PROBLEM 11-3a

Statement: Table P11-1 shows kinematic and geometric data for several slider-crank linkages of the type and orientation shown in Figure P11-1. The point locations are defined as described in the text. For row *a* in the table, solve for forces and torques at the position shown. Also, compute the shaking force and the shaking torque. Consider the coefficient of friction μ between slider and ground to be zero.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

Link 2 (O_2 to A) $a := 4.00 \cdot in$ Link 3 (A to B) $b := 12.00 \cdot in$

Offset $c := 0.00 \cdot in$ Friction: $\mu := 0$

Crank angle and motion: $\theta_2 := 45 \cdot deg$ $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ $\theta_3 := 166.40 \cdot deg$

Coupler point: $R_{P3} := 0.0 \cdot in$ $\delta_{RP3} := 0.0 \cdot deg$

Mass: $m_2 := 0.002 \cdot blob$ $m_3 := 0.020 \cdot blob$ $m_4 := 0.060 \cdot blob$

Moment of inertia: $I_{G2} := 0.10 \cdot blob \cdot in^2$ $I_{G3} := 0.20 \cdot blob \cdot in^2$

Mass center: $R_{CG2} := 2.00 \cdot in$ $\delta_2 := 0 \cdot deg$ $R_{CG3} := 5.00 \cdot in$ $\delta_3 := 0 \cdot deg$

Force and torque: $F_{P3} := 0 \cdot lbf$ $\delta_{FP3} := 0 \cdot deg$ $T_3 := 20 \cdot lbf \cdot in$

Accelerations: $\alpha_2 := 20 \cdot rad \cdot sec^{-2}$ $a_{G2} := 203.96 \cdot in \cdot sec^{-2}$ $\theta_{AG2} := 213.69 \cdot deg$

$\alpha_3 := -2.40 \cdot rad \cdot sec^{-2}$ $a_{G3} := 371.08 \cdot in \cdot sec^{-2}$ $\theta_{AG3} := 200.84 \cdot deg$

$a_{G4} := 357.17 \cdot in \cdot sec^{-2}$ $\theta_{AG4} := 180.0 \cdot deg$

Solution: See Figure P11-1, Table P11-1, and Mathcad file P1103a.

1. Calculate the *x* and *y* components of the position vectors.

$R_{12x} := R_{CG2} \cdot cos(\theta_2 + 180 \cdot deg)$ $R_{12x} = -1.414\ in$

$R_{12y} := R_{CG2} \cdot sin(\theta_2 + 180 \cdot deg)$ $R_{12y} = -1.414\ in$

$R_{32x} := R_{CG2} \cdot cos(\theta_2)$ $R_{32x} = 1.414\ in$

$R_{32y} := R_{CG2} \cdot sin(\theta_2)$ $R_{32y} = 1.414\ in$

$R_{23x} := R_{CG3} \cdot cos(\theta_3)$ $R_{23x} = -4.860\ in$

$R_{23y} := R_{CG3} \cdot sin(\theta_3)$ $R_{23y} = 1.176\ in$

$R_{43x} := (b - R_{CG3}) \cdot cos(\theta_3 + 180 \cdot deg)$ $R_{43x} = 6.804\ in$

$R_{43y} := (b - R_{CG3}) \cdot sin(\theta_3 + 180 \cdot deg)$ $R_{43y} = -1.646\ in$

$R_{P3x} := R_{P3} \cdot cos(\theta_3 + 180 \cdot deg + \delta_{RP3})$ $R_{P3x} = 0.000\ in$

$R_{P3y} := R_{P3} \cdot sin(\theta_3 + 180 \cdot deg + \delta_{RP3})$ $R_{P3y} = 0.000\ in$

2. Calculate the *x* and *y* components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$a_{G2x} := a_{G2} \cdot cos(\theta_{AG2}) \qquad a_{G2x} = -169.705\ in \cdot sec^{-2}$$

$$a_{G2y} := a_{G2} \cdot sin(\theta_{AG2}) \qquad a_{G2y} = -113.136\ in \cdot sec^{-2}$$

$$a_{G3x} := a_{G3} \cdot cos(\theta_{AG3}) \qquad a_{G3x} = -346.803\ in \cdot sec^{-2}$$

$$a_{G3y} := a_{G3} \cdot sin(\theta_{AG3}) \qquad a_{G3y} = -132.015\ in \cdot sec^{-2}$$

$$a_{G4x} := a_{G4} \cdot cos(\theta_{AG4}) \qquad a_{G4x} = -357.170\ in \cdot sec^{-2}$$

