Logic Design I (17.341)

Fall 2011

Lecture Outline

Class # 01

September 12, 2011

Dohn Bowden

Today's Lecture

- Administrative
- General Course Overview
- Main Logic Topic
- Lab
- Homework

Course Admin

Administrative

- Admin for tonight ...
 - Attendance/Introductions/Backgrounds
 - Syllabus
 - Textbook
 - 17.341 Web Site
 - Email List creation
 - Course Objectives

Attendance/Introductions/Backgrounds

- Attendance ...
 - When called ... please introduce yourself
 - Include the following
 - Knowledge of electronics/logic design
 - Education
 - Work Experience
 - Other notable work/engineering/hobbies
 - Future Plans

My Background

- Education
- Work Experience
- Other notable work/engineering/hobbies
- Future Plans

Syllabus

- Syllabus
 - Hard copies available
 - Electronic copy available on the class website
 - Web Address on syllabus
 - Updates will be posted on the class website

Syllabus Review

Week	Date	Topics	Chapter	Lab Report Due
1	09/12/11	Introduction to digital systems, number systems, and Binary Codes	1	
2	09/19/11	Boolean algebra	2	
3	09/28/11	Boolean algebra (continued)	3	
4	10/03/11	Examination 1		
X	10/10/11	No Class - Holiday		
5	10/17/11	Application of Boolean Algebra. Lab lecture	4	
6	10/24/11	Karnaugh Maps	5	
7	10/31/11	Multi-Level Gate Circuits. NAND and NOR Gates	7	1
8	11/07/11	Examination 2		
9	11/14/11	Combinational Circuit Design and Simulation Using Gates	8	2
10	11/23/11	Multiplexers, Decoders. Encoder, and PLD	9	
11	11/28/11	Introduction to VHDL	10	3
12	12/05/11	Examination 3		
13	12/12/11	Review		4
14	12/19/11	Final Exam		

Class Hours

- Monday evenings ... 6 9 PM
 - Lectures ... BL-313
 - Access to Software (for homework assignments)
 - Computer Labs ... BL-420
 - ... *OR* ...
 - Engineering Lab ... EB-321
- Call/email if you will not be in class ...
- I am available for extra help Before / After class
 - If possible ... please schedule in advance so I will ensure that I am available

Textbook

- Roth & Kinney, "Fundamentals of Logic Design, 6th Ed", CENGAGE Learning, 2010
 - Also available as an eBook
- Anh Tran, "Experiments in Logic Design" (To be handed out in class)

Software (Optional)

- If you want to purchase the software ...
 - Capilano Computing Systems Ltd., "LogicWorks 5: Interactive Circuit Design Software", Addison Wesley, 2004

... *OR* ...

- Capilano Computing Systems Ltd., "LogicWorks 4: Interactive Circuit Design Software", Addison Wesley, 1999
- Otherwise either LogicWorks 4 or 5 is available in the computer laboratory (BL - 420) ... OR ... Engineering Laboratory (EB - 321)

Software (included in the text)

- Computer-aided Logic Design Program ...
 - LogicAid
 - Included on the text's CD
- Logic Simulator
 - SimUaid
 - Also included on the text's CD

Access to Computer Lab and/or Engineering Lab

- To gain access to the Computer Lab (BL-420) ... and/OR ... the Engineering Lab (EB-321) ...
 - You will need to have an access cards
 - Access cards are given out at Access Services
 - Arrangements are done through the Continuing Education Office
 - Then I will need your UMS# (will be on your Access Card)
 - I will then send your name and UMS# to the Room POC

Course Web Site

The Course Homepage is at:

http://faculty.uml.edu/dbowden

- This website will contain the following:
 - Syllabus
 - Reference documents
 - Such as the textbook material
 - Links
 - Class lectures (will be placed on the web site <u>AFTER</u> the lecture)
 - Homework

Email Distribution List

- I will be creating a class email list
- Email me at:

Dohn_Bowden@uml.edu

- This will ensure that your correct email address or addresses are included
- The email list will allow me to provide information to each of you
- Let me know if you have a problem with your email in the "To" block of email messages

- What do you want to get out of this class?
- My goals for the course ...

Course Evaluations

How they are used

Questions?

Course Objectives and Introduction

- Design Logic circuits that ...
 - Are efficient
 - Minimal components to perform the operations required
 - At minimal cost
- Efficient and minimal cost go hand and hand
 - Efficient circuits will cost less due to a reduction in components

- Logic Design I ...
 - Will center on combinational logic circuits
- Logic Design II ...
 - Will be geared towards sequential logic circuits
- Differences between combinational and sequential circuits ...
 - Will be discussed shortly

- Towards the end of the semester we will start looking at ...
 - VHDL ...
 - VHSIC Hardware Descriptive Language
 - VHSIC ...
 - Very High Speed Integrated Circuit
- Logic Design II will explore VHDL in greater detail

- VHDL hardware description language will be used for ...
 - The design of combinational logic
 - Sequential logic
 - And ... simple digital systems

- We will be looking at ...
 - Boolean algebra ...
 - The basic mathematical tool needed to analyze and synthesize switching circuits
- We will also design circuits of logic gates that ... have ...
 - A specified relationship between signals at the input and output terminals

- In Logic Design II ...
 - We will evaluate sequential switching circuits
 - Look at the Logical properties of ...
 - Flip-flops ... which ...
 - Serve as memory devices in sequential switching circuits

- By combining flip-flops with circuits of logic gates ... we will ...
 - Design counters
 - Adders
 - Sequence detectors
 - And similar circuits

Chapter 1 ...

Introduction and Number Systems/Conversions

Introduction to

Digital Systems and Switching Circuits

- Fundamentals principles of Logic Design ...
 - Has not changed over the years ... however ...
 - There have been advances in technology used to ...
 - Implement digital systems

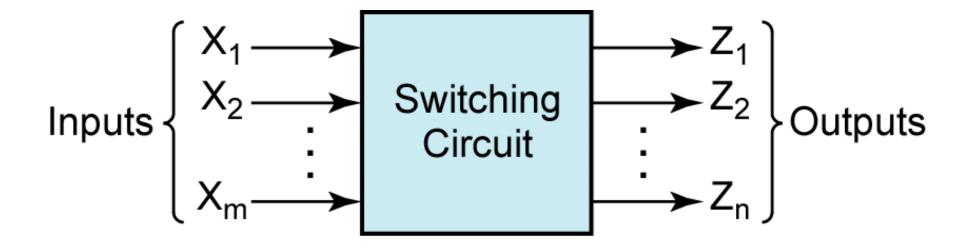
- Design of digital systems ... three components ...
 - System design
 - Logic design ... and ...
 - Circuit design
- This course will focus in on the Logic Design component
- The three components in a little more detail ...

