PROBLEM 12-1a

Statement: A system of two coplanar arms on a common shaft, as shown in Figure 12-1, is to be designed. For row *a* in Table P12-1, find the shaking force of the linkage when run unbalanced and design a counterweight to statically balance the system. Work in a any consistent unit system you prefer.

Given: Masses and radii:

$m_1 := 0.20 \cdot kg \qquad r_1 := 1.25 \cdot m \qquad \theta_1 := 30 \cdot deg$

$m_2 := 0.40 \cdot kg \qquad r_2 := 2.25 \cdot m \qquad \theta_2 := 120 \cdot deg$

Solution: See Figure 12-1, Table P12-1, and Mathcad file P1201a.

1. Resolve the position vectors into *xy* components in the arbitrary coordinate system associated with the freeze-frame position of the linkage chosen for analysis.

$R_{1x} := r_1 \cdot \cos(\theta_1) \qquad R_{1x} = 1.083\, m \qquad R_{1y} := r_1 \cdot \sin(\theta_1) \qquad R_{1y} = 0.625\, m$

$R_{2x} := r_2 \cdot \cos(\theta_2) \qquad R_{2x} = -1.125\, m \qquad R_{2y} := r_2 \cdot \sin(\theta_2) \qquad R_{2y} = 1.949\, m$

2. Solve equation 12.2c for the mass-radius product components.

$mR_{bx} := -m_1 \cdot R_{1x} - m_2 \cdot R_{2x} \qquad mR_{bx} = 0.233\, kg \cdot m$

$mR_{by} := -m_1 \cdot R_{1y} - m_2 \cdot R_{2y} \qquad mR_{by} = -0.904\, kg \cdot m$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required.

$\theta_b := atan2(mR_{bx}, mR_{by}) \qquad \theta_b = -75.524\, deg$

$mR_b := \sqrt{mR_{bx}^2 + mR_{by}^2} \qquad mR_b = 0.934\, kg \cdot m$

PROBLEM 12-5a

Statement: A system of three non coplanar weights is arranged on a shaft generally as shown in Figure 12-3. For row *a* in Table P12-2, find the shaking forces and shaking moment when run unbalanced at 100 rpm and specify the *mR* product and angle of the counterweights in planes *A* and *B* needed to dynamically balance the system. The correction planes are 20 units apart. Work in a any consistent unit system you prefer.

Given: Masses and radii:

$m_1 := 0.20 \cdot kg$ $\quad r_1 := 1.25 \cdot m$ $\quad \theta_1 := 30 \cdot deg$ $\quad l_1 := 2 \cdot m$

$m_2 := 0.40 \cdot kg$ $\quad r_2 := 2.25 \cdot m$ $\quad \theta_2 := 120 \cdot deg$ $\quad l_2 := 8 \cdot m$

$m_3 := 1.24 \cdot kg$ $\quad r_3 := 5.50 \cdot m$ $\quad \theta_3 := -30 \cdot deg$ $\quad l_3 := 17 \cdot m$

Distance between correction planes: $l_B := 20 \cdot m$

Solution: See Figure 12-3, Table P12-2, and Mathcad file P1205a.

1. Resolve the position vectors into *xy* components in the arbitrary coordinate system associated with the freeze-frame position of the linkage chosen for analysis.

$R_{1x} := r_1 \cdot cos(\theta_1)$ $\quad R_{1x} = 1.083\, m$ $\quad R_{1y} := r_1 \cdot sin(\theta_1)$ $\quad R_{1y} = 0.625\, m$

$R_{2x} := r_2 \cdot cos(\theta_2)$ $\quad R_{2x} = -1.125\, m$ $\quad R_{2y} := r_2 \cdot sin(\theta_2)$ $\quad R_{2y} = 1.949\, m$

$R_{3x} := r_3 \cdot cos(\theta_3)$ $\quad R_{3x} = 4.763\, m$ $\quad R_{3y} := r_3 \cdot sin(\theta_3)$ $\quad R_{3y} = -2.750\, m$

2. Solve equations 12.4e for summation of moments about *O*.

$$mR_{Bx} := \frac{-(m_1 \cdot R_{1x}) \cdot l_1 - (m_2 \cdot R_{2x}) \cdot l_2 - (m_3 \cdot R_{3x}) \cdot l_3}{l_B} \qquad mR_{Bx} = -4.862\, kg \cdot m$$

$$mR_{By} := \frac{-(m_1 \cdot R_{1y}) \cdot l_1 - (m_2 \cdot R_{2y}) \cdot l_2 - (m_3 \cdot R_{3y}) \cdot l_3}{l_B} \qquad mR_{By} = 2.574\, kg \cdot m$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane *B*.

$\theta_B := atan2(mR_{Bx}, mR_{By})$ $\quad \theta_B = 152.101\, deg$

$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2}$ $\quad mR_B = 5.501\, kg \cdot m$

4. Solve equations 12.4c for forces in *x* and *y* directions in plane *A*.

$mR_{Ax} := -m_1 \cdot R_{1x} - m_2 \cdot R_{2x} - m_3 \cdot R_{3x} - mR_{Bx}$ $\quad mR_{Ax} = -0.811\, kg \cdot m$

$mR_{Ay} := -m_1 \cdot R_{1y} - m_2 \cdot R_{2y} - m_3 \cdot R_{3y} - mR_{By}$ $\quad mR_{Ay} = -0.069\, kg \cdot m$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane *A*.

