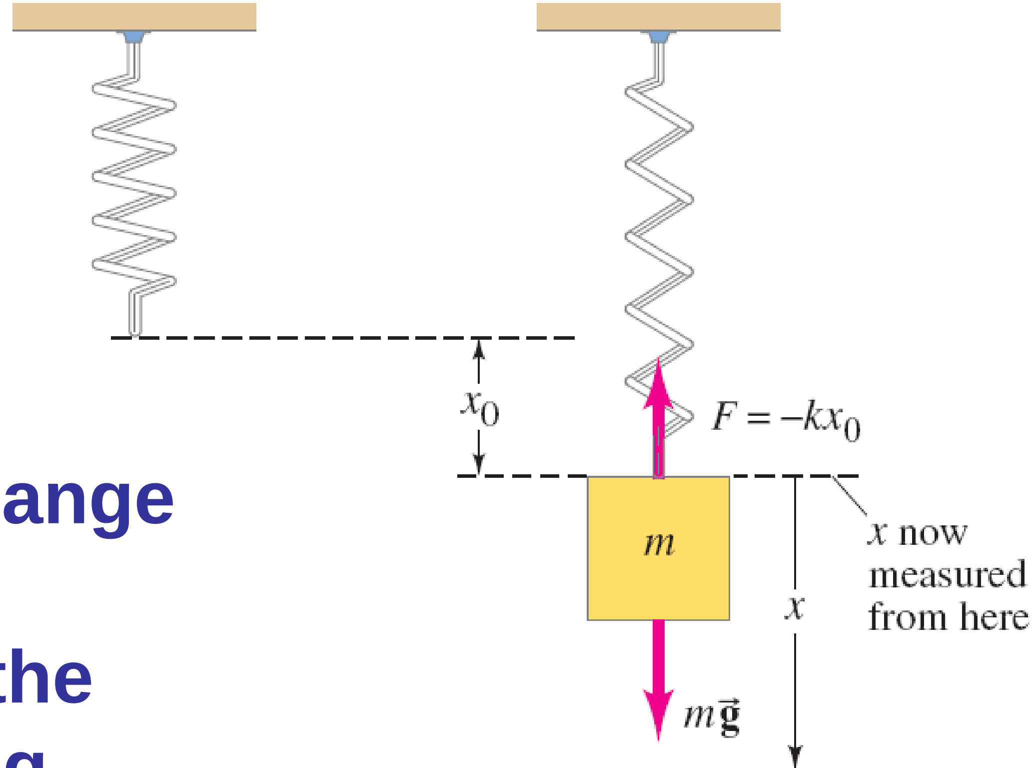
- The **minus** sign on the force indicates that it is a **restoring** force—it is directed to restore the mass to its **equilibrium** position.
- k is the **spring constant.**
- The force is **not constant**, so the acceleration is not constant either.

14-1 Oscillations of a Spring



- **Displacement** is measured from the **equilibrium** point.
- **Amplitude** is the **maximum** displacement.
- A **cycle** is a full to-and-fro motion.
- **Period** is the **time** required to complete one cycle.
- **Frequency** is the **number** of cycles completed per second.

14-1 Oscillations of a Spring



If the spring is hung **vertically**, the only change is in the **equilibrium position**, which is at the point where the spring force equals the gravitational force.

14-2 Simple Harmonic Motion

Any vibrating system where the restoring force is proportional to the negative of the displacement is in **simple harmonic motion (SHM)**, and is often called a **simple harmonic oscillator (SHO)**.

Substituting $F = kx$ into Newton's second law gives the equation of motion:

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0,$$

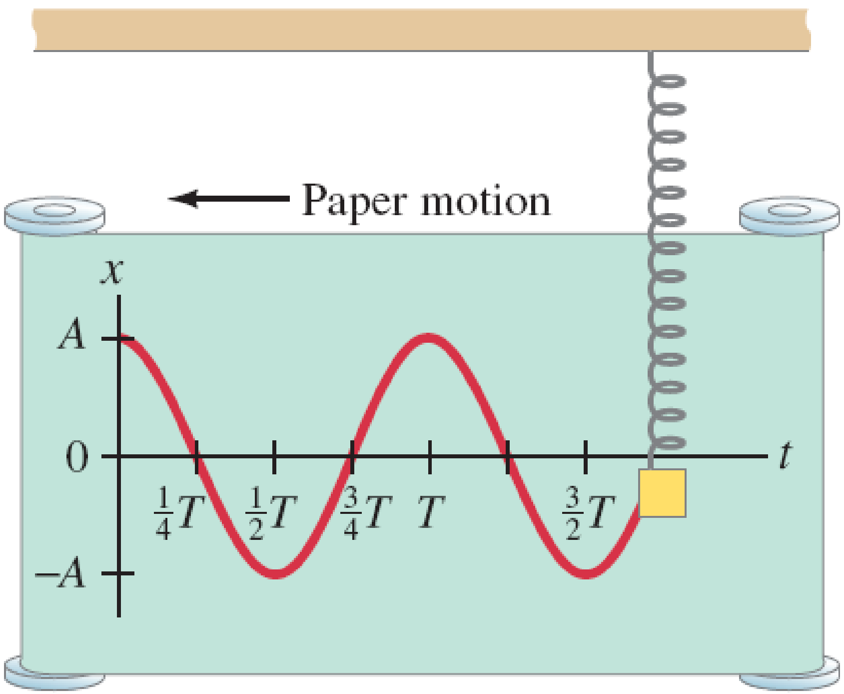
with solutions of the form:

$$x = A\cos(\omega t + \phi).$$

14-2 Simple Harmonic Motion

Substituting, we verify that this solution does indeed satisfy the equation of motion, with:

$$\omega^2 = \frac{k}{m}.$$



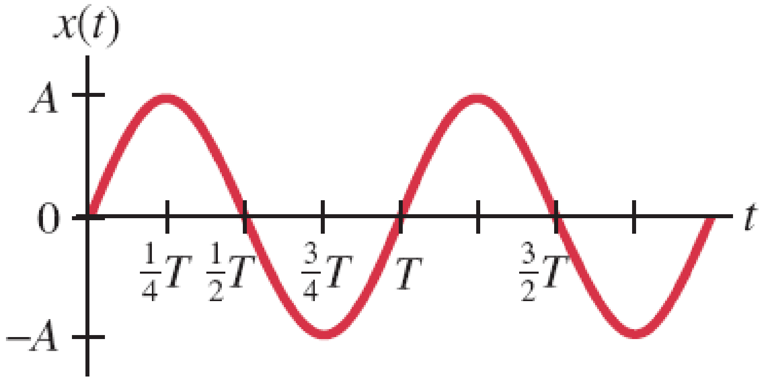
The constants A and φ will be determined by initial conditions; A is the amplitude, and φ gives the phase of the motion at $t = 0$.

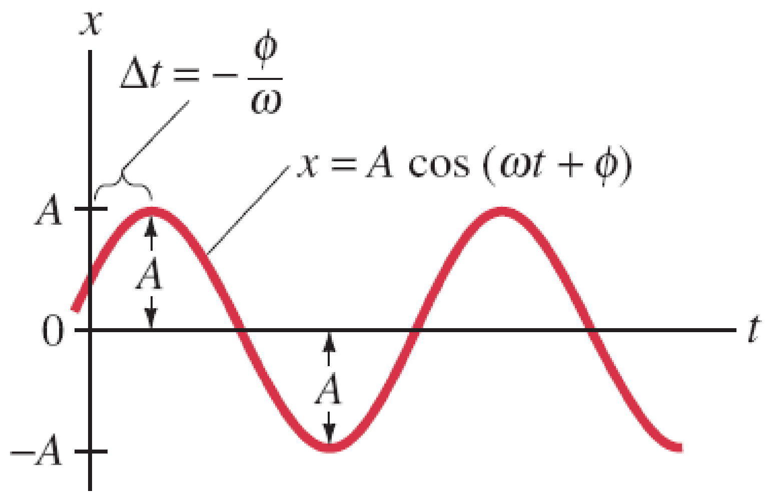
14-2 Simple Harmonic Motion

The velocity can be found by differentiating the displacement:

$$v = \frac{dx}{dt} = \frac{d}{dt}[A\cos(\omega t + \phi)] = -\omega A \sin(\omega t + \phi).$$