3. Calculate the *x* and *y* components of the external force at *P* on link 3 in the CGS.

$$F_{P3x} := F_{P3} \cdot cos(\delta_{FP3}) \qquad F_{P3x} = 0.000\ lbf$$

$$F_{P3y} := F_{P3} \cdot sin(\delta_{FP3}) \qquad F_{P3y} = 0.000\ lbf$$

4. Substitute these given and calculated values into the matrix equation 11.10g, modified for this problem. Note that Mathcad requires that all elements in a matrix have the same dimension. Thus, the matrix and array in equation 11.10g will be made dimensionless and the dimensions will be put back in after solving it.

$$C := \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{-R_{12y}}{in} & \frac{R_{12x}}{in} & \frac{-R_{32y}}{in} & \frac{R_{32x}}{in} & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{R_{23y}}{in} & \frac{-R_{23x}}{in} & \frac{-R_{43y}}{in} & \frac{R_{43x}}{in} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{pmatrix}$$

$$F := \begin{bmatrix} m_2 \cdot a_{G2x} \cdot lbf^{-1} \\ m_2 \cdot a_{G2y} \cdot lbf^{-1} \\ I_{G2} \cdot \alpha_2 \cdot lbf^{-1} \cdot in^{-1} \\ (m_3 \cdot a_{G3x} - F_{P3x}) \cdot lbf^{-1} \\ (m_3 \cdot a_{G3y} - F_{P3y}) \cdot lbf^{-1} \\ (I_{G3} \cdot \alpha_3 - R_{P3x} \cdot F_{P3y} + R_{P3y} \cdot F_{P3x} - T_3) \cdot lbf^{-1} \cdot in^{-1} \\ m_4 \cdot a_{G4x} \cdot lbf^{-1} \\ 0 \end{bmatrix} \qquad R := C^{-1} \cdot F$$

$F_{12x} := R_1 \cdot lbf$ $\quad F_{12x} = -28.7\,lbf$ $\quad F_{12y} := R_2 \cdot lbf$ $\quad F_{12y} = 5.87\,lbf$

$F_{32x} := R_3 \cdot lbf$ $\quad F_{32x} = 28.4\,lbf$ $\quad F_{32y} := R_4 \cdot lbf$ $\quad F_{32y} = -6.10\,lbf$

$F_{43x} := R_5 \cdot lbf$ $\quad F_{43x} = 21.4\,lbf$ $\quad F_{43y} := R_6 \cdot lbf$ $\quad F_{43y} = -8.74\,lbf$

$F_{14y} := R_7 \cdot lbf$ $\quad F_{14y} = -8.74\,lbf$

$T_{12} := R_8 \cdot lbf \cdot in$ $\quad T_{12} = 99.6\,lbf \cdot in$

5. Calculate the shaking force and shaking torque using equations 11.15.

$\mathbf{F_{21}} := -F_{12x} - j \cdot F_{12y}$ $\quad \mathbf{F_{41}} := -j \cdot F_{14y}$

$\mathbf{F_s} := \mathbf{F_{21}} + \mathbf{F_{41}}$ $\quad \mathbf{F_s} = 28.706 + 2.867j\,lbf$

Magnitude: $F_s := |\mathbf{F_s}|$ $\quad F_s = 28.848\,lbf$

Angle: $\theta_{Fs} := arg(\mathbf{F_s})$ $\quad \theta_{Fs} = 5.703\,deg$

$\mathbf{T_s} := -T_{12}$ $\quad \mathbf{T_s} = -99.6\,lbf \cdot in$

PROBLEM 11-12

Statement: Figure P11-5b shows a fourbar linkage and its dimensions in meters. The steel crank, coupler, and rocker have uniform cross sections of 50 mm diameter. In the instantaneous position shown, the crank O_2A has ω = -10 rad/sec and α = 10 rad/sec^2. There is a horizontal force at P of F = 300 N. Find all pin forces and the torque needed to drive the crank at this instant.

