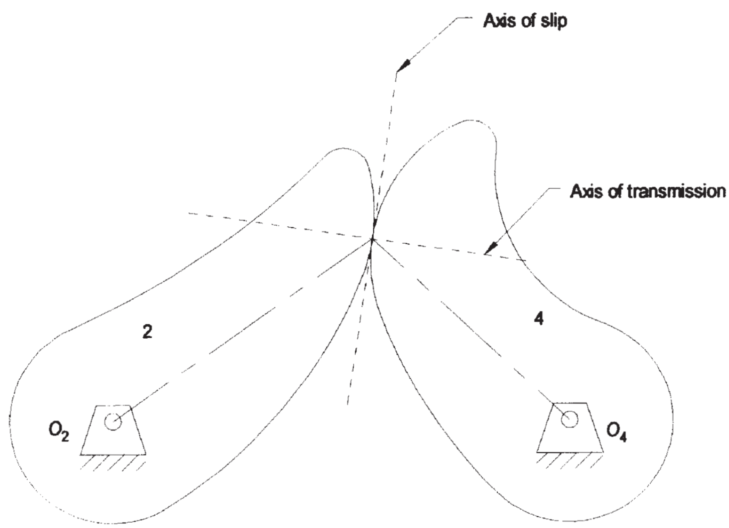
## PROBLEM 8-2

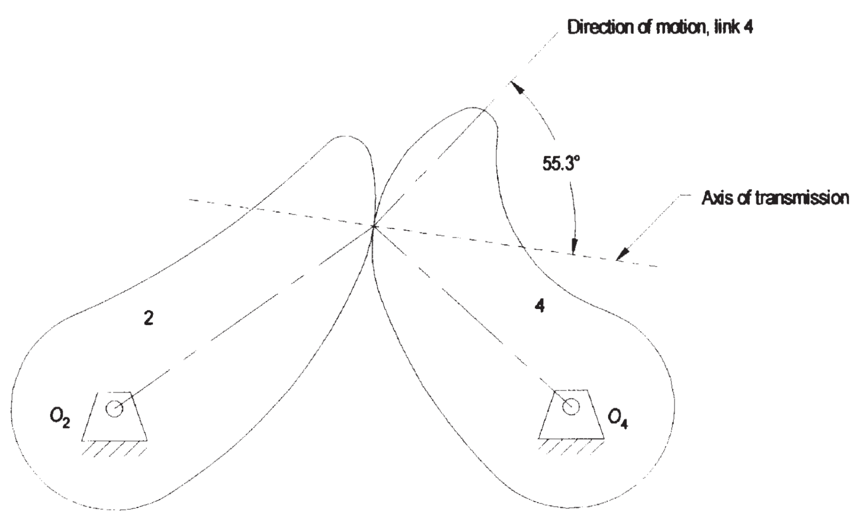
***Statement:*** Figure P8-1 shows the cam and follower from Problem 6-65. Using graphical methods, find the pressure angle at the position shown.

***Solution:*** See Figure P8-1 and Mathcad file P0802.

1. Draw the cam and follower to scale.




2. As defined in Section 8.6 and Figure 8-41, the pressure angle is the angle between the direction of motion of the follower and the direction of the axis of transmission. Establish those two directions and measure the angle between them.



## PROBLEM 8-7

***Statement:*** Design a double-dwell cam to move a follower from 0 to 2.5 in in 60 deg, dwell for 120 deg, fall 2.5 i in 30 deg and dwell for the remainder. The total cycle must take 4 sec. Choose suitable programs for rise and fall to minimize accelerations. Plot the *s v a j* diagrams.

***Given:***

| RISE | DWELL | FALL | DWELL |
|---|---|---|---|
| $\beta_1 := 60 \cdot deg$ | $\beta_2 := 120 \cdot deg$ | $\beta_3 := 30 \cdot deg$ | $\beta_4 := 150 \cdot deg$ |
| $h_1 := 2.5 \cdot in$ | $h_2 := 0 \cdot in$ | $h_3 := 2.5 \cdot in$ | $h_4 := 0 \cdot in$ |

Cycle time: $\tau := 4 \cdot sec$

***Solution:*** See Mathcad file P0807.

1. The camshaft turns $2\pi$ rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \qquad \omega = 1.571 \frac{rad}{sec}$$

2. From Table 8-3, the motion program with lowest acceleration that does not have infinite jerk is the modified trapezoidal. The modified trapezoidal motion is defined in local coordinates by equations 8.15 through 8.19. The numerical constants in these SCCA equations are given in Table 8-2.

$$b := 0.25 \qquad c := 0.50 \qquad d := 0.25$$

$$C_v := 2.0000 \qquad C_a := 4.8881 \qquad C_j := 61.426$$

3. The SCCA equations for the rise or fall interval ($\beta$) are divided into 5 subintervals. These are:

for 0 <= $x$ <= $b$/2 where, for these equations, $x$ is a local coordinate that ranges from 0 to 1,

$$y_1(x) := C_a \cdot \left[ x \cdot \frac{b}{\pi} - \left(\frac{b}{\pi}\right)^2 \cdot \sin\left(\frac{\pi}{b} \cdot x\right) \right] \qquad y'_1(x) := C_a \cdot \frac{b}{\pi} \cdot \left(1 - \cos\left(\frac{\pi}{b} \cdot x\right)\right)$$

$$y''_1(x) := C_a \cdot \sin\left(\frac{\pi}{b} \cdot x\right) \qquad y'''_1(x) := C_a \cdot \frac{\pi}{b} \cdot \cos\left(\frac{\pi}{b} \cdot x\right)$$

for $b$/2 <= $x$ <= (1 - $d$)/2

$$y_2(x) := C_a \cdot \left[ \frac{x^2}{2} + b \cdot \left(\frac{1}{\pi} - \frac{1}{2}\right) \cdot x + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2}\right) \right] \qquad y'_2(x) := C_a \cdot \left[ x + b \cdot \left(\frac{1}{\pi} - \frac{1}{2}\right) \right]$$