- System design ...
 - Breaking the over-all system into subsystems ... and ...
 - Specifying the characteristics of each subsystem
- Example ... system design of a digital computer ...
 - Number and type of memory units
 - arithmetic units
 - Input output devices ... as well as ...
 - The interconnection and control of these subsystems

- Logic design ... what we will center on this semester ...
 - Determining how to interconnect basic logic building blocks to perform a specific function
- Example ...
 - Determine the interconnection of logic gates and flip-flops required to perform binary addition

- Circuit design ...
 - The interconnection of specific components such as ...
 - Resistors
 - Diodes
 - Transistors
 - To form a gate, flip-flop, or other logic building blocks

- Digital systems ... utilize switching circuits
- Switching circuits ...
 - Have one or more inputs ... and ...
 - One or more outputs ...
 - Which take on *discrete values*



Introduction to Logic Design

- Logic circuits have ...
 - A specified relationship between signals at the input terminals of the circuit ... and ...
 - Signals at the output terminals of the circuit

Introduction to Logic Design

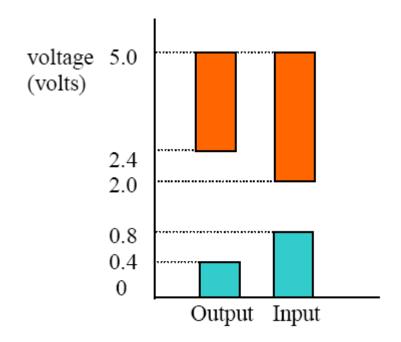
- We shall apply switching theory ...to ...
 - The solution of logic design problems
- We will learn both ...
 - The basic theory of switching circuits ... and ...
 - How to apply it

Digital and Analog Systems ...

Digital Systems

- Digital systems ...
 - Signals can assume only discrete values
 - The output voltage of a digital system might be constrained to take on only two values such as ...
 - 0 volts ... and ...
 - 5 volts

Digital Systems



Analog Systems

- Analog systems ...
 - Signals may vary continuously over a specified range
 - The output voltage from an analog system might be allowed to assume any value in the ...
 - Range -10 volts to +10 volts

Switching Circuit

- There are two types of switching circuits ...
 - Combinational ... and ...
 - Sequential

Combinational Circuits

- Combinational circuit ...
 - The output values depend only on the present value of the inputs and not on past values

Sequential Circuits

- Sequential circuits ...
 - The outputs depend on both ...
 - The present and past input values
- Sequential circuit have memory ... because ...
 - It "remembers" something about the past sequence of inputs
- Combinational circuit has no memory

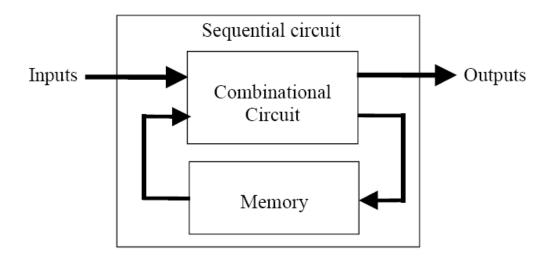
Sequential Circuits

- Sequential circuits are composed of ...
 - A combinational circuit ... with ...
 - Added memory elements

Combinational versus Sequential Circuits

Combinational circuits: Outputs depend on present inputs

Sequential circuits: Outputs depend on present inputs as well as past inputs



Combinational / Sequential Circuits

- Combinational circuits are easier to design than sequential circuits
- Therefore we will start with those circuits

- Combinational circuits are constructed by using logic gates
- We must determine how to interconnect these gates in order to ...
 - Convert the circuit input signals to the desired output

- The relationship between the input and output ...
 - Can be described mathematically
 - Using Boolean algebra
- We shall explore ...
 - The basic laws and theorems of Boolean algebra ... and ...
 - The behavior of circuits of logic gates

- Basic steps in designing combinational circuits ...
 - Starting with a problem statement
 - Derive a table or the algebraic logic equations which ...
 - Describes the circuit outputs as a function of the circuit inputs
 - Simplify the logic equations using algebraic methods and other methods such as ...
 - Karnaugh map and Quine-McCluskey procedure

- Basic steps ... continued ...
 - Implementation of the simplified logic equations using ...
 - Gates
 - Programmable logic devices

Designing Sequential Circuits

- Designing sequential circuits ...
 - Will be explored next semester in Logic Design II

Switching Devices

- The switching devices are two-state devices ...
 - The output can assume only two different discrete values
- Examples of switching devices ...
 - Relays
 - Diodes
 - Transistors
 - · When used in cut-off or saturated state

Binary Numbers

- We will use binary numbers ...
 - Because the outputs of most switching devices assume only two different values
- Binary numbers and number systems will be discussed ...
 - Before proceeding to the design of switching circuits

Number Systems and Conversions

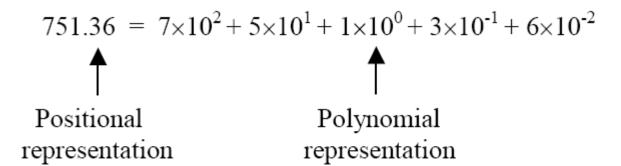
- Positional notation ...
 - For base 10 numbers ...
 - Each digit is multiplied by an appropriate power of 10 depending on its position in the number
- For example ...

$$953.78_{10} = 9x10^2 + 5x10^1 + 3x10^0 + 7x10^{-1} + 8x10^{-2}$$

= $900 + 50 + 3 + 0.7 + 0.08$
= 953.78_{10}

Positional/Polynomial Representations

Number representation



- Positional notation ...
 - For base 2 numbers ...
 - Each digit is multiplied by an appropriate power of 2 depending on its position in the number
- For example ...

$$1011.11_{2} = 1x2^{3} + 0x2^{2} + 1x2^{1} + 1x2^{0} + 1x2^{-1} + 1x2^{-2}$$

$$= 8 + 0 + 2 + 1 + \frac{1}{2} + \frac{1}{4}$$

$$= 11.75_{10}$$

- The decimal and binary points separates the ...
 - Positive and negative powers

$$953.78_{10} = 9x10^2 + 5x10^1 + 3x10^0 + 7x10^{-1} + 8x10^{-2}$$

$$1011.11_2 = 1x2^3 + 0x2^2 + 1x2^1 + 1x2^0 + 1x2^{-1} + 1x2^{-2}$$

- The *base* of a number system ... such as ...
 - Base 10

- ... or ...
- Base 2

• Is also called the *Radix*

- Any positive integer ... R (R > 1) ... can be ...
 - Chosen as the *radix* or *base* of a number system
- If the *base* is R ...then ... R digits (0, 1, . . . , R 1) are used
- For example ...
 - If R = 8 ... then ... the required digits are ...
 - » 0, 1, 2, 3, 4, 5, 6, and 7

- For bases greater than 10 ...
 - More than 10 symbols are needed to represent the digits
- For example ... in hexadecimal (base 16) ...
 - A represents 10₁₀
 - B represents 11₁₀
 - C represents 12₁₀
 - D represents 13₁₀
 - E represents 14₁₀
 - F represents 15₁₀

Numbering Systems

• Examples of five number systems ...