Given: Link lengths:

Link 2 (O_2 to A) $a := 0.86 \cdot m$ Link 3 (A to B) $b := 1.85 \cdot m$

Link 4 (B to O_4) $c := 0.86 \cdot m$ Link 1 (O_2 to O_4) $d := 2.22 \cdot m$

Coupler point: $R_{pa} := 1.33 \cdot m$ $\delta_3 := 0 \cdot deg$ $F := 300 \cdot N$ $T_4 := 0 \cdot N \cdot m$

Crank angle and motion: $\theta_2 := -36 \cdot deg$ $\omega_2 := -10 \cdot rad \cdot sec^{-1}$ $\alpha_2 := 10 \cdot rad \cdot sec^{-2}$

Link cross-section dims:
$d_{link} := 50 \cdot mm$

Material specific weight: $\gamma_s := 0.3 \cdot lbf \cdot in^{-3}$

Solution: See Figure P11-5b and Mathcad file P1112.

1. Use program FOURBAR to determine the position, velocity, and acceleration of links 3 and 4.

$\theta_3 := 46.028 \cdot deg$ $\omega_3 := 3.285 \cdot rad \cdot sec^{-1}$ $\alpha_3 := -109.287 \cdot rad \cdot sec^{-2}$

$\theta_4 := 106.189 \cdot deg$ $\omega_4 := 11.417 \cdot rad \cdot sec^{-1}$ $\alpha_4 := -43.426 \cdot rad \cdot sec^{-2}$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

Links 2 and 4: $R_{CG2} := 0.5 \cdot a$ $R_{CG2} = 0.430\,m$ $R_{CG4} := 0.5 \cdot c$ $R_{CG4} = 0.430\,m$

Link 3: $R_{CG3} := 0.5 \cdot b$ $R_{CG3} = 0.925\,m$

3. Determine the mass and moment of inertia of each link.

$$m_2 := \frac{\pi \cdot d_{link}^{\,2}}{4} \cdot a \cdot \frac{\gamma_s}{g} \qquad m_3 := \frac{\pi \cdot d_{link}^{\,2}}{4} \cdot b \cdot \frac{\gamma_s}{g} \qquad m_4 := \frac{\pi \cdot d_{link}^{\,2}}{4} \cdot c \cdot \frac{\gamma_s}{g}$$

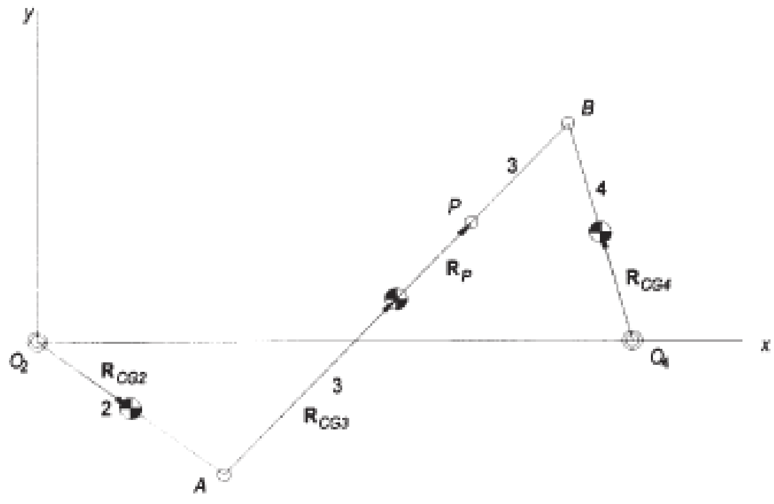
$m_2 = 14.022\,kg$ $m_3 = 30.164\,kg$ $m_4 = 14.022\,kg$

$$I_{G2} := \frac{m_2}{12} \cdot \left(\frac{3}{4} \cdot d_{link}^{\,2} + a^2\right) \qquad I_{G2} = 0.866\,kg \cdot m^2$$

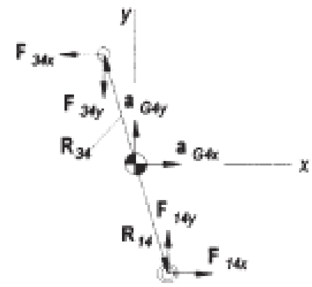
$$I_{G3} := \frac{m_3}{12} \cdot \left(\frac{3}{4} \cdot d_{link}^{\,2} + b^2\right) \qquad I_{G3} = 8.608\,kg \cdot m^2$$

$$I_{G4} := \frac{m_4}{12} \cdot \left(\frac{3}{4} \cdot d_{link}^{\,2} + c^2\right) \qquad I_{G4} = 0.866\,kg \cdot m^2$$