$$y''_2(x) := C_a \qquad y'''_2(x) := 0$$

for (1 - $d$)/2 <= $x$ <= (1 + $d$)/2

$$y_3(x) := C_a \cdot \left[ \left(\frac{b}{\pi} + \frac{c}{2}\right) \cdot x + \left(\frac{d}{\pi}\right)^2 + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2}\right) - \frac{(1-d)^2}{8} - \left(\frac{d}{\pi}\right)^2 \cdot \cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \right]$$

$$y'_3(x) := C_a \cdot \left[ \frac{b}{\pi} + \frac{c}{2} + \frac{d}{\pi} \cdot \sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \right]$$

$$y''_3(x) := C_a \cdot \cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \qquad y'''_3(x) := -C_a \cdot \frac{\pi}{d} \cdot \sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right]$$

for $(1 + d)/2 <= x <= 1 - b/2$

$$y_4(x) := C_a \cdot \left[ -\frac{x^2}{2} + \left( \frac{b}{\pi} + 1 - \frac{b}{2} \right) \cdot x + \left( 2 \cdot d^2 - b^2 \right) \cdot \left( \frac{1}{\pi^2} - \frac{1}{8} \right) - \frac{1}{4} \right]$$

$$y'_4(x) := C_a \cdot \left( -x + \frac{b}{\pi} + 1 - \frac{b}{2} \right) \qquad y''_4(x) := -C_a \qquad y'''_4(x) := 0$$

for $1 - b/2 <= x <= 1$

$$y_5(x) := C_a \cdot \left[ \frac{b}{\pi} \cdot x + \frac{2 \cdot \left( d^2 - b^2 \right)}{\pi^2} + \frac{(1 - b)^2 - d^2}{4} - \left( \frac{b}{\pi} \right)^2 \cdot sin\left[ \frac{\pi}{b} \cdot (x - 1) \right] \right]$$

$$y'_5(x) := C_a \cdot \frac{b}{\pi} \cdot \left[ 1 - cos\left[ \frac{\pi}{b} \cdot (x - 1) \right] \right]$$

$$y''_5(x) := C_a \cdot sin\left[ \frac{\pi}{b} \cdot (x - 1) \right] \qquad y'''_5(x) := C_a \cdot \frac{\pi}{b} \cdot cos\left[ \frac{\pi}{b} \cdot (x - 1) \right]$$

4. **The above equations can be used for a rise or fall by using the proper values of θ, β, and h. To plot the *SVAJ* curves, first define a range function that has a value of one between the values of *a* and *b* and zero elsewhere.**

$$R(x,x1,x2) := if[(x > x1) \wedge (x \le x2), 1, 0]$$

5. **The global *SVAJ* equations are composed of four intervals (rise, dwell, fall, and dwell). The local equations above must be assembled into a single equation each for *S*, *V*, *A*, and *J* that applies over the range 0 <= θ <= 360 deg.**

6. **Write the local *svaj* equations for the first interval, $0 <= \theta <= \beta_1$. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.**

**For $0 <= \theta <= \beta_1$ (Rise)**

$$s_1(x) := h_1 \cdot \left( R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right)$$

$$v_1(x) := \frac{h_1}{\beta_1} \cdot \left( R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x) \right)$$

$$a_1(x) := \frac{h_1}{\beta_1^2} \cdot \left( R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x) \right)$$

$$j_1(x) := \frac{h_1}{\beta_1^3} \cdot \left( R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x) \right)$$

7. Write the local *svaj* equations for the second interval, $\beta_1 <= \theta <= \beta_1 + \beta_2$. For this interval, the value of *S* is the value of *S* at the end of the previous interval and the values of *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 <= \theta <= \beta_1 + \beta_2$

$$s_2(x) := h_1 \qquad v_2(x) := 0 \qquad a_2(x) := 0 \qquad j_2(x) := 0$$

8. Write the local *svaj* equations for the third interval, $\beta_1 + \beta_2 <= \theta <= \beta_1 .+ \beta_2 + \beta_3$.

For $\beta_1 + \beta_2 <= \theta <= \beta_1 .+ \beta_2 + \beta_3$

$$s_3(x) := h_3\left[1 - \left(R\left(x,0,\frac{b}{2}\right)\cdot y_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y_5(x)\right)\right]$$

$$v_3(x) := \frac{h_3}{\beta_3}\cdot\left(R\left(x,0,\frac{b}{2}\right)\cdot y'_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y'_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y'_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y'_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y'_5(x)\right)$$

$$a_3(x) := \frac{h_3}{\beta_3^{\,2}}\cdot\left(R\left(x,0,\frac{b}{2}\right)\cdot y''_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y''_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y''_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y''_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y''_5(x)\right)$$

$$j_3(x) := \frac{h_3}{\beta_3^{\,3}}\cdot\left(R\left(x,0,\frac{b}{2}\right)\cdot y'''_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y'''_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y'''_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y'''_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y'''_5(x)\right)$$

9. Write the local *svaj* equations for the fourth interval, $\beta_1 + \beta_2 + \beta_3 <= \theta <= \beta_1 .+ \beta_2 + \beta_3 + \beta_4$. For this interval, the values of *S*, *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 + \beta_2 + \beta_3 <= \theta <= \beta_1 .+ \beta_2 + \beta_3 + \beta_4$