These figures illustrate the effect of φ:





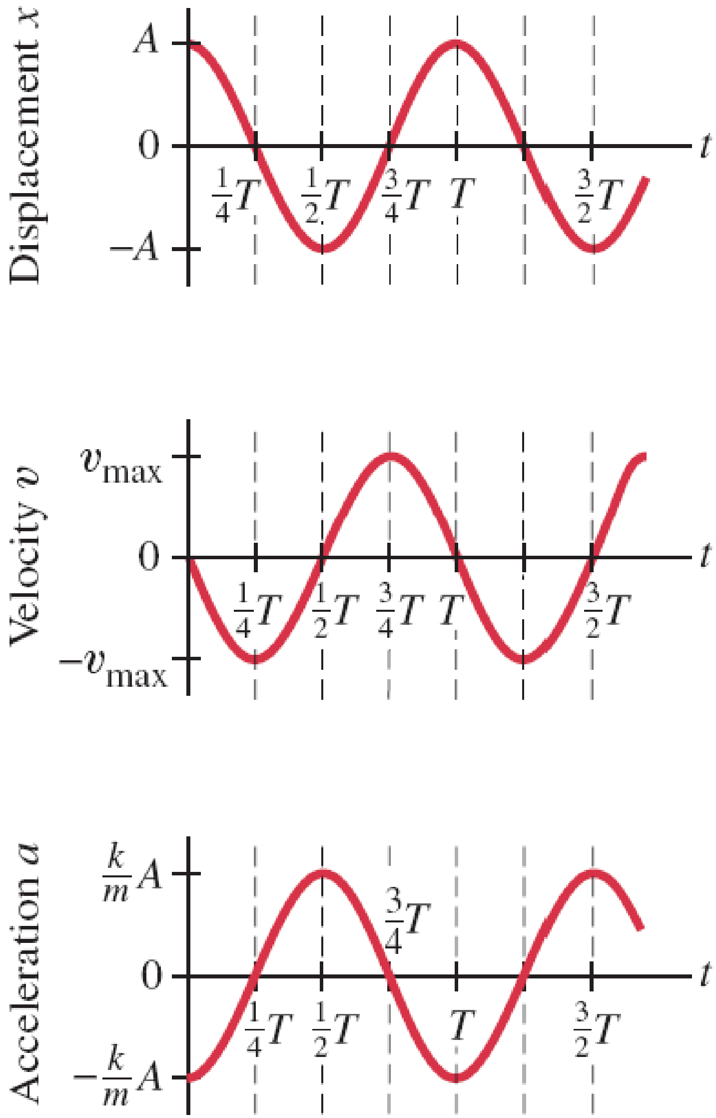
14-2 Simple Harmonic Motion

Because $\omega = 2\pi f = \sqrt{k/m}$, **then**

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}},$$

$$T = 2\pi\sqrt{\frac{m}{k}}.$$

14-2 Simple Harmonic Motion



The velocity and acceleration for simple harmonic motion can be found by differentiating the displacement:

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = -\omega^2 A \cos(\omega t + \phi).$$

14-3 Energy in the Simple Harmonic Oscillator

We already know that the **potential energy** of a spring is given by:

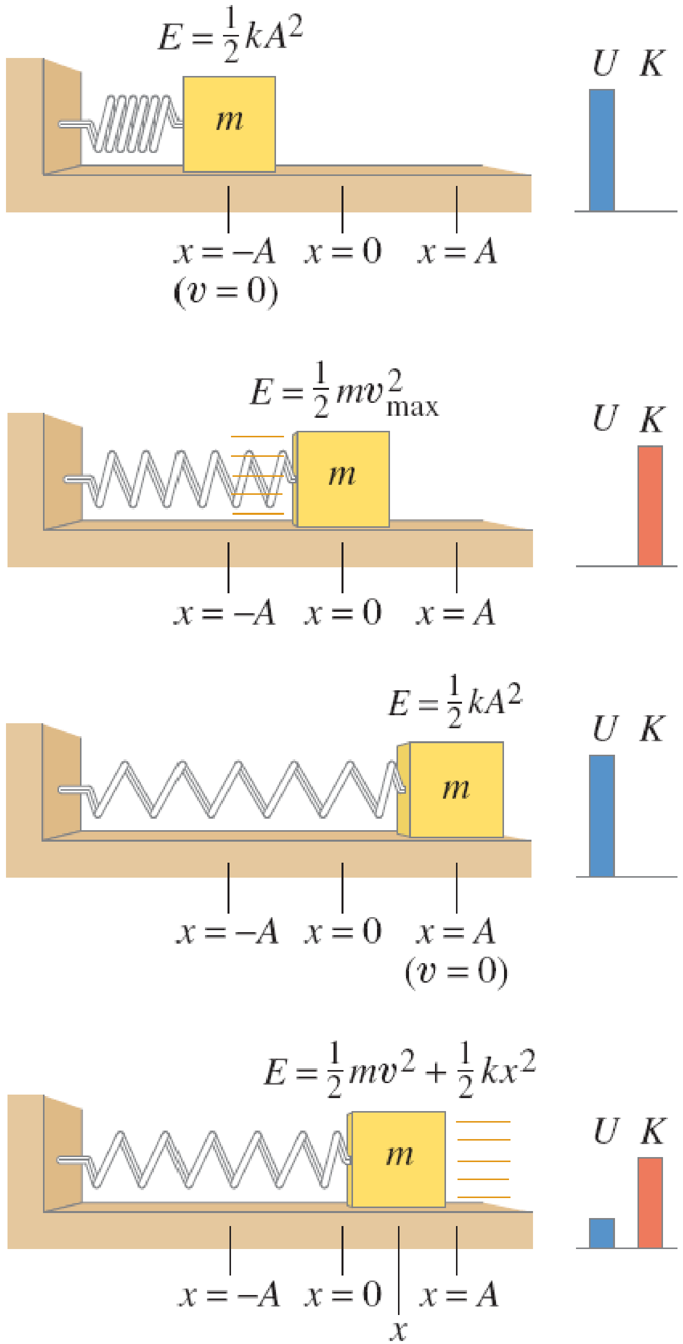
$$U = -\int F\,dx = \tfrac{1}{2}kx^2.$$

The **total mechanical energy** is then:

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2.$$

The total mechanical energy will be **conserved**, as we are assuming the system is **frictionless**.

14-3 Energy in the Simple Harmonic Oscillator



If the mass is at the **limits** of its motion, the energy is all **potential**.

If the mass is at the **equilibrium** point, the energy is all **kinetic**.

We know what the potential energy is at the turning points:

$$E = \frac{1}{2}kA^2.$$

14-3 Energy in the Simple Harmonic Oscillator

The **total energy** is, therefore, $\frac{1}{2}kA^2$.

And we can write:

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2.$$

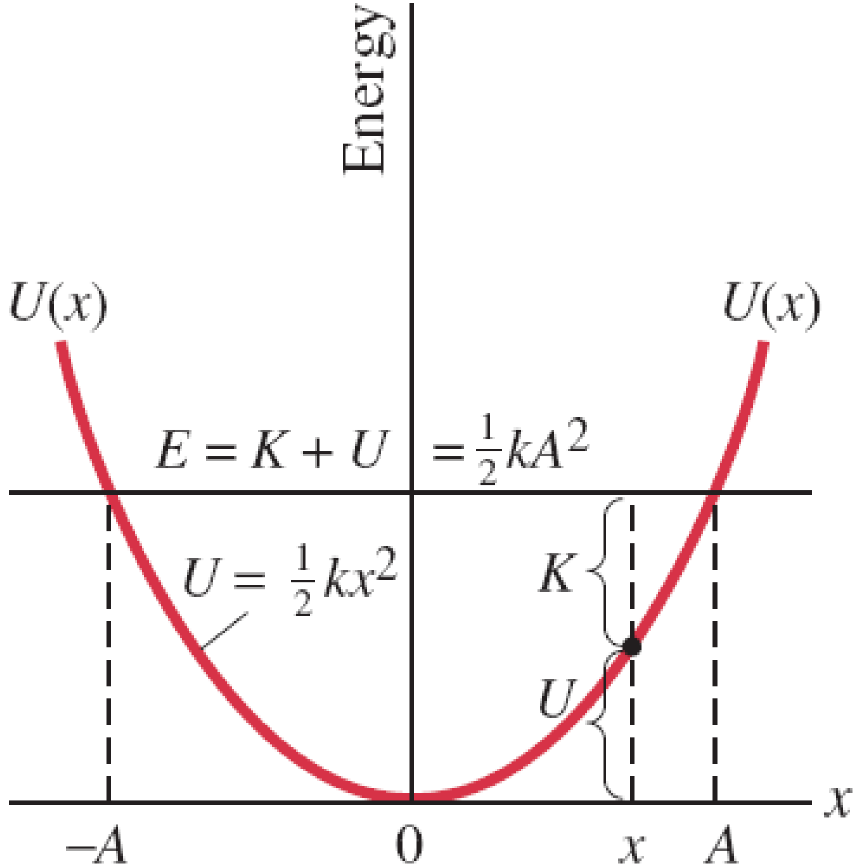
This can be solved for the **velocity** as a function of **position**:

$$v = \pm v_{\text{max}}\sqrt{1 - \frac{x^2}{A^2}},$$

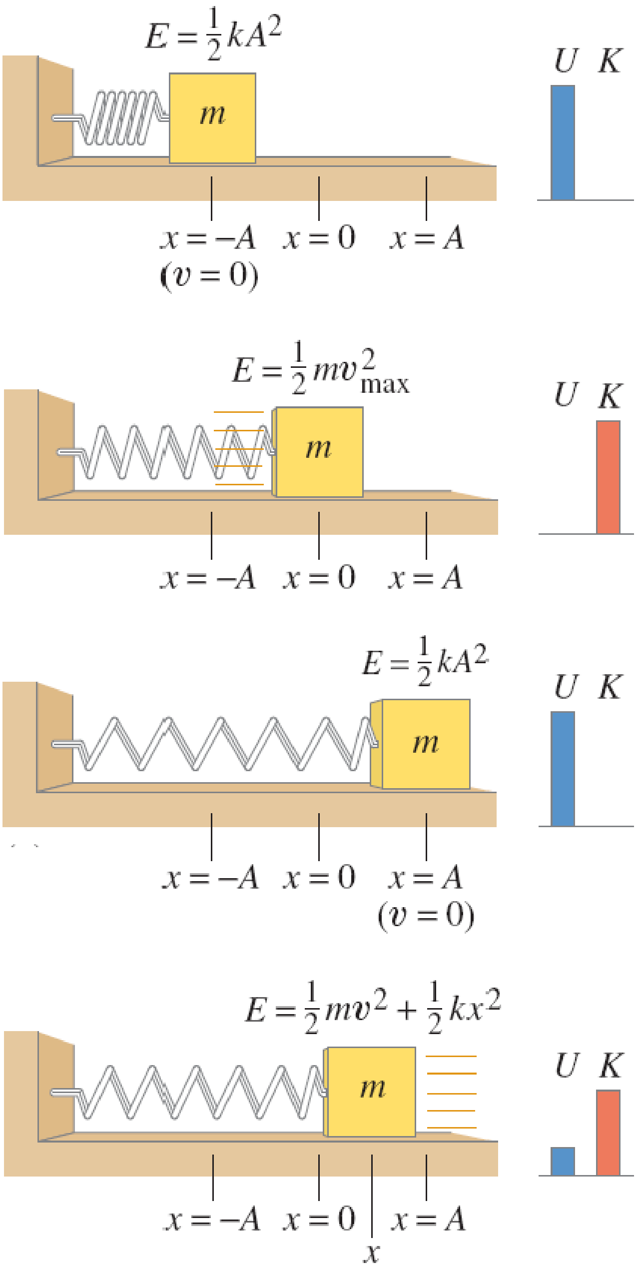
where $$v_{\text{max}}^2 = (k/m)A^2.$$

14-3 Energy in the Simple Harmonic Oscillator

This graph shows the potential energy function of a spring. The total energy is constant.