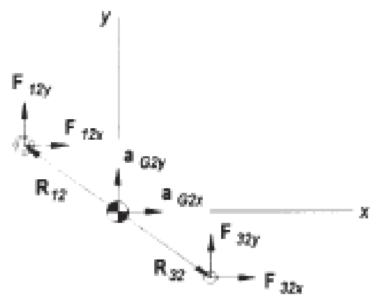
4. Set up an LNCS xy coordinate system at the CG of each link, and draw all applicable vectors acting on the system as shown in Figure 11-3. Draw a free-body diagram of each moving link as shown in Figure 11-3.



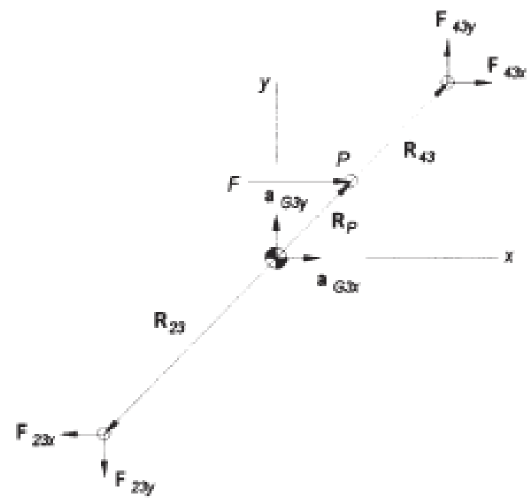
(a) The complete linkage with GCS



(d) FBD of Link 4



(b) FBD of Link 2



(c) FBD of Link 3

5. Calculate the *x* and *y* components of the position vectors.

$$R_{12x} := R_{CG2} \cdot cos(\theta_2 + 180 \cdot deg) \qquad R_{12x} = -0.348\,m$$

$$R_{12y} := R_{CG2} \cdot sin(\theta_2 + 180 \cdot deg) \qquad R_{12y} = 0.253\,m$$

$$R_{32x} := R_{CG2} \cdot cos(\theta_2) \qquad R_{32x} = 0.348\,m$$

$$R_{32y} := R_{CG2} \cdot sin(\theta_2) \qquad R_{32y} = -0.253\,m$$

$$R_{23x} := R_{CG3} \cdot cos(\theta_3 + 180 \cdot deg) \qquad R_{23x} = -0.642\,m$$

$$R_{23y} := R_{CG3} \cdot sin(\theta_3 + 180 \cdot deg) \qquad R_{23y} = -0.666\,m$$

$$R_{43x} := (R_{CG3} - b) \cdot cos(\theta_3 + 180 \cdot deg) \qquad R_{43x} = 0.642\,m$$

$$R_{43y} := (R_{CG3} - b) \cdot sin(\theta_3 + 180 \cdot deg) \qquad R_{43y} = 0.666\,m$$

$R_{34x} := R_{CG4} \cdot cos(\theta_4)$ $\qquad$ $R_{34x} = -0.120\, m$

$R_{34y} := R_{CG4} \cdot sin(\theta_4)$ $\qquad$ $R_{34y} = 0.413\, m$

$R_{14x} := R_{CG4} \cdot cos(\theta_4 + 180 \cdot deg)$ $\qquad$ $R_{14x} = 0.120\, m$

$R_{14y} := R_{CG4} \cdot sin(\theta_4 + 180 \cdot deg)$ $\qquad$ $R_{14y} = -0.413\, m$

$R_{Px} := (R_{pa} - R_{CG3}) \cdot cos(\theta_3)$ $\qquad$ $R_{Px} = 0.281\, m$

$R_{Py} := (R_{pa} - R_{CG3}) \cdot sin(\theta_3)$ $\qquad$ $R_{Py} = 0.291\, m$

6. Calculate the *x* and *y* components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{a_{G2}} := R_{CG2} \cdot \alpha_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) - a \cdot \omega_2^2 \cdot (cos(\theta_2) + j \cdot sin(\theta_2))$$