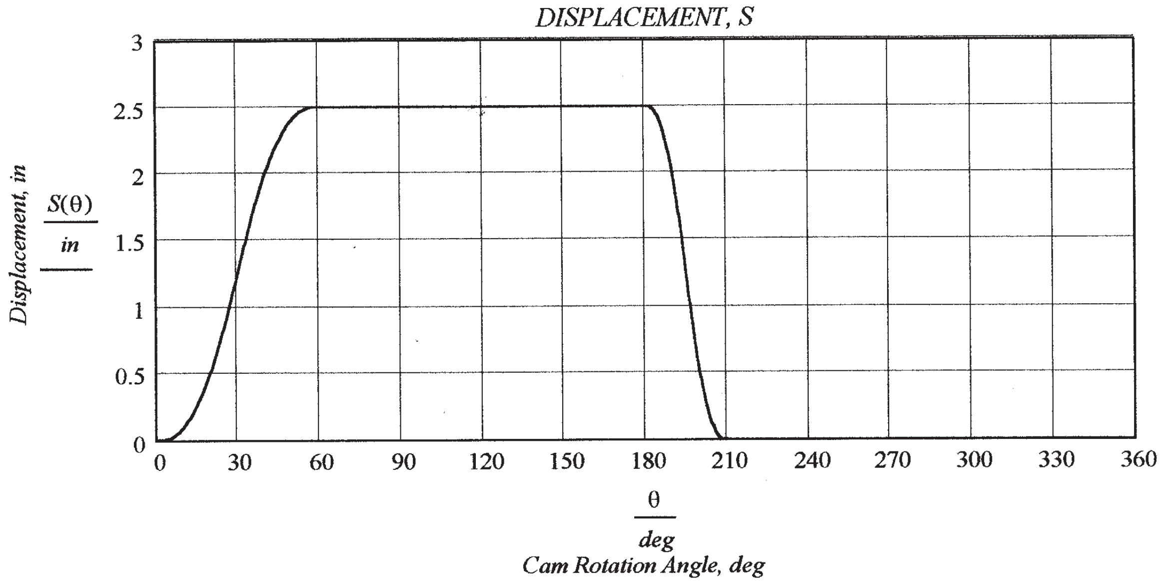
$$s_4(x) := 0 \qquad v_4(x) := 0 \qquad a_4(x) := 0 \qquad j_4(x) := 0$$

10. Write the complete global equation for the displacement and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

Let $\theta_1 := \beta_1 \qquad \theta_2 := \theta_1 + \beta_2 \qquad \theta_3 := \theta_2 + \beta_3 \qquad \theta_4 := \theta_3 + \beta_4$

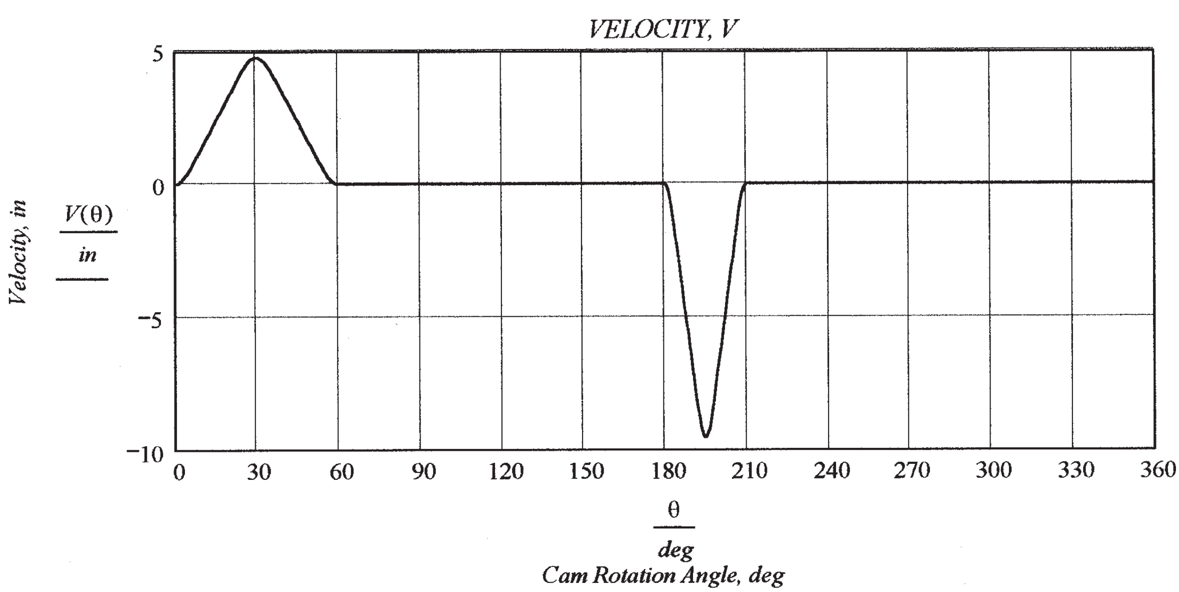
$$S(\theta) := s_1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot s_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \ldots \\ + R(\theta,\theta_2,\theta_3)\cdot s_3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot s_4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right)$$