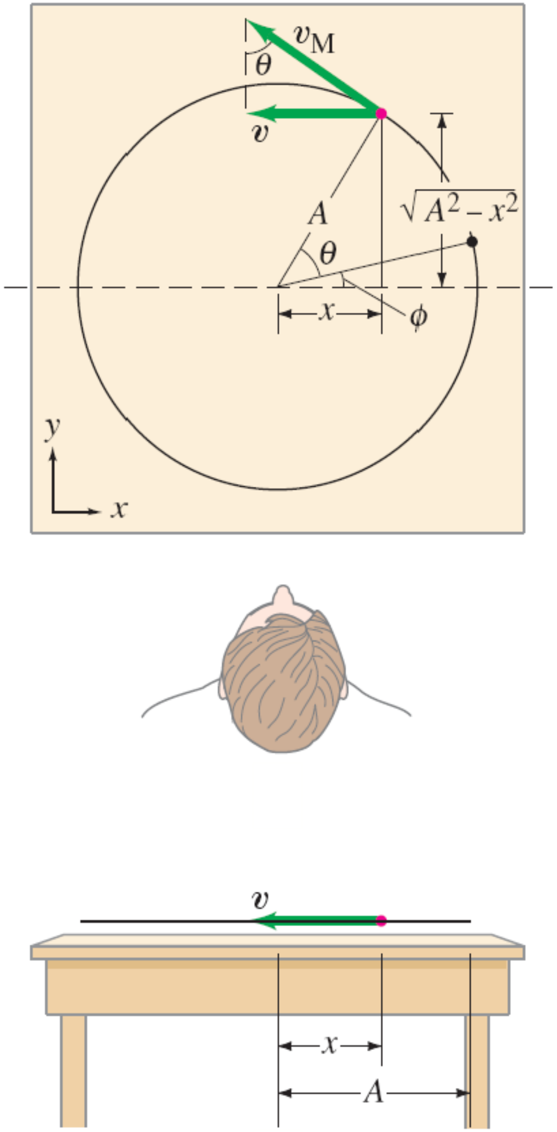
14-3 Energy in the Simple Harmonic Oscillator



Conceptual Example 14-8: Doubling the amplitude.

Suppose this spring is stretched twice as far (to $x = 2A$).What happens to (a) the energy of the system, (b) the maximum velocity of the oscillating mass, (c) the maximum acceleration of the mass?