$a_{G2x} := Re(\mathbf{a_{G2}})$ $\qquad$ $a_{G2x} = -67.048 \frac{m}{sec^2}$

$a_{G2y} := Im(\mathbf{a_{G2}})$ $\qquad$ $a_{G2y} = 54.028 \frac{m}{sec^2}$

$$\mathbf{a_A} := a \cdot \alpha_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) - a \cdot \omega_2^2 \cdot (cos(\theta_2) + j \cdot sin(\theta_2))$$

$$\mathbf{a_{CG3A}} := R_{CG3} \cdot \alpha_3 \cdot (-sin(\theta_3) + j \cdot cos(\theta_3)) \ldots$$
$$+ -R_{CG3} \cdot \omega_3^2 \cdot (cos(\theta_3) + j \cdot sin(\theta_3))$$

$\mathbf{a_{G3}} := \mathbf{a_A} + \mathbf{a_{CG3A}}$ $\qquad$ $a_{G3x} := Re(\mathbf{a_{G3}})$ $\qquad$ $a_{G3x} = 1.302 \frac{m}{sec^2}$

$a_{G3y} := Im(\mathbf{a_{G3}})$ $\qquad$ $a_{G3y} = -19.864 \frac{m}{sec^2}$

$$\mathbf{a_{G4}} := R_{CG4} \cdot \alpha_4 \cdot (-sin(\theta_4) + j \cdot cos(\theta_4)) - c \cdot \omega_4^2 \cdot (cos(\theta_4) + j \cdot sin(\theta_4))$$

$a_{G4x} := Re(\mathbf{a_{G4}})$ $\qquad$ $a_{G4x} = 49.187 \frac{m}{sec^2}$

$a_{G4y} := Im(\mathbf{a_{G4}})$ $\qquad$ $a_{G4y} = -102.448 \frac{m}{sec^2}$

7. Calculate the *x* and *y* components of the external force at *P* in the CGS.

$F_{Px} := F$ $\qquad$ $F_{Py} := 0 \cdot N$

8. Substitute these given and calculated values into the matrix equation 11.9. Note that Mathcad requires that all elements in a matrix have the same dimension. Thus, the matrix and array in equation 11.9 will be made dimensionless and the dimensions will be put back in after solving it.

$$C := \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{-R_{12y}}{m} & \frac{R_{12x}}{m} & \frac{-R_{32y}}{m} & \frac{R_{32x}}{m} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{R_{23y}}{m} & \frac{-R_{23x}}{m} & \frac{-R_{43y}}{m} & \frac{R_{43x}}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{R_{34y}}{m} & \frac{-R_{34x}}{m} & \frac{-R_{14y}}{m} & \frac{R_{14x}}{m} & 0 \end{pmatrix}$$

$$F := \begin{bmatrix} m_2 \cdot a_{G2x} \cdot N^{-1} \\ m_2 \cdot a_{G2y} \cdot N^{-1} \\ I_{G2} \cdot \alpha_2 \cdot N^{-1} \cdot m^{-1} \\ \left(m_3 \cdot a_{G3x} - F_{Px}\right) \cdot N^{-1} \\ \left(m_3 \cdot a_{G3y} - F_{Py}\right) \cdot N^{-1} \\ \left(I_{G3} \cdot \alpha_3 - R_{Px} \cdot F_{Py} + R_{Py} \cdot F_{Px}\right) \cdot N^{-1} \cdot m^{-1} \\ m_4 \cdot a_{G4x} \cdot N^{-1} \\ m_4 \cdot a_{G4y} \cdot N^{-1} \\ \left(I_{G4} \cdot \alpha_4 - T_4\right) \cdot N^{-1} \cdot m^{-1} \end{bmatrix} \qquad R := C^{-1} \cdot F$$

$F_{12x} := R_1 \cdot N$ $\quad F_{12x} = -1246\,N$ $\qquad F_{12y} := R_2 \cdot N$ $\quad F_{12y} = 940\,N$

$F_{32x} := R_3 \cdot N$ $\quad F_{32x} = 306\,N$ $\qquad F_{32y} := R_4 \cdot N$ $\quad F_{32y} = -183\,N$

$F_{43x} := R_5 \cdot N$ $\quad F_{43x} = 45.1\,N$ $\qquad F_{43y} := R_6 \cdot N$ $\quad F_{43y} = -782\,N$

$F_{14x} := R_7 \cdot N$ $\quad F_{14x} = 735\,N$ $\qquad F_{14y} := R_8 \cdot N$ $\quad F_{14y} = -2219\,N$

$T_{12} := R_9 \cdot N \cdot m$ $\quad T_{12} = 7.14\,N \cdot m$