System	Radix (base)	Digits
Binary	2	0, 1
Ternary	3	0, 1, 2
Octal	8	0, 1, 2, 3, 4, 5, 6, 7
Decimal	10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Hexadecimal	16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Numbering Systems

Base	10	2	4	8	16
	0	0	0	0	0
	1	1	1	1	1
	2	10	2	2	2
	3	11	3	3	3
	4	100	10	4	4
	5	101	11	5	5
	6	110	12	6	6
	7	111	13	7	7
	8		20	10	8
	9		21	11	9
	10		22	12	A
	11		23	13	В
	12		30	14	C
	13		31	15	D
	14		32	16	E
	15		33	17	F
	16			20	10

Power Series Expansion

Power Series Expansion

- Power series expansion ...
 - If the arithmetic indicated in the power series expansion is done in base 10 ... then ... the result is the decimal equivalent

$$147.3_8 = 1 \times 8^2 + 4 \times 8^1 + 7 \times 8^0 + 3 \times 8^{-1} = 103.375_{10}$$
base 10 decimal

Power Series Expansion

- The power series expansion can be used to convert to <u>any base</u> ...
 - Converting 147₁₀ to base 3 would be written as ...

$$147_{10} = 1 \text{ x } (101)^2 + (11) \text{ x } (101)^1 + (21) \text{ x } (101)^0$$
base 3

- » Where all the numbers on the right hand side are base 3 numbers ... MUST USE base 3 arithmetic
- Then ... the result is base 3 equivalent
- However ... this is not very convenient if the arithmetic is being done by hand

Example

Convert Base 8 to Base 10

$$(356.1)_8 = (?)_{10}$$

 $(356.1)_8 = (3 \times 10^2 + 5 \times 10^1 + 6 \times 10^0 + 1 \times 10^{-1})_8$
 $(3 \times 8^2 + 5 \times 8^1 + 6 \times 8^0 + 1 \times 8^{-1})_{10} = (238.125)_{10}$

Conversion Table

ANY BASE Power Series Expansion
BASE 10

Division Method

- Division Method ...
 - The conversion of a decimal integer N to base R

$$N = (a_0 a_1 a_2 \dots a_n)_R$$

- The base R equivalent of a decimal integer N can be determined by ...
 - Dividing N by R ... we get ...
 - Quotient Q₁ and remainder is a₀
 - Then we divide Q₁ by R ... we get ...
 - Quotient Q₂ and remainder a₁
 - The process is continued until we finally obtain a_n

- The remainder obtained at each step is one of the desired digits
 - The least significant digit is determined first ... a₀
 - The most significant digit is determined last ... a_n

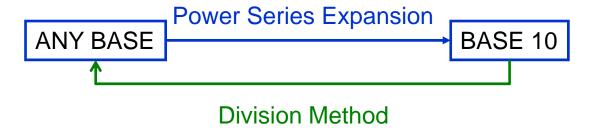
$$N = (a_n ... a_2 a_1 a_0)_R$$

• Example ... Convert 53₁₀ to binary

$$2 / 53$$

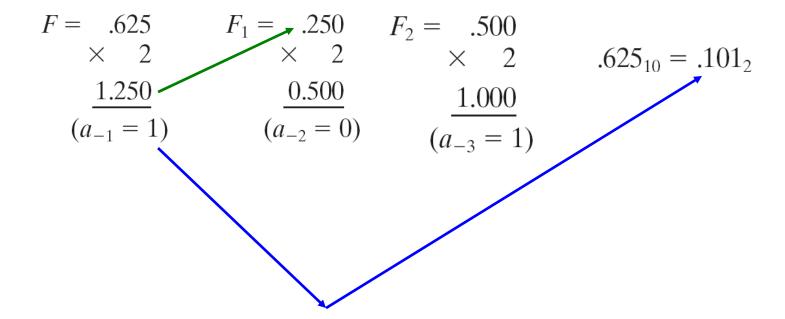
 $2 / 26$ rem. = $1 = a_0$
 $2 / 13$ rem. = $0 = a_1$
 $2 / 6$ rem. = $1 = a_2$ $53_{10} = 110101_2$
 $2 / 3$ rem. = $0 = a_3$
 $2 / 1$ rem. = $1 = a_4$
 0 rem. = $1 = a_5$

Conversion Table



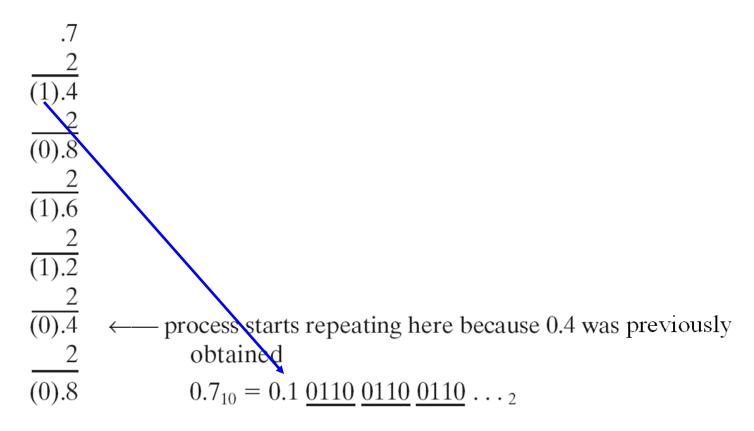
- Successive Multiplications ...
 - Conversion of a decimal fraction F to base R
 - Accomplished by using successive multiplications by R
- The integer part obtained at each step is one of the desired digits
- The most significant digit is obtained first

• EXAMPLE ... Convert 0.625₁₀ to binary

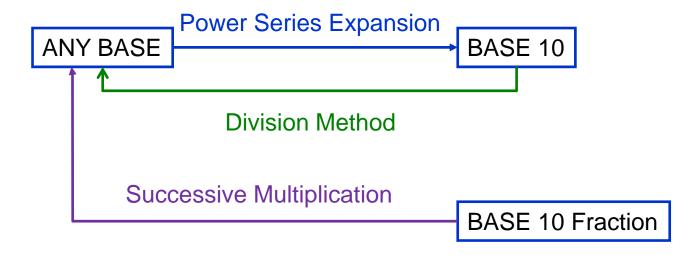


- Successive Multiplications ...
 - This process does not always terminate ...
 - If it does not terminate ...
 - The result is a repeating fraction

• EXAMPLE ... Convert 0.7₁₀ to binary



Conversion Table



Conversions Between

Two Bases

Conversions Between Two Bases

- Conversion between two bases other than decimal can be done directly by ...
 - Using the procedures provided thus far ...
 - However ... the arithmetic operations would have to be carried out using a base other than 10
 - It is generally easier to convert to decimal first ... and then ...
 convert the decimal number to the new base
- Therefore ... to Convert between two bases other than decimal ...
 - convert to decimal first ... then ... convert the decimal number to the new base