$\theta := 0\cdot deg, 0.5\cdot deg .. 360\cdot deg$



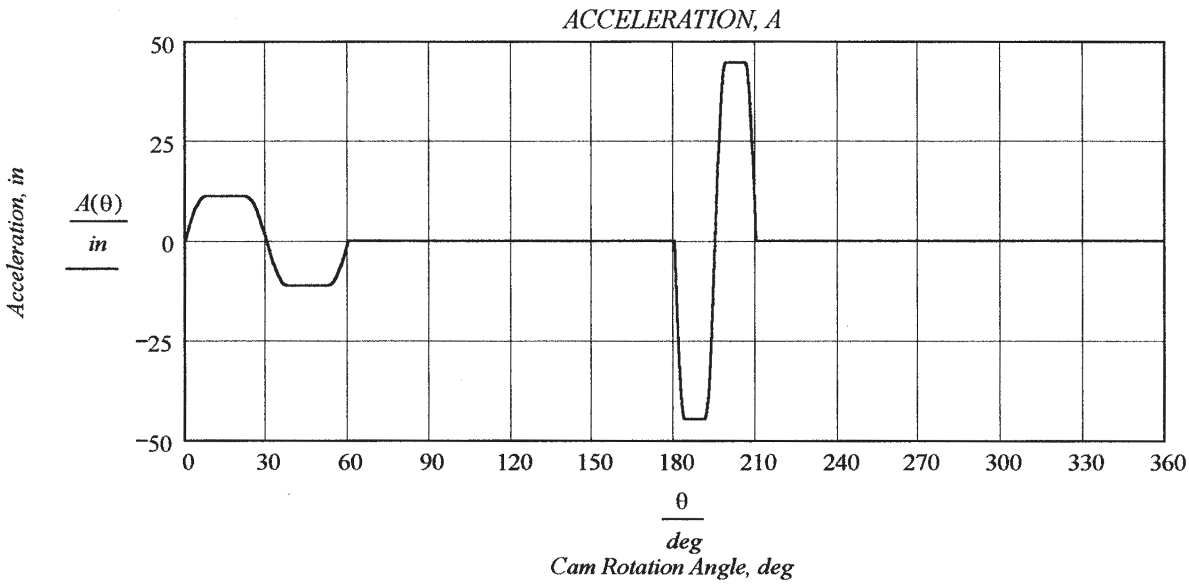

11. Write the complete global equation for the velocity and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$V(\theta) := v_1\left(\frac{\theta}{\theta_1}\right) + R(\theta, \theta_1, \theta_2)\cdot v_2\left(\frac{\theta - \theta_1}{\theta_2 - \theta_1}\right) \ldots$$
$$+ R(\theta, \theta_2, \theta_3)\cdot v_3\left(\frac{\theta - \theta_2}{\theta_3 - \theta_2}\right) + R(\theta, \theta_3, \theta_4)\cdot v_4\left(\frac{\theta - \theta_3}{\theta_4 - \theta_3}\right)$$



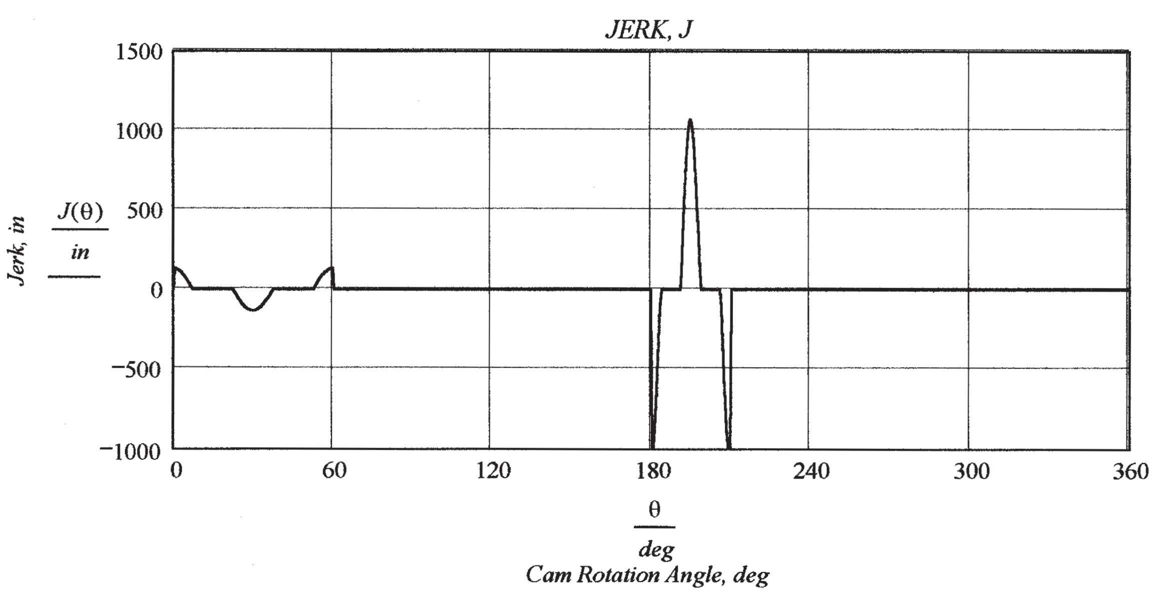

12. Write the complete global equation for the acceleration and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$A(\theta) := a_1\left(\frac{\theta}{\theta_1}\right) + R(\theta, \theta_1, \theta_2)\cdot a_2\left(\frac{\theta - \theta_1}{\theta_2 - \theta_1}\right) \ldots$$
$$+ R(\theta, \theta_2, \theta_3)\cdot a_3\left(\frac{\theta - \theta_2}{\theta_3 - \theta_2}\right) + R(\theta, \theta_3, \theta_4)\cdot a_4\left(\frac{\theta - \theta_3}{\theta_4 - \theta_3}\right)$$




13. Write the complete global equation for the jerk and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$J(\theta) := j1\left(\frac{\theta}{\theta_1}\right) + R(\theta, \theta_1, \theta_2)\cdot j2\left(\frac{\theta - \theta_1}{\theta_2 - \theta_1}\right) \dots$$
$$+ R(\theta, \theta_2, \theta_3)\cdot j3\left(\frac{\theta - \theta_2}{\theta_3 - \theta_2}\right) + R(\theta, \theta_3, \theta_4)\cdot j4\left(\frac{\theta - \theta_3}{\theta_4 - \theta_3}\right)$$



## PROBLEM 8-9

***Statement:*** Design a single-dwell cam to move a follower from 0 to 2.0 in in 60 deg, fall 2.0 in in 90 deg and dwell for the remainder. The total cycle must take 2 sec. Choose suitable programs for rise and fall to minimize velocities. Plot the *SVAJ* diagrams.

***Given:***

| RISE/FALL | DWELL |
|---|---|
| $\beta := 150 \cdot deg$ | $\beta_3 := 210 \cdot deg$ |
| $h := 2.0 \cdot in$ | $h_3 := 0.0 \cdot in$ |

Cycle time: $\tau := 2 \cdot sec$

***Solution:*** See Mathcad file P0809.

1. The camshaft turns $2\pi$ rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \qquad \omega = 3.142 \frac{rad}{sec}$$

2. Use a two-segment polynomial. Let the rise and fall, together, be one segment and the dwell be the second segment. Then, the boundary conditions are:

at
$\theta = 0$: $s = 0$, $v = 0$, $a = 0$
$\theta = \beta_1$: $s = h$, $v = 0$
$\theta = \beta$: $s = 0$, $v = 0$, $a = 0$

This is a minimum set of 8 BCs. The v = 0 condition at $\theta = \beta_1$ is required to keep the displacement from over-shooting the lift, *h*. Define the total lift, the rise interval, the fall interval, and the ratio of rise to the total interval.

