1. Let $K \in \mathcal{K}_n$. Prove that
\[ S(K)D(K) \geq V_n(K) \frac{n\omega_n}{\omega_{n-1}}. \]
Recall that $D(K)$ is the diameter of $K$.
*Hint:* Think about cylinders.

2. Let $\Delta$ denote a unit edge equilateral triangle. Compute the mixed area $A(\Delta, -\Delta)$.

3. Let $Q$ denote a regular hexagon in $\mathbb{R}^2$ with unit edge lengths, and let $S$ denote a square with unit edge lengths. Suppose that both $Q$ and $S$ have some edges parallel to the $x$-axis. Compute the mixed area $A(Q, S)$.

4. Let $K, L \in \mathcal{K}_2$, and suppose that $W_u(K) \leq W_u(L)$ for every direction $u$. Prove that, if $M \in \mathcal{K}_2$ is centrally symmetric, then
\[ A(K, M) \leq A(L, M). \]
*Hint:* First, consider the case for which $M$ is a polygon, and recall from Chapter 14 that every centrally symmetric polygon is a zonotope.

5. Find examples of $K, L, M \in \mathcal{K}_2$ such that $W_u(K) \leq W_u(L)$ for every direction $u$, while $A(K, M) > A(L, M)$.
*Hint:* Evidently $M$ can’t be centrally symmetric!

6. This is a problem about averaging, and does not (necessarily) use mixed areas. Let $K \in \mathcal{K}_2$.
   (a) Prove that, if $u$ and $v$ are unit vectors in $\mathbb{R}^2$ such that $u \perp v$, then
   \[ A(K) \leq W_u(K)W_v(K). \]
   (b) Prove that, if $u$ and $v$ are unit vectors in $\mathbb{R}^2$ such that $u \perp v$, then
   \[ \sqrt{A(K)} \leq \frac{W_u(K) + W_v(K)}{2}. \]
   (c) Now use an averaging argument on the result in part (b) to prove that
   \[ \pi \sqrt{A(K)} \leq \text{Perimeter}(K). \]
Incidently, the inequality in part (c) is weak, in the sense that equality never holds unless $K$ is a point. We will prove a better version of this inequality in class next week.