Conversions Between Two Bases

EXAMPLE ... Convert 231.34 to base 7

Base 4 to base 10

$$231.3_4 = 2 \times 16 + 3 \times 4 + 1 + \frac{3}{4} = 45.75_{10}$$

$$7 / 45$$
 .75
 $7 / 6$ rem. 3 7
0 rem. 6 (5) .25

 $45.75_{10} = 63.5151..._{7}$

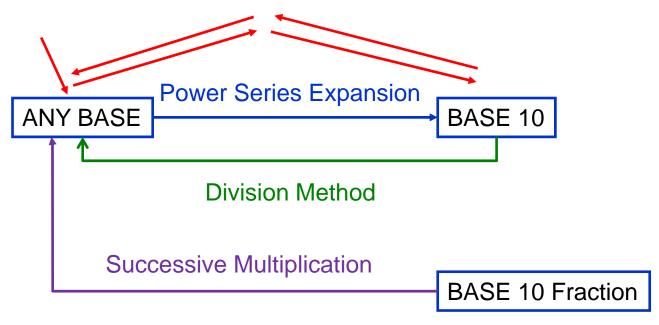
Base 10 to base 7

(5) .25 $\frac{7}{(1) .75}$

 $\overline{(1).75}$

Note that you need to Convert the integer ... Then the fraction ... Then combine!

Conversion Table



Any non-Base 10 to another non-Base 10 ...

Convert to Base 10 ... then ...

Convert to other non-Base 10

Binary to Hexadecimal Conversions

Binary ⇔ **Hexadecimal Conversion**

- Conversion from binary to hexadecimal ... and ... conversely ...
 - Convert by inspection ...
 - Each hexadecimal digit corresponds to exactly four binary digits (bits)
 - Starting at the binary point ...
 - The bits are divided into groups of four
 - Each group is replaced by a hexadecimal digit
 - Extra 0's are added at each end of the bit string as needed to fill out the groups of four bits

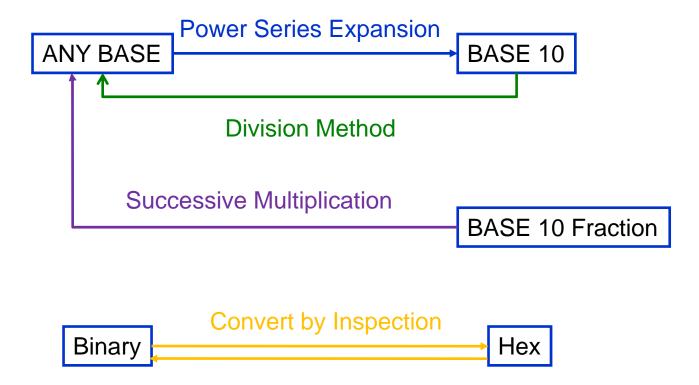
Binary ⇔ **Hexadecimal Conversion**

• Example ... Convert 1001101.010111₂ to hexadecimal

$$1001101.010111_2 = \underbrace{\frac{0100}{4}} \quad \underbrace{\frac{1101}{D}} \cdot \underbrace{\frac{0101}{5}} \quad \underbrace{\frac{1100}{C}} = 4D.5C_{16}$$

Added zeros ... leading and trailing

Conversion Table

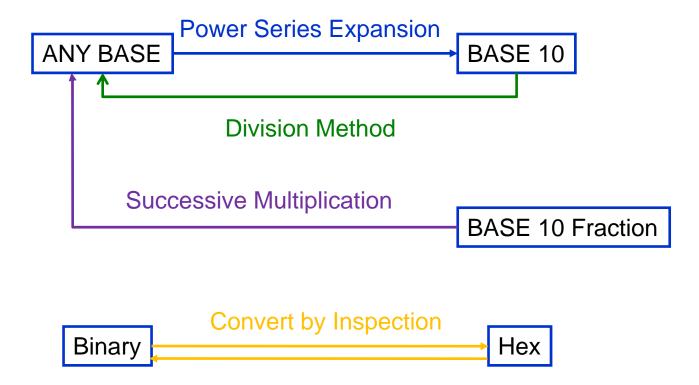


Summary ...

Summary

- The Power Series Expansion can be used to convert to <u>any base</u>
- Division Method ... the conversion of a decimal integer N to base R $N = (a_0 a_1 a_2 a_n)_R$
- Successive Multiplications ... Conversion of a decimal fraction F to base R
- Conversion between two bases other than decimal ... convert to decimal first ... then ... convert the decimal number to the new base
- Conversion from binary to hexadecimal ... and ... conversely ...
 convert by inspection ... Each hexadecimal digit corresponds to
 exactly four binary digits (bits)

Conversion Table



Binary Arithmetic

Binary Arithmetic

- Arithmetic operations in digital systems are ...
 - Usually performed in binary
 - The design of logic circuits to perform binary arithmetic is much easier than for decimal

Binary Addition

Binary Arithmetic - Addition

The addition table for binary numbers is ...

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 0$$
 and carry 1 to next column

Carrying 1 to a column is equivalent to adding 1 to that column

Binary Arithmetic - Addition

• Example ... Add 13₁₀ and 11₁₀ in binary

$$13_{10} = 1101$$
 $11_{10} = 1011$
 $11000 = 24_{10}$

Binary Subtraction

The subtraction table for binary numbers is ...

$$0 - 0 = 0$$

0 - 1 = 1 and borrow 1 from the next column

$$1 - 0 = 1$$

$$1 - 1 = 0$$

 Borrowing 1 from a column is equivalent to subtracting a 1 from that column

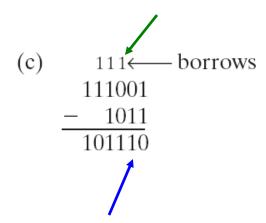
• Example ... of binary subtraction ...

111001 - 1011

(c)
$$111 \leftarrow$$
 borrows 111001 $- 1011$ 101110

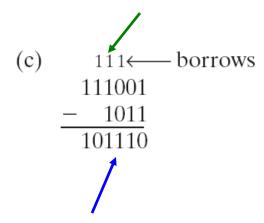
Starting with the rightmost column ...

$$1 - 1 = 0$$



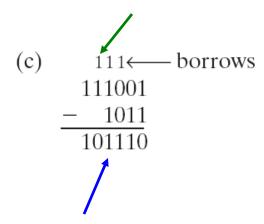
- Second column ... borrow from the third column
 - Rather than borrow immediately ... place a 1 over the third column to indicate that a borrow is necessary
 - Actual borrowing when we get to the third column
 - Similar to the way borrow propagate in a computer
 - Now because of borrowed 1 ... the second column becomes 10 ... and ...

$$10 - 1 = 1$$



- In order to borrow 1 from the third column ... we must ...
 - Borrow 1 from the fourth column
 - Column 3 then becomes 10 ...
 - Subtracting off the borrow yields 1 ... and ...

$$1 - 0 = 1$$



- Column 4 ... subtract off the borrow leaving 0
- In order to complete the subtraction ... we must ...
- Borrow from column 5 ... which gives 10 in column 4 ... and ...

$$10 - 1 = 1$$

(c)
$$111 \leftarrow$$
 borrows 111001 $- 1011$ 101110

• Column 5 ... subtract off the borrow leaving 0

$$0 - 0 = 0$$

(c)
$$111 \leftarrow$$
 borrows 111001 $- 1011$ 101110

Column 6 ... subtract 0 from 1

Resulting in ...

$$111001 - 1011 = 101110$$

Binary Arithmetic - Subtraction

Two other examples of binary subtractions ...