Total lift: $h = 2.000\ in$

Rise interval: $\beta_1 := 60 \cdot deg$ $\qquad A := \frac{\beta_1}{\beta} \qquad A = 0.400$

Fall interval: $\beta_2 := 90 \cdot deg$

3. Use the 8 BCs and equation 8.23 to write 8 equations in *s*, *v*, and *a* similar to those in example 8-9 but with 8 terms in the equation for *s* (the highest term will be seventh degree).

For $\theta = 0$: $s = v = a = 0$

$$0 := c_0 \qquad 0 := c_1 \qquad 0 := c_2$$

For $\theta = \beta_1$: $s = h$, $v = 0$

$$h := c_3 \cdot A^3 + c_4 \cdot A^4 + c_5 \cdot A^5 + c_6 \cdot A^6 + c_7 \cdot A^7$$

$$0 := 3 \cdot c_3 \cdot A^2 + 4 \cdot c_4 \cdot A^3 + 5 \cdot c_5 \cdot A^4 + 6 \cdot c_6 \cdot A^5 + 7 \cdot c_7 \cdot A^6$$

For $\theta = \beta$: $s = v = a = 0$

$$0 := c_3 + c_4 + c_5 + c_6 + c_7$$

$$0 := 3 \cdot c_3 + 4 \cdot c_4 + 5 \cdot c_5 + 6 \cdot c_6 + 7 \cdot c_7$$

$$0 := 6 \cdot c_3 + 12 \cdot c_4 + 20 \cdot c_5 + 30 \cdot c_6 + 42 \cdot c_7$$

4. Solve for the unknown polynomial coefficients. Note that $C_0$ through $C_2$ are zero

$$C := \begin{pmatrix} A^3 & A^4 & A^5 & A^6 & A^7 \\ 3\cdot A^2 & 4\cdot A^3 & 5\cdot A^4 & 6\cdot A^5 & 7\cdot A^6 \\ 1 & 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 6 & 7 \\ 6 & 12 & 20 & 30 & 42 \end{pmatrix} \qquad H := \begin{pmatrix} h \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{pmatrix} := C^{-1}\cdot H$$

$c_3 = 289.352\,in$ $\quad c_4 = -1229.745\,in$ $\quad c_5 = 1953.125\,in$

$c_6 = -1374.421\,in$ $\quad c_7 = 361.690\,in$

5. Write the *SVAJ* equations for the rise/fall segment.

$$S(\theta) := c_3\cdot\left(\frac{\theta}{\beta}\right)^3 + c_4\cdot\left(\frac{\theta}{\beta}\right)^4 + c_5\cdot\left(\frac{\theta}{\beta}\right)^5 + c_6\cdot\left(\frac{\theta}{\beta}\right)^6 + c_7\cdot\left(\frac{\theta}{\beta}\right)^7$$

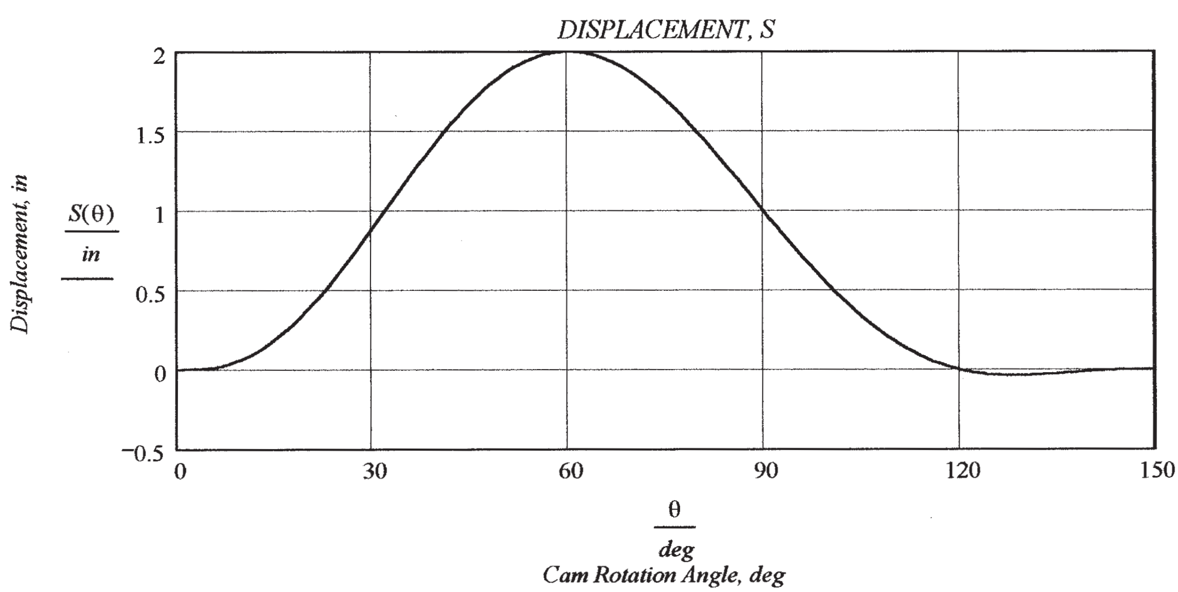
$$V(\theta) := \frac{1}{\beta}\cdot\left[3\cdot c_3\cdot\left(\frac{\theta}{\beta}\right)^2 + 4\cdot c_4\cdot\left(\frac{\theta}{\beta}\right)^3 + 5\cdot c_5\cdot\left(\frac{\theta}{\beta}\right)^4 + 6\cdot c_6\cdot\left(\frac{\theta}{\beta}\right)^5 + 7\cdot c_7\cdot\left(\frac{\theta}{\beta}\right)^6\right]$$

