92.490/651 Supplemental Exercises **due in class on 5/3/2010**

1. Let $K \in \mathscr{K}_n$. Prove that

$$S(K)D(K) \geq V_n(K)\frac{n\omega_n}{\omega_{n-1}}$$

Recall that $D(K)$ is the diameter of K.
Hint: Think about cylinders.

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2. Let Δ denote a unit edge equilateral triangle. Compute the mixed area $A(\Delta, -\Delta)$.

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3. Let Q denote a regular hexagon in $\mathbb{R}^2$ with unit edge lengths, and let S denote a square with unit edge lengths. Suppose that both Q and S have some edges parallel to the x-axis. Compute the mixed area $A(Q, S)$.

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4. Let $K, L \in \mathscr{K}_2$, and suppose that $W_u(K) \leq W_u(L)$ for every direction u. Prove that, if $M \in \mathscr{K}_2$ is centrally symmetric, then

$$A(K, M) \leq A(L, M).$$

Hint: First, consider the case for which M is a polygon, and recall from Chapter 14 that every centrally symmetric polygon is a zonotope.

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5. Find examples of $K, L, M \in \mathscr{K}_2$ such that $W_u(K) \leq W_u(L)$ for every direction u, while

$$A(K, M) > A(L, M).$$

Hint: Evidently M can't be centrally symmetric!

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6. This is a problem about averaging, and does not (necessarily) use mixed areas. Let $K \in \mathscr{K}_2$.

(a) Prove that, if u and v are unit vectors in $\mathbb{R}^2$ such that $u \perp v$, then

$$A(K) \leq W_u(K)W_v(K).$$

(b) Prove that, if u and v are unit vectors in $\mathbb{R}^2$ such that $u \perp v$, then

$$\sqrt{A(K)} \leq \frac{W_u(K) + W_v(K)}{2}.$$

(c) Now use an averaging argument on the result in part (b) to prove that

$$\pi\sqrt{A(K)} \leq Perimeter(K).$$

Incidently, the inequality in part (c) is weak, in the sense that equality never holds unless K is a point. We will prove a better version of this inequality in class next week.