(a)
$$1 \leftarrow -$$
 (indicates 11101 a borrrow -10011 from the 1010 3rd column)

(b)
$$1111 \leftarrow$$
 borrows 10000 $11 \over 1101$

Binary Multiplication

• The multiplication table for binary numbers is ...

$$0 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 0 = 0$$

$$1 \times 1 = 1$$

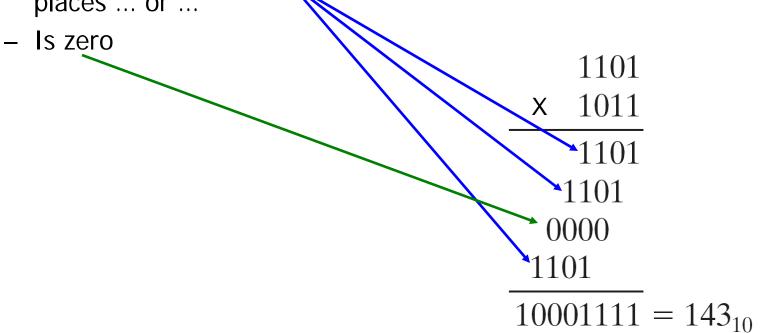
• Example ... multiply 13₁₀ by 11₁₀ in binary ...

$$13_{10} = 1101_2$$

 $11_{10} = 1011_2$... therefore ...

Note that each partial product is either ...

 The multiplicand (1101) shifted over the appropriate number of places ... or ...



- When adding up long columns of binary numbers ...
 - The sum of the bits in a single column can exceed 11₂
 - Therefore ... the carry to the next column can be greater than 1
- To avoid carries greater than 1 ...
 - Add the partial products one at a time

• Example ... multiplication adding partial products one at a time

1111	multiplicand
x <u>1101</u>	multiplier
1111	1st partial product
0000	2nd partial product
(01111)	sum of first two partial products
<u>1111 </u>	3rd partial product
(1001011)	sum after adding 3rd partial product
<u>1111 </u>	4th partial product
11000011	final product (sum after adding 4th partial
	product)

Binary Division

Binary Arithmetic - Division

- Binary division is similar to decimal division ... except ...
 - There are only two possible quotient digits ...

» 0 and 1

Binary Arithmetic - Division

- Start division by ...
 - Comparing the divisor with the upper bits of the dividend
 - If we cannot subtract without getting a negative result ...
 - We move one place to the right and try again
 - If we can subtract ...
 - We place a 1 for the quotient above the number we subtracted from ... and ...
 - » Append the next dividend bit to the end of the difference ... and ...
 - » Repeat this process with this modified difference until we run out of bits in the dividend

Binary Arithmetic - Division

• Example ... Divide 145_{10} (10010001₂) by 11_{10} (1011₂)

$$\begin{array}{c|c}
 & 1101 \\
\hline
 & 1011 \\
\hline
 & 1011 \\
\hline
 & 1110 \\
\hline
 & 1011 \\
\hline
 & 1101 \\
\hline
 & 1101 \\
\hline
 & 1101 \\
\hline
 & 1011 \\
\hline
 & 1011 \\
\hline
 & 1011 \\
\hline
 & 101 \\
\hline$$

Representation of Negative Numbers

Negative Numbers

- Thus far ... we have been working with unsigned positive numbers
- Need to be able to work with negative numbers as well
- Three (3) Systems for representing negative numbers in binary ...
 - Sign and Magnitude system
 - 1's complement
 - 2's complement

Negative Numbers - Sign and Magnitude

Negative Numbers - Sign and Magnitude

- Sign and Magnitude system ...
 - Most significant bit is the sign
- Example ...

$$-5_{10} = 1101_2$$

- 1's Complement ...
 - A negative number ... N
 - Word length of ... n bits
 - 1's complement ... \overline{N}
- 1's complement is defined as ...

$$\overline{N} = (2^n - 1) - N$$

• Example ... 1's Complement ... $\overline{N} = (2^n - 1) - N$... for n = 4

$$-5_{10} = (2^4 - 1) - 5 = 16 - 1 - 5 = 10_{10}$$

Recall ...

=
$$2/10$$

= $2/5$ rem = $0 = a_0$
= $2/2$ rem = $1 = a_1$
= $2/1$ rem = $0 = a_2$
= 0 rem = $1 = a_3$

Therefore ...

$$10_{10} = 1010_2$$

... So ...

 $-5_{10} = 1010_2$ using 1's complement method

- For 1's complement in the 4 bit system ...
 - 1111 represents a minus 0 ... and ...
 - Minus 8 has no representation

	Positive		Negative Integers		
	Integers		Sign and	2's Complement	1's Complement
+N	(all systems)	-N	Magnitude	W*	N
+0	0000	-0	1000		1111
+1	0001	-1	1001	1111	1110
+2	0010	-2	1010	1110	1101
+3	0011	-3	1011	1101	1100
+4	0100	-4	1100	1100	1011
+5	0101	-5	1101	1011	1010
+6	0110	-6	1110	1010	1001
+7	0111	-7	1111	1001	1000
		-8		1000	

- An alternate way to form the 1's complement is to ...
 - Complement N bit-by-bit by replacing 0's with 1's and 1's with 0's
- So ... for N = 5 ... the 1's complement of N is found ...

N = 0101 by using the alternate method above replacing 0's with 1's and 1's with 0's

 $\overline{N} = 1010 \dots$ which is the same as previous example

- 2's Complement ...
 - A negative number ... N
 - Word length of ... n bits
 - 2's complement ... N*
- 2's complement is defined as ...

$$N^* = 2^n - N$$

• Example ... 2's Complement ... $N^* = 2^n - N$... for n = 4

$$-5_{10} = 2^4 - 5 = 16 - 5 = 11_{10}$$

Recall ...

=
$$2/11$$

= $2/5$ rem = $1 = a_0$
= $2/2$ rem = $1 = a_1$
= $2/1$ rem = $0 = a_2$
= 0 rem = $1 = a_3$

Therefore ...

$$11_{10} = 1011_2$$

... So ...

 $-5_{10} = 1011_2$ using 2's complement method

- Alternate methods obtaining 2' s complement ...
 - Complementing N bit-by-bit and then adding 1
 - Or ...
 - Starting at the right ... complement all bits to the left of the first

- Example ... if ... $N = 0101100 ... then ... N^* = 1010100 ...$
- By either ...