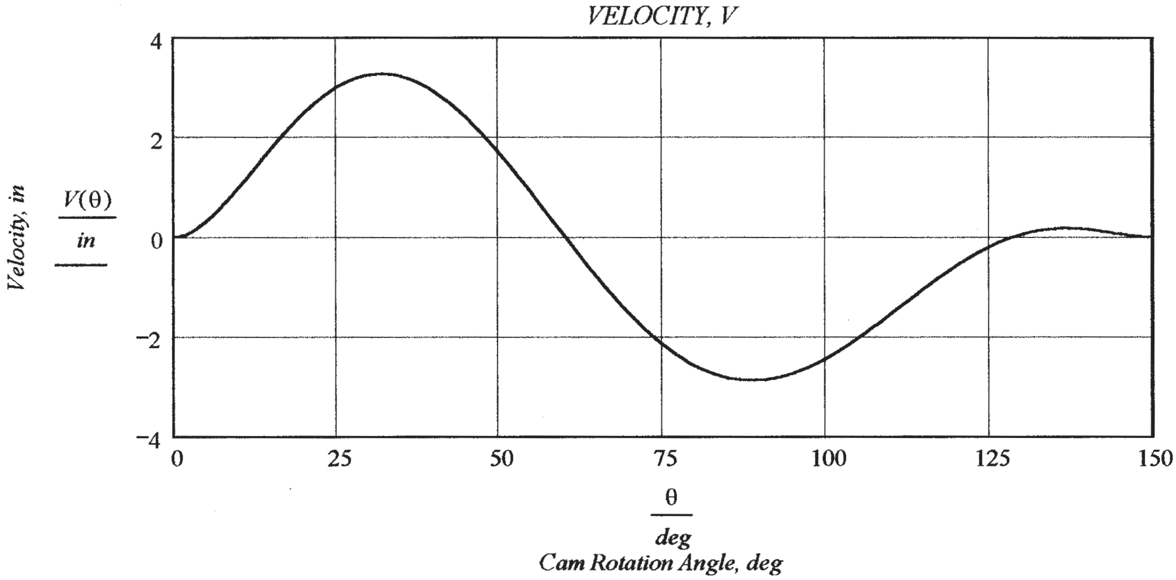
$$A(\theta) := \frac{1}{\beta^2}\cdot\left[6\cdot c_3\cdot\left(\frac{\theta}{\beta}\right) + 12\cdot c_4\cdot\left(\frac{\theta}{\beta}\right)^2 + 20\cdot c_5\cdot\left(\frac{\theta}{\beta}\right)^3 + 30\cdot c_6\cdot\left(\frac{\theta}{\beta}\right)^4 + 42\cdot c_7\cdot\left(\frac{\theta}{\beta}\right)^5\right]$$

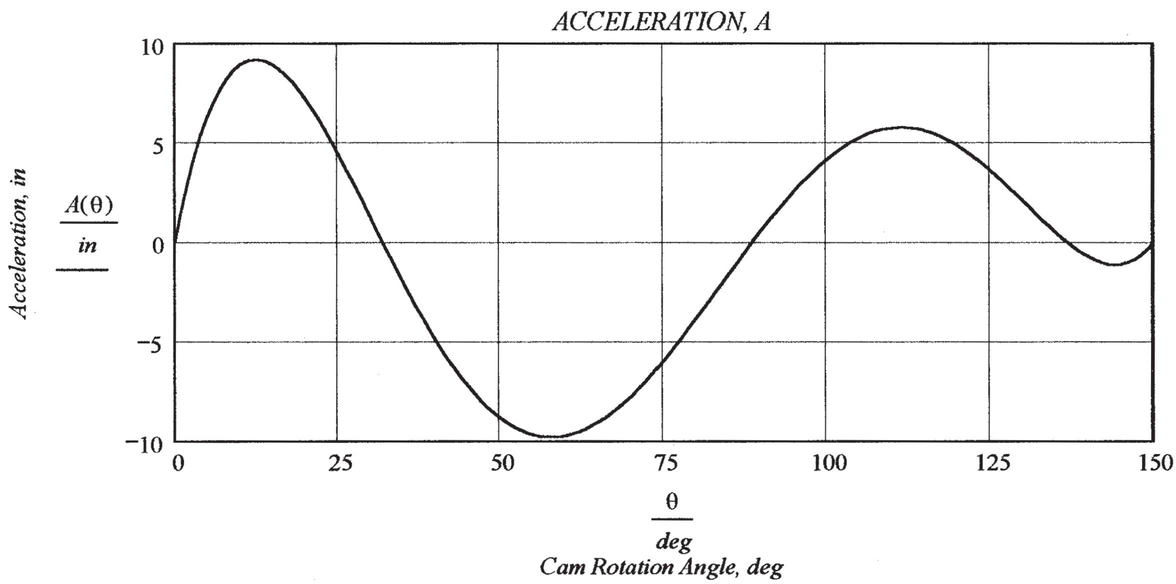
$$J(\theta) := \frac{1}{\beta^3}\cdot\left[6\cdot c_3 + 24\cdot c_4\cdot\left(\frac{\theta}{\beta}\right) + 60\cdot c_5\cdot\left(\frac{\theta}{\beta}\right)^2 + 120\cdot c_6\cdot\left(\frac{\theta}{\beta}\right)^3 + 210\cdot c_7\cdot\left(\frac{\theta}{\beta}\right)^4\right]$$