$\theta_A := atan2(mR_{Ax}, mR_{Ay})$ $\quad \theta_A = -175.160\, deg$

$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2}$ $\quad mR_A = 0.814\, kg \cdot m$

PROBLEM 12-29

Statement: Figure P12-11b shows a fourbar linkage and its dimensions in inches. All links have a uniform 0.5-in wide x 0.2-in thick cross-section and are made from steel. Link 3 has squared ends that extend 0.25 in from the pivot point centers. Links 2 and 4 have rounded ends that have a radius of 0.25 in. Design counterweights to force balance the linkage using the method of Berkof and Lowen.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

Link 2 (O_2 to A) $L_2 := 2.75 \cdot in$ Link 3 (A to B) $L_3 := 3.26 \cdot in$

Link 4 (B to O_4) $L_4 := 2.75 \cdot in$ Link 1 (O_2 to O_4) $L_1 := 4.46 \cdot in$

Link cross section: $h := 0.50 \cdot in$ $t := 0.20 \cdot in$ $r := 0.25 \cdot in$

Mass center: $b_2 := 0.5 \cdot L_2$ $b_3 := 0.5 \cdot L_3$ $b_4 := 0.5 \cdot L_4$

$\phi_2 := 0 \cdot deg$ $\phi_3 := 0 \cdot deg$ $\phi_4 := 0 \cdot deg$

Mass density of steel: $\rho := 7.8 \cdot 10^3 \cdot kg \cdot m^{-3}$

Solution: See Figure P12-11 and Mathcad file P1229.

1. Calculate the mass of each link.

$$m_2 := \left(L_2 \cdot h + \pi \cdot r^2\right) \cdot t \cdot \rho \qquad m_3 := \left(L_3 + 2 \cdot r\right) \cdot h \cdot t \cdot \rho \qquad m_4 := \left(L_4 \cdot h + \pi \cdot r^2\right) \cdot t \cdot \rho$$

$$m_2 = 2.294 \times 10^{-4}\, blob \qquad m_3 = 2.744 \times 10^{-4}\, blob \qquad m_4 = 2.294 \times 10^{-4}\, blob$$

2. Solve equations 12.8c and 12.8d for the total mR product components for links 2 and 4.

$$mb_{2x} := m_3 \cdot \left(b_3 \cdot \frac{L_2}{L_3} \cdot cos(\phi_3) - L_2\right) \qquad mb_{2x} = -3.773 \times 10^{-4}\, blob \cdot in$$

$$mb_{2y} := m_3 \cdot \left(b_3 \cdot \frac{L_2}{L_3} \cdot sin(\phi_3)\right) \qquad mb_{2y} = 0.000\, blob \cdot in$$

$$mb_{4x} := -m_3 \cdot \left(b_3 \cdot \frac{L_4}{L_3} \cdot cos(\phi_3)\right) \qquad mb_{4x} = -3.773 \times 10^{-4}\, blob \cdot in$$

$$mb_{4y} := -m_3 \cdot \left(b_3 \cdot \frac{L_4}{L_3} \cdot sin(\phi_3)\right) \qquad mb_{4y} = 0.000\, blob \cdot in$$

3. Determine the additional mR product components for links 2 and 4.

$$mR_{2x} := mb_{2x} - m_2 \cdot b_2 \cdot cos(\phi_2) \qquad mR_{2x} = -6.927 \times 10^{-4}\, blob \cdot in$$

$$mR_{2y} := mb_{2y} - m_2 \cdot b_2 \cdot sin(\phi_2) \qquad mR_{2y} = 0.000\, blob \cdot in$$

$$mR_{4x} := mb_{4x} - m_4 \cdot b_4 \cdot cos(\phi_4) \qquad mR_{4x} = -6.927 \times 10^{-4}\, blob \cdot in$$

$$mR_{4y} := mb_{4y} - m_4 \cdot b_4 \cdot sin(\phi_4) \qquad mR_{4y} = 0.000\, blob \cdot in$$

4. Solve equations 12.2d and 12.2e for the position angle and additional mass-radius product required.

$$mR_{b2} := \sqrt{mR_{2x}^2 + mR_{2y}^2} \qquad mR_{b2} = 6.927 \times 10^{-4}\, blob \cdot in$$

$$\theta_{b2} := atan2(mR_{2x}, mR_{2y}) \qquad \theta_{b2} = 180.000\, deg$$

$$mR_{b4} := \sqrt{mR_{4x}^2 + mR_{4y}^2} \qquad mR_{b4} = 6.927 \times 10^{-4}\, blob \cdot in$$

$$\theta_{b4} := atan2(mR_{4x}, mR_{4y}) \qquad \theta_{b4} = 180.000\, deg$$