0101100 ... Complement to get ... 1010011

1010011 ... Then add 1

+ 0000001

1010100 ... Equals N*

Or ... complement all bits to the left of the first 1 ... and get ...

0101100

1010100 ... Equals N*

Signed Binary Integers

(Word Length n = 4)

Signed Binary Integers (Word Length n = 4)

	Positive Integers (all systems)		Negative Integers			
+ <i>N</i>		-N	Sign and Magnitude	2's Complement N*	1's Complement \bar{N}	
+0	0000	-0	1000		1111	
+1	0001	-1	1001	1111	1110	
+2	0010	-2	1010	1110	1101	
+3	0011	-3	1011	1101	1100	
+4	0100	-4	1100	1100	1011	
+5	0101	-5	1101	1011	1010	
+6	0110	-6	1110	1010	1001	
+7	0111	-7	1111	1001	1000	
		-8		1000		

Determine Magnitude of Negative Numbers

Magnitude of Negative Numbers

- Sign and Magnitude system ...
 - Most significant bit is the sign
 - Remainder is the magnitude
- 1's Complement system ...
 - Take the 1's complement of \overline{N} $N = (2^n - 1) - \overline{N}$
- 2's Complement system ...
 - Take the 2's complement of N* $N = 2^n N^*$

- The addition is carried out just as if all the numbers were positive ...
 - Any carry from the sign position is ignored
 - This will always yield the correct result except when an overflow occurs
- Overflow ... has occurred if ...
 - The correct representation of the sum (including sign) requires more than n bits (for word length of n bits)

- A general rule for detecting overflow when adding two n-bit signed binary numbers (1's or 2's complement) to get an n-bit sum is ...
- An overflow occurs if ...
 - Adding two positive numbers gives a negative answer ... or ...
 - If adding two negative numbers gives a positive answer

1. Addition of two positive numbers, sum $< 2^{n-1}$

$$\begin{array}{r}
 +3 & 0011 \\
 +4 & 0100 \\
 +7 & 0111
 \end{array}$$
 (correct answer)

2. Addition of two positive numbers, sum $\geq 2^{n-1}$

$$\begin{array}{ccc} +5 & & 0101 \\ \hline +6 & & 0110 \\ \hline & 1011 & & \longleftarrow \text{wrong answer because of overflow (+11 requires} \\ & & 5 \text{ bits including sign)} \end{array}$$

3. Addition of positive and negative numbers (negative number has greater magnitude)

$$\begin{array}{ccc}
 +5 & 0101 \\
 -6 & 1010 \\
 \hline
 -1 & 1111 & (correct answer)
 \end{array}$$

4. Same as case 3 except positive number has greater magnitude

5. Addition of two negative numbers, $|\text{sum}| \le 2^{n-1}$

6. Addition of two negative numbers, $|\text{sum}| > 2^{n-1}$

- Addition of 1's complement numbers is similar to 2's complement except ...
 - Instead of discarding the last carry ...
 - It is added to the n-bit sum in the position furthest to the right
 - This is referred to as an end-around carry

- RECALL ... A general rule for detecting overflow when adding two nbit signed binary numbers (1's or 2's complement) to get an n-bit sum is ...
- An overflow occurs if ...
 - Adding two positive numbers gives a negative answer ... or ...
 - If adding two negative numbers gives a positive answer

- Same as 2's complement!
- **1.** Addition of two positive numbers, sum $< 2^{n-1}$

2. Addition of two positive numbers, sum $\geq 2^{n-1}$

3. Addition of positive and negative numbers (negative number with greater magnitude)

4. Same as case 3 except positive number has greater magnitude

5. Addition of two negative numbers, $|\operatorname{sum}| < 2^{n-1}$

$$\begin{array}{c|c}
-3 & 1100 \\
\underline{-4} & \underline{1011} \\
\hline
 & (1) & 0111 \\
\hline
 & \underline{)} & (end-around carry) \\
\hline
 & 1000 & (correct answer, no overflow)
\end{array}$$

6. Addition of two negative numbers, $|\text{sum}| \ge 2^{n-1}$

$$\begin{array}{c|c}
-5 & 1010 \\
\underline{-6} & 1001 \\
\hline
 & (1) & 0011 \\
\hline
 & \rightarrow 1 & (end-around carry) \\
\hline
 & 0100 & (wrong answer because of overflow)
\end{array}$$

Examples of Addition of 1's Complement and 2's Complement

Numbers

1's Complement ... n = 8

1. Add -11 and -20 in 1's complement.

$$+11 = 00001011$$
 $+20 = 00010100$

taking the bit-by-bit complement,

-11 is represented by 11110100 and -20 by 11101011

$$\begin{array}{ccc}
11110100 & (-11) \\
 & & 11101011 & +(-20) \\
\hline
(1) 11011111 & (end-around carry) \\
\hline
 & 111000000 = -31
\end{array}$$

The addition produced a carry out of the furthest left bit position, but there is no overflow because the answer can be correctly represented by eight bits (including sign)

2's Complement ... n = 8

2. Add -8 and +19 in 2's complement

$$+ 8 = 00001000$$

complementing all bits to the left of the first 1, -8, is represented by 11111000

The addition produced a carry out of the furthest left bit position, but there is no overflow because the answer can be correctly represented by eight bits (including sign)

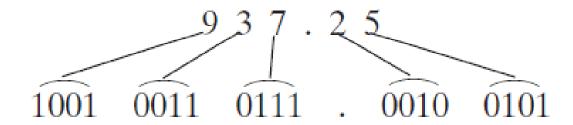
Binary Codes

Binary Code

- Most computers work internally with binary numbers ... however ...
 - Input-output equipment generally use decimal numbers
- Most logic circuits only accept two-valued signals ... therefore ...
 - Decimal numbers must be coded in terms of binary signals
 - In the simplest form of binary code, each decimal digit is replaced by its binary equivalent. For example, 937.25 is represented by:

Binary Code

- In the simplest form of binary code ...
 - Each decimal digit is replaced by its binary equivalent
- Example ... 937.25 is represented by ...



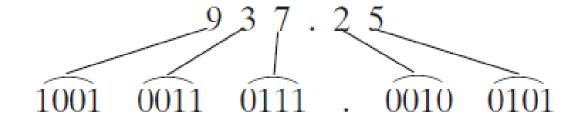
 The result is quite different than that obtained by converting the number as a whole into binary

Binary-Coded Decimal (BCD)

Binary-Coded Decimal - (BCD)

 The following representation is referred to as binary-coded-decimal (BCD) ... or ...