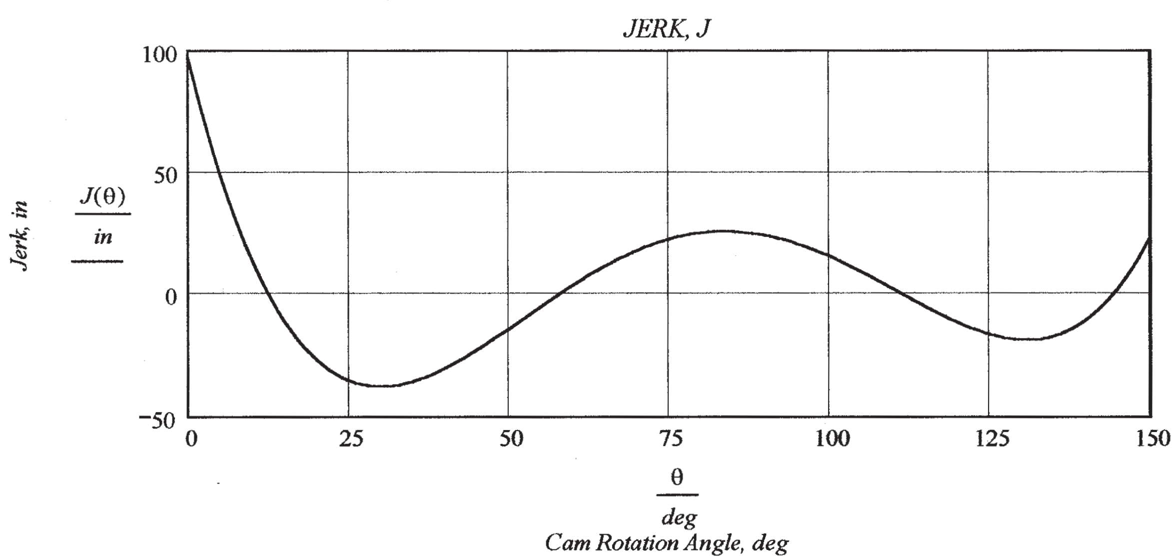
6. Plot the displacement, velocity, acceleration, and jerk over the interval $0 <= \theta <= \beta$.

$\theta := 0\cdot deg, 1\cdot deg .. \beta$

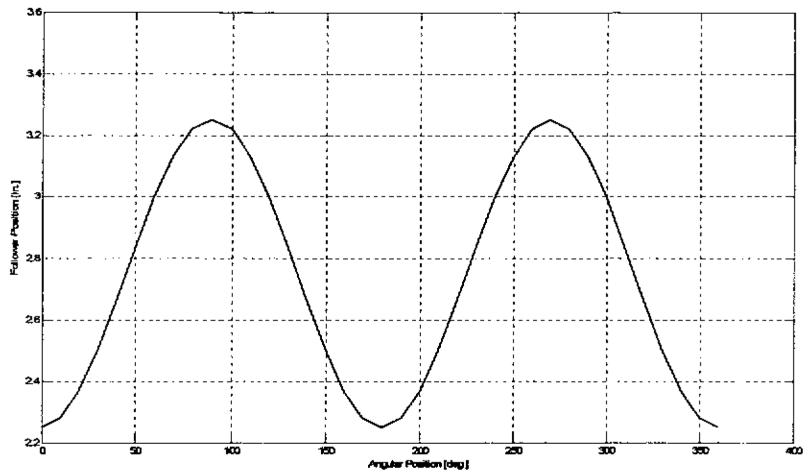



VELOCITY, V
4
2
0
−2
−4
Velocity, in
V(θ)
in
0
25
50
75
100
125
150
θ
deg
Cam Rotation Angle, deg