14-4 Simple Harmonic Motion Related to Uniform Circular Motion

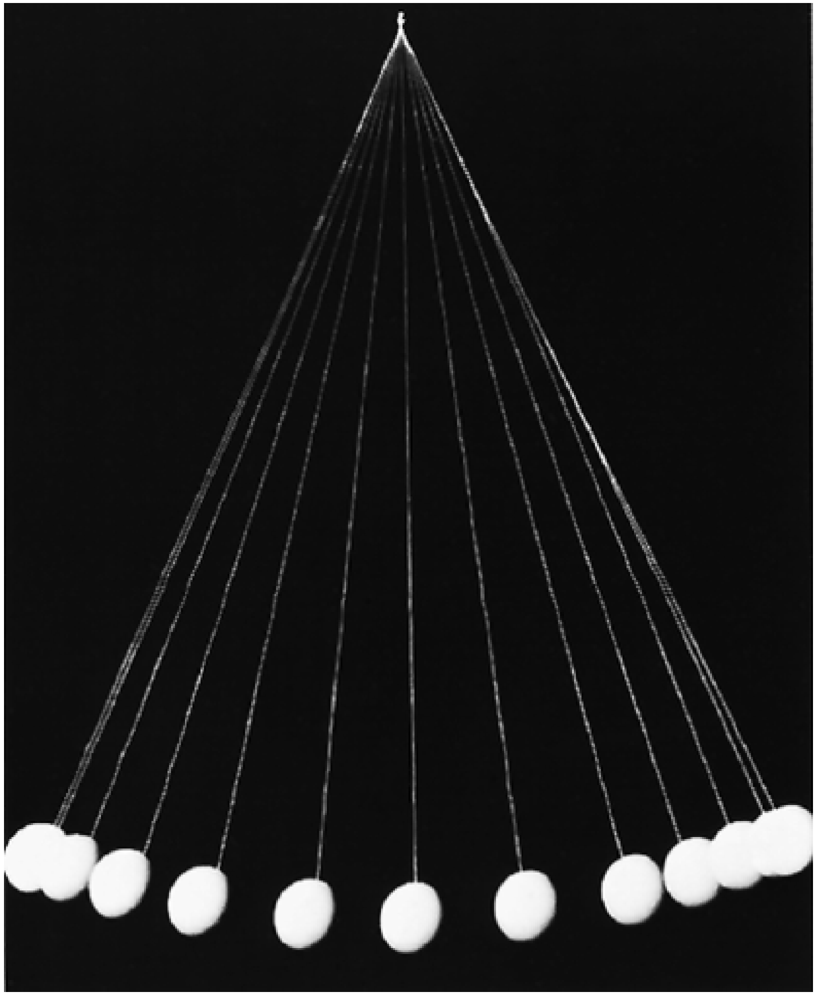


If we look at the projection onto the *x* axis of an object moving in a circle of radius *A* at a constant speed v_M, we find that the *x* component of its velocity varies as:

$$v = v_M \sqrt{1 - \frac{x^2}{A^2}}.$$

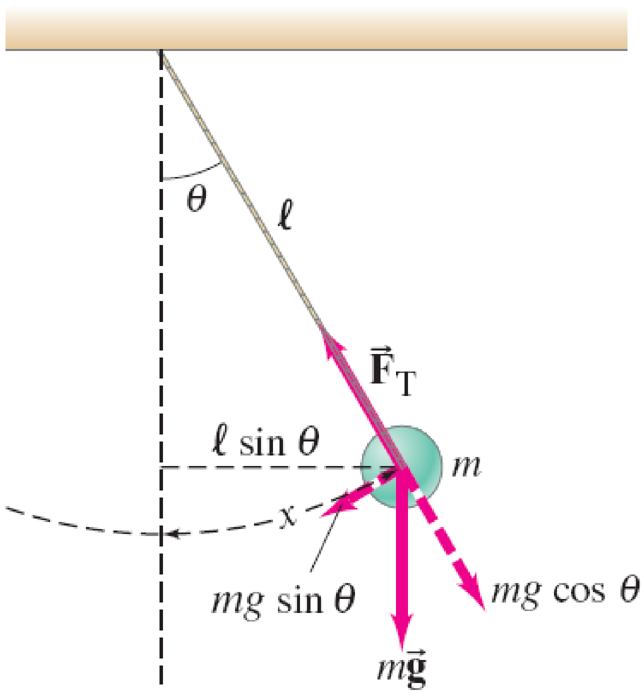
This is identical to SHM.

14-5 The Simple Pendulum



A **simple pendulum** consists of a mass at the end of a lightweight cord. We assume that the cord does not **stretch**, and that its mass is **negligible**.

14-5 The Simple Pendulum



In order to be in SHM, the restoring force must be **proportional** to the negative of the **displacement**. Here we have: $F = -mg\sin\theta,$

which is proportional to **sin *θ*** and not to ***θ*** itself.

However, if the angle is **small**, **sin *θ* ≈ *θ***.

14-5 The Simple Pendulum

Therefore, for small angles, we have:

$$F \approx -\frac{mg}{L}x,$$

where $x = L\theta.$

The period and frequency are:

$$T = 2\pi\sqrt{\frac{L}{g}},$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}.$$

14-5 The Simple Pendulum

So, as long as the cord can be considered **massless** and the amplitude is **small**, the period does not depend on the **mass**.

Summary of Chapter 14

- For SHM, the restoring force is proportional to the displacement.
- The period is the time required for one cycle, and the frequency is the number of cycles per second.
- Period for a mass on a spring: $T = 2\pi \sqrt{\frac{m}{k}}.$
- SHM is sinusoidal.
- During SHM, the total energy is continually changing from kinetic to potential and back.

Summary of Chapter 14

- A simple pendulum approximates SHM if its amplitude is not large. Its period in that case is:

$$T = 2\pi\sqrt{\frac{L}{g}}.$$

- When friction is present, the motion is damped.

- If an oscillating force is applied to a SHO, its amplitude depends on how close to the natural frequency the driving frequency is. If it is close, the amplitude becomes quite large. This is called resonance.