- 8-4-2-1 BCD



- There are only ten decimal digits ... therefore ...
 - 1010 through 1111 are not valid BCD codes

Other Binary Codes

Binary Codes

• Binary-coded-decimal (BCD) ... or ... 8-4-2-1 BCD

•	6-3-1-1 Code	Decimal Digit	8-4-2-1 Code (BCD)	6-3-1-1 Code	Excess-3 Code	2-out-of-5 Code	Gray Code
		0	0000	0000	0011	00011	0000
•	Excess-3 Code	1	0001	0001	0100	00101	0001
		2	0010	0011	0101	00110	0011
		3	0011	0100	0110	01001	0010
•	2-out-of-5 Code	4	0100	0101	0111	01010	0110
		5	0101	0111	1000	01100	1110
		6	0110	1000	1001	10001	1010
•	Gray Code	7	0111	1001	1010	10010	1011
		8	1000	1011	1011	10100	1001
		9	1001	1100	1100	11000	1000

- The 8-4-2-1 (BCD) code and the 6-3-1-1 code are weighted codes
- 4-bit weighted codes have weights that are ...

$$- w_3 \dots w_2 \dots w_1 \dots and \dots w_0$$

- The code $a_3a_2a_1a_0$ represents a decimal number N ...
 - Where $N = w_3 a_3 + w_2 a_2 + w_1 a_1 + w_0 a_0$

Decimal Digit	8-4-2-1 Code (BCD)	6-3-1-1 Code	Excess-3 Code	2-out-of-5 Code	Gray Code
0	0000	0000	0011	00011	0000
1	0001	0001	0100	00101	0001
2	0010	0011	0101	00110	0011
3	0011	0100	0110	01001	0010
4	0100	0101	0111	01010	0110
5	0101	0111	1000	01100	1110
6	0110	1000	1001	10001	1010
7	0111	1001	1010	10010	1011
8	1000	1011	1011	10100	1001
9	1001	1100	1100	11000	1000

• Example ... the weights for the 6-3-1-1 code are ...

$$w_3 = 6$$
, $w_2 = 3$, $w_1 = 1$, and $w_0 = 1$

The binary code 1011 thus represents the decimal digit ...

$$N = 6*1 + 3*0 + 1*1 + 1*1 = 8_{10}$$

Excess-3 Code

Excess-3 Code

- Excess-3 code is
 - Obtained from the 8-4-2-1 code by ...
 - Adding 3 (0011) to each of the codes

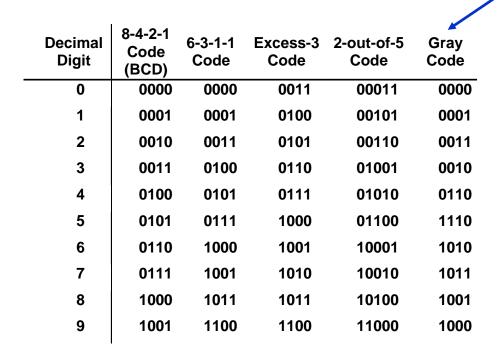
	Decimal Digit	8-4-2-1 Code (BCD)	6-3-1-1 Code	Excess-3 Code	2-out-of-5 Code	Gray Code
_	0	0000	0000	0011	00011	0000
	1	0001	0001	0100	00101	0001
	2	0010	0011	0101	00110	0011
	3	0011	0100	0110	01001	0010
	4	0100	0101	0111	01010	0110
	5	0101	0111	1000	01100	1110
	6	0110	1000	1001	10001	1010
	7	0111	1001	1010	10010	1011
	8	1000	1011	1011	10100	1001
	9	1001	1100	1100	11000	1000
		1				

- 2-out-of-5 code ...
 - 2 out of the 5 bits are 1 for every valid code combination
- This code has useful error-checking properties ...
 - If any one of the bits in a code combination is changed due to a malfunction of the logic circuitry ...
 - The number of 1 bits is no longer exactly two

	8-4-2-1									
Decimal Digit	Code (BCD)	6-3-1-1 Code	Excess-3 Code	2-out-of-5 Code	Gray Code					
0	0000	0000	0011	00011	0000					
1	0001	0001	0100	00101	0001					
2	0010	0011	0101	00110	0011					
3	0011	0100	0110	01001	0010					
4	0100	0101	0111	01010	0110					
5	0101	0111	1000	01100	1110					
6	0110	1000	1001	10001	1010					
7	0111	1001	1010	10010	1011					
8	1000	1011	1011	10100	1001					
9	1001	1100	1100	11000	1000					

- 2-out-of-5 code is not a weighted code
 - The decimal value of a coded digit cannot ...
 - Be computed by a simple formula when a non-weighted code is used

- Gray code ...
 - Codes for successive decimal digits differ in exactly one bit
 - A Gray code is often used when translating an analog quantity, such as a shaft position, into digital form
 - A small change in the analog quantity will change only one bit in the code ... which ...
 - Gives more reliable operation than if two or more bits changed at a time
- Gray Code is not a weighted code



- Gray code is not a weighted code
 - The decimal value of a coded digit cannot ...
 - Be computed by a simple formula when a non-weighted code is used

(American Standard Code for Information Interchange)

- ASCII Code (American Standard Code for Information Interchange)
 - Many applications of computers require the processing of data which contains ...
 - Numbers, letters, and other symbols such as punctuation marks
 - In order to transmit such alphanumeric data to or from a computer or store it internally in a computer ...
 - Each symbol must be represented by a binary code

- A 7-bit code ... therefore ...
 - 2⁷ (128) different code combinations are available ...
 - They can represent ...
 - Letters
 - Numbers
 - And other symbols