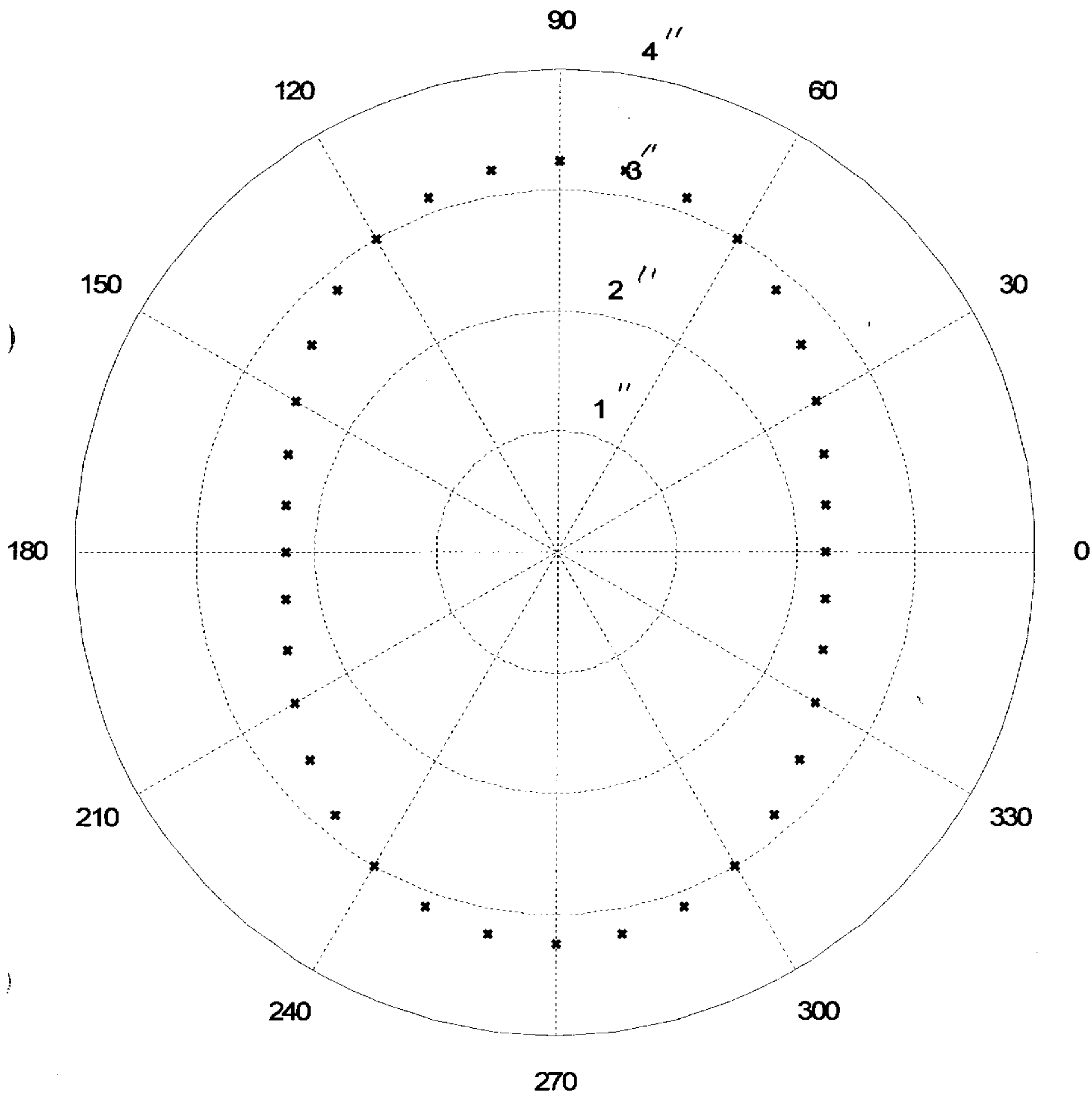

ACCELERATION, A
10
5
0
−5
−10
Acceleration, in
A(θ)
in
0
25
50
75
100
125
150
θ
deg
Cam Rotation Angle, deg


JERK, J
100
50
0
−50
Jerk, in
J(θ)
in
0
25
50
75
100
125
150
θ
deg
Cam Rotation Angle, deg

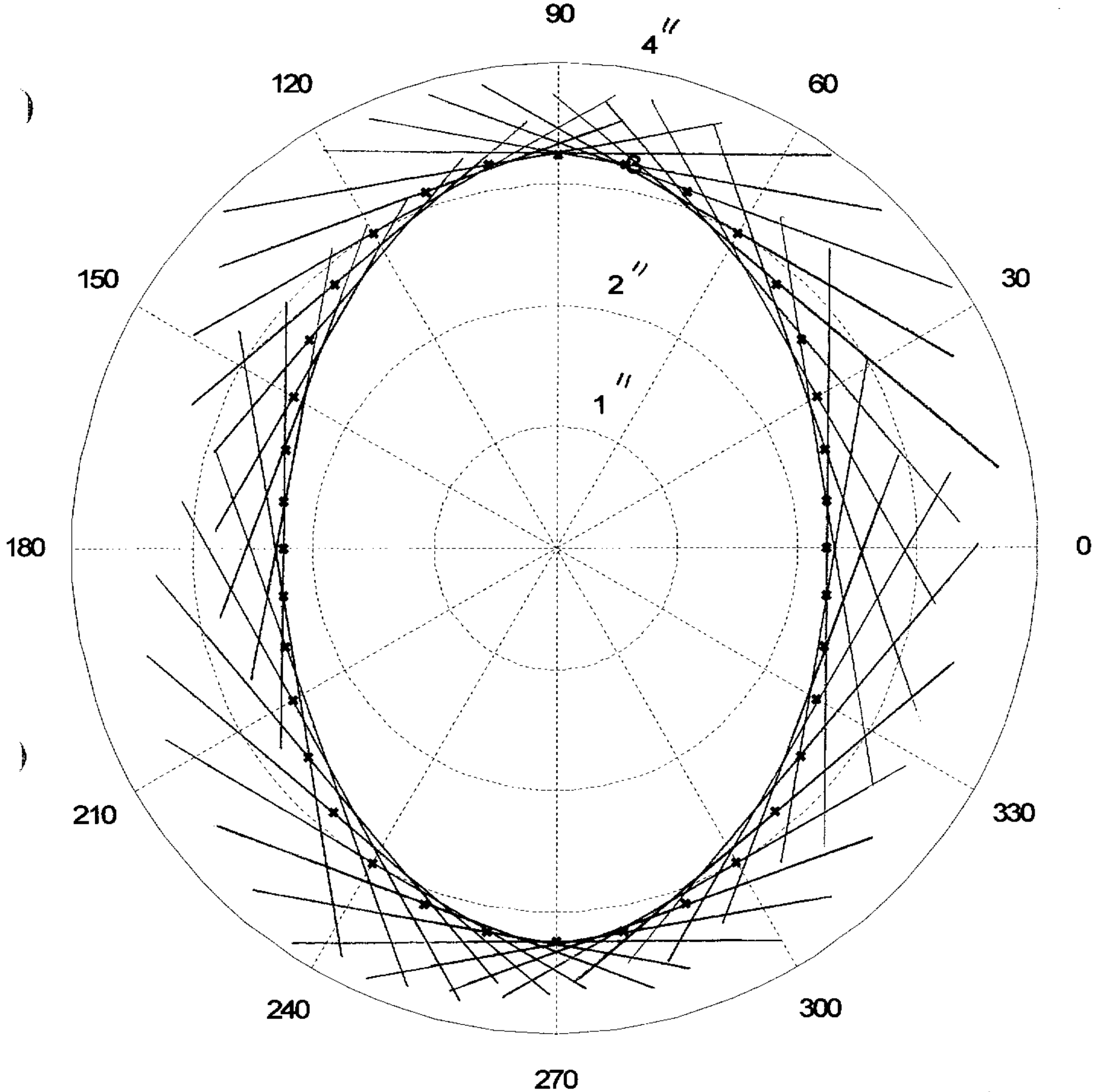
# SOLUTION



Follower Position [in.]
Angular Position [deg]
3.6
3.4
3.2
3
2.8
2.6
2.4
2.2
0
50
100
150
200
250
300
350
400

90
4"
120
60
3"
2"
150
30
1"
180
0
210
330
240
300
270




The cam shape is inscribed within the tangent lines. It should be smooth and touch all the lines, otherwise the cam is undercut.