ASCII Code						ASCII Code									ASCII Code								
Character	A_6	A ₅	A_4	A_3	A ₂	A ₁	A_0	Character	A_6	A_5	A_4	A_3	A ₂	A ₁	A_0	Character	A_6	A_5	A_4	A ₃	A ₂	A ₁	A ₀
space	0	1	0	0	0	0	0	@	1	0	0	0	0	0	0	,	1	1	0	0	0	0	0
. i	0	1	0	0	0	0	1	Α	1	0	0	0	0	0	1	a	1	1	0	0	0	0	1
u	0	1	0	0	0	1	0	В	1	0	0	0	0	1	0	b	1	1	0	0	0	1	0
#	0	1	0	0	0	1	1	C	1	0	0	0	0	1	1	C	1	1	0	0	0	1	1
\$	0	1	0	0	1	0	0	D	1	0	0	0	1	0	0	d	1	1	0	0	1	0	0
%	0	1	0	0	1	0	1	E	1	0	0	0	1	0	1	e	1	1	0	0	1	0	1
&	0	1	0	0	1	1	0	F	1	0	0	0	1	1	0	f	1	1	0	0	1	1	0
,	0	1	0	0	1	1	1	G	1	0	0	0	1	1	1	g	1	1	0	0	1	1	1
(0	1	0	1	0	0	0	Н	1	0	0	1	0	0	0	ĥ	1	1	0	1	0	0	0
)	0	1	0	1	0	0	1	1	1	0	0	1	0	0	1	i	1	1	0	1	0	0	1
*	0	1	0	1	0	1	0	J	1	0	0	1	0	1	0	j	1	1	0	1	0	1	0
+	0	1	0	1	0	1	1	K	1	0	0	1	0	1	1	k	1	1	0	1	0	1	1
,	0	1	0	1	1	0	0	L	1	0	0	1	1	0	0		1	1	0	1	1	0	0
_	0	1	0	1	1	0	1	M	1	0	0	1	1	0	1	m	1	1	0	1	1	0	1
	0	1	0	1	1	1	0	N	1	0	0	1	1	1	0	n	1	1	0	1	1	1	0
/	0	1	0	1	1	1	1	0	1	0	0	1	1	1	1	0	1	1	0	1	1	1	1
0	0	1	1	0	0	0	0	Р	1	0	1	0	0	0	0	р	1	1	1	0	0	0	0
1	0	1	1	0	0	0	1	Q	1	0	1	0	0	0	1	q	1	1	1	0	0	0	1
2	0	1	1	0	0	1	0	R	1	0	1	0	0	1	0	r	1	1	1	0	0	1	0
3	0	1	1	0	0	1	1	S	1	0	1	0	0	1	1	S	1	1	1	0	0	1	1
4	0	1	1	0	1	0	0	Т	1	0	1	0	1	0	0	t	1	1	1	0	1	0	0
5	0	1	1	0	1	0	1	U	1	0	1	0	1	0	1	u	1	1	1	0	1	0	1
6	0	1	1	0	1	1	0	V	1	0	1	0	1	1	0	٧	1	1	1	0	1	1	0
7	0	1	1	0	1	1	1	W	1	0	1	0	1	1	1	W	1	1	1	0	1	1	1
8	0	1	1	1	0	0	0	X	1	0	1	1	0	0	0	X	1	1	1	1	0	0	0
9	0	1	1	1	0	0	1	Υ	1	0	1	1	0	0	1	y	1	1	1	1	0	0	1
:	0	1	1	1	0	1	0	Z	1	0	1	1	0	1	0	Z	1	1	1	1	0	1	0
;	0	1	1	1	0	1	1	[1	0	1	1	0	1	1	{	1	1	1	1	0	1	1
<	0	1	1	1	1	0	0	\	1	0	1	1	1	0	0		1	1	1	1	1	0	0
=	0	1	1	1	1	0	1]	1	0	1	1	1	0	1	}	1	1	1	1	1	0	1
>	0	1	1	1	1	1	0	٨	1	0	1	1	1	1	0	~	1	1	1	1	1	1	0
?	0	1	1	1	1	1	1	_	1	0	1	1	1	1	1	delete	1	1	1	1	1	1	1

Lab

LAB OVERVIEW

- Structure ...
 - There are four experiments in this course
 - Circuits analyzed/designed in each experiment are simulated using the software package ...
 - LogicWorks 4 ... OR ...
 - LogicWorks 5

LAB OVERVIEW

- Circuits can be analyzed/designed at ...
 - Home ... OR ... at UMass Labs
 - In the computer laboratory (BL 420) ... OR ...
 - Engineering Laboratory (EB 321)
 - Either LogicWorks 4 or 5 is available

Access to Labs

- To gain access to BL-420 and EB-321, which has LogicWorks4 and 5 installed
 - You will need to have an access cards
 - Access cards are given out on the South Campus at Access Services
 - Arrangements are done through the Continuing Education Office
 - Then I will need your UMS# (will be on your Access Card)
 - I will then send your name and UMS# to the Room POC

- A report is required for each experiment
 - Detailed requirements will be provided
- You are also required to wire a given sequential circuit

- Labs to be performed are ...
 - Experiment 1 ... Analysis of Digital Circuits
 - Experiment 2 ... Two-level Circuits
 - Experiment 3 ... Binary Code Converter
 - Experiment 4 ... Design with Decoders and Multiplexers

- Lab Policies ...
 - 1. All experiments in this course should be done independently
 - 2. Reports should be submitted on the due date
 - Points will be deducted for late lab reports
 - 3. Circuits that are not designed according to requirements will not be accepted
 - 4. Additional report and design requirements are described in the laboratory notes

- Mixture of breadboarding and Simulation software ... there are benefits to both
- Benefits of hands-on labs/breadboarding ...
 - Use of ...
 - Components
 - Test equipment
 - Knowledge of Test equipment is a foundation for hardware troubleshooting
 - ** Learn troubleshooting techniques
 - ** Will greatly enhance the class material
 - Solving Lab Problems will enforce the course material

- Basic lab knowledge/techniques
 - Use of a breadboard
 - Learn the identification systems for components
 - Resistors
 - Capacitors
 - Integrated circuits
 - Application of data sheets

- Benefits of LogicWorks Simulation software ...
 - An interactive circuit design tool intended for teaching and learning digital logic
 - Gives you the power, speed and flexibility to create and test an unlimited number of circuit elements on-screen
 - You can study advanced concepts much more quickly and clearly using on-screen simulation than you can by spending time wiring up expensive and damage-prone parts in a lab
- Problems encountered during lab performance ...
 - Knowledge gained from troubleshooting

• Lab grade ...

Lab Report Content

100 points

- Cover page, folder, disk
- Conforms to requirements
- Technical adequacy

Adjustment to grade (deductions ... up to 30 points)

Neatness and legibility (-10 points)

• Templates (-20 points)

Late submitted reports (up to -30 points)

Additional Requirements - Laboratory Reports

- 1. Lab Reports
 - Report form for each lab will be available on the course web pages
 - Electronic report submission is an alternative to hard copies ...
 - PDF format only
 - Sent via email NLT than 11:59 PM on the due date
 - Send your schematics to me via email
- 2. Write your report using the given report template
- 3. A cover page (second page of report template) is required

Additional Requirements - Laboratory Reports

- 4. Include a schematic diagram for each of your designs in the report (see report template)
 - Use the given template for the schematic diagram to draw your diagram
 - Report will not be accepted if the schematic template is not used or if the template is changed in either contents or location, or in both
 - Twenty (out of 100) points will be deducted after the schematic diagram is redrawn using the template and without any change of the template
 - Remember to fill in your name

Additional Requirements - Laboratory Reports

- 5. Email your schematic diagram(s)
- 6. Include a grade sheet for each report

Lab Materials

• Lab requirements and booklets will be provided in the coming weeks

Questions?

Next Week ...

Next Week Topics

- Boolean Algebra ...
 - Chapter 2 ... Boolean Algebra
 - Pages 27 55

Home Work

Homework

- 1. Send an email with your email address or addresses (for class distribution list)
- 2. Send me your UMS# (will be on your Access Card) so I can get access to BL-420 and EB-321 (computer labs), if you currently do not have access
- 3. Read ...
 - Chapter 1 ... pages 1 23
 - Chapter 2 ... pages 27 55

Homework

4. Solve the following Chapter 1 problems ...

```
• 1.1 ... (a)
```

- 1.2 ... (b)
- 1.5 ... (a), (b), (c), and (d)
- 1.6 ... (a)
- 1.7 ... (a) and (d)
- 1.8
- 1.9
- 1.16 ... (b)